Reflection of Production and Energy Inter-Sectoral Relations in the Model of Analysis of Territorial Multisector Objects (ATMO)

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Abstract. The aim of the work is creation of a block of intersectoral production balance as part of the Analysis of Territorial Multisector Objects model. For one branch of industry it is achieved by uniting activity of manufacturing and trading enterprises meeting regional needs, including import and export. The concept of technological coefficient is corrected and the notion of logistic coefficient is introduced. The main results are as follows: the influence of government purchases and consumption in the sphere of small business is defined; methods of using R&D funds to change resource and material consumption in production and trade are presented. Each variable in the model has three dimensions, compiled by parent and child coordinates. The variables are split into manufacturing and trading components with indices. Macroeconomic technological and investment restrictions of complete demand satisfaction for economic entities in the region are developed. The service consumption variable is an indicator of full coverage of regional needs in industrial products. It is positive when other needs of regional industry, trade and households, as well as in other regions and abroad, are satisfied. Framework conditions affecting regional economy through fuel and energy complex by decreasing energy and electrical intensity in industry and trade and by the growth of their exports are developed with reference to payment for its consumption in households. The importance of work is defined by the model implementation by simple software of object-oriented languages, using OLAP (data hypercube) to store variables, and by practical and market orientation of this software.

Keywords: Leontief’s model, production, intersectoral, balance, industrial, trade, region, resource intensity, material intensity, fuel and energy complex (FEC).

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Prezentarea relaţiilor operaţionale inter-industrie şi energie în modelul de analiză a obiectelor multispectrale teritoriale (ATMO)

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Rezumat. Scopul lucrării: construirea unui bloc de echilibru de producție intersectorial ca parte a modelului AOTM - Analiza obiectelor teritoriale multisectoriale. Acest lucru a fost realizat prin ajustarea coeficientului tehnologic și introducerea unui coeficient logistic care să reflecte oferta de produse către întreprinderile comerciale. Principalul rezultat ale lucrării constă în următoarele - metode de utilizare a fondurilor de cercetare și dezvoltare pentru a modifica consumul de resurse și materiale în producție și comerț. Variabilele au trei dimensiuni, compilate de coordonatele părinte și copil: „an-lună”, „regiune-regiune”, „industrie-corporație”. De asemenea, conțin componente de fabricație și tranzacționare, cu indici legați de industrie. Există constrângerii tehnologice și de investiții cu privire la nevoile subiecților regiunii, luând în considerare ratele de creștere industrială și economică. Variabila consumului necesar este definită ca indicator al acoperirii cerințelor produsului. Este pozitiv, când sunt satisfăcute nevoile industriei regionale, comerțului și gospodăriilor, precum și ale altor regiuni și din străinătate, indicând o creștere a bunăstării populației. Au fost dezvoltate condițiile care determină activitățile economiei regionale create de complexul de combustibil și energie. Se iau în considerare tendințele de diminuare a energiei electrice și a intensității energetice în industrie și comerț și a creșterii exporturilor de energie - luând în considerare compensarea acestora pentru consumul casnic. Semnificația lucrării este determinată de simpla implementare software a modelului în limbaje orientate obiect (clase, moștenire), folosind OLAP (hypercube de date) pentru a stoca variabile, precum și orientarea practică și de piată a acestui software.

Cuvinte-cheie: model Leontief, producție, intersectorial, echilibru, industrial, comerț, regiune, intensitatea resurselor, consumul de materiale, complexul de combustibil și energie (CCE).

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Отражение производственных и энергетических межотраслевых связей в модели анализа территориальных мультисекторных объектов (ATMO)

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Аннотация. Цель работы: построение блока межотраслевого продукционного баланса как части модели ATMO – Анализ Территориальных Мультисекторных Объектов. Это достигнуто объединением для отрасли деятельности производственных и торговых предприятий, удовлетворяющих региональных потребностей в своей продукции с учетом ввоза и вывоза. Также для этого скорректированы понятия технологического коэффициента и введен логистический коэффициент, отражающий поставки продукции торговым предприятиям. Основными результатами работы являются следующие. Отражено влияние госзакупок со стороны государства и потребления в сфере малого бизнеса. Приведены методики задействования средств НИР на изменение в ресурсо- и материалоемкости в производстве и торговле. Каждая переменная модели имеет три измерения, составленных родительской и дочерней координатами: «год-месяц», «регион-район», «отрасль-корпорация». Переменные разделяются на производственную и торговую компоненты, с индексами, привязанными к исходному отраслевому. Выработаны макроэкономические технологические и инвестиционные ограничения полного удовлетворения спроса экономических субъектов региона с учетом промышленного и экономического роста. Индикатором полного покрытия потребностей региона в продукции отрасли служит переменная сервисного обеспечения (в сфере услуг). Она положительна, когда другие нужды региональных промышленности, торговли и домохозяйств, а также в других регионах и зарубежом, удовлетворены, свидетельствуя о росте благосостояния населения региона. Разработаны рамочные условия, влияющие на региональную экономику со стороны ТЭК через уменьшение энергетической и электроемкости в промышленности и торговле, и роста их экспорта – с учетом оплаты за его счет потребления в домохозяйствах. Значимость работы определяется простой программной реализации модели объектно-ориентированными языками (с их классами и наследованием), с использованием OLAP (гиперкуб данных) для хранения переменных, а также практической и рыночной ориентированностью этого ПО.

Ключевые слова: леонтьевская модель, продукционный, межотраслевой, баланс, промышленный, торговый, регион, ресурсоемкость, материалоемкость, топливно-энергетический комплекс (ТЭК).

Introduction

The urgent problem of economics is the lack of practices of applying economic and mathematical tools allowing for the specifics of market economy in industry, including a high-tech one.

To fill this gap, a model for analyzing territorial multisectoral objects (ATMO model) was developed.

The model includes the following blocks:
• interindustry production balance;
• interindustry financial balance;
• interregional trade balance;
• interregional rental balance.

These blocks describe the movement of commodity and financial flows for a system of enterprises of different industries in several regions - within each region and between them. Thus, functioning of the production and trade regional industrial system is modeled, with the consideration of belonging of the enterprises to specific corporations, and presence in the region of trading enterprises participating together with them in the exchange of goods with other regions and countries.

The most important of the blocks of the developed model is the block of interindustry production balance. This block is based on Leontief’s interindustry balance (IIB) model, which is used for industrial planning.

II. REASEARCH OBJECTIVE

In the block of the interindustry production balance, an attempt was made to correct a number of features of the most detailed of the available IIB models. According to the review given below they include the following features:
• fragmentary accounting of the contribution of imported goods including those imported from abroad, as well as the region in the output of its products both in the model equations and in the technological coefficients;
• denial of counter flows of exported and imported products including export and import from abroad, and mixing them in one indicator with a sign that coincides with the sign of the balance of import-export in the region;
• ignoring such a cluster of the regional economy as trade enterprises importing products for industry and households, as well as specific consumption of products for this import;
• inattention to the influence of the state on
production and trade activities in the form of government procurement for the supply of products;

- rudimentary implementation of mechanisms for changing technological coefficients depending on the amount of investment in scientific and technological developments and related equipment;
- not imparting a non-equilibrium character to the balance equations, where the imbalances contain supplies to the rest of the regional agents excluding manufacturing and trade enterprises, government agencies, households, partners outside the region and export ones which comprise small business of providing services;
- almost complete lack of accounting for the impact of energy on regional economic processes, with the exception of the general dependence of GDP growth on the activities of the fuel and energy complex (FEC) and extrapolation of CO₂ emissions to the efficiency of energy resources use utilizing IIB models.

The developed block of the intersectoral production balance is intended to get rid of the indicated drawbacks for a more adequate modeling of the economic ties of business agents of the regional branches of industry.

III. CHARACTERISTICS OF AVAILABLE IIB MODELS

In [1], where \( x_{ij} \) denotes the supply of products of the \( i \)-th industry to the \( j \)-th one, \( y_i \) is the final demand for goods of this industry, and \( z_j \) is the total output of the industry, functioning of each industry \( j \) is expressed as follows:

\[
z_j = \sum_y x_{ij} + y_i. \tag{1}
\]

The volume of purchases \( q_j \) of industry \( j \) is the sum of intermediate purchases by enterprises, population \( (v_j) \) and imports \( (m_j) \):

\[
q_j = \sum_x x_{ij} + v_j + m_j. \tag{2}
\]

Technological coefficients are introduced as a set of technical conditions:

\[
a_{ij} = x_{ij} / q_j. \tag{3}
\]

These coefficients are redefined on the basis of \( p_{ij} \), which is the share of resources purchased from industry \( j \), as well as on the "regional trade coefficient", \( r_{ij} \), which is the percentage of this purchase made in the region:

\[
a_q = p_q \cdot r_q, a_q = (x_q / q_j) \cdot (r_q - m_q) / x_q. \tag{4}
\]

where \( a_{ij} \) is purchases from industry \( j \) without regard to the location of the industry, and \( m_{ij} \) is the import of products from sector \( i \) to sector \( j \).

Technological factor reflects changes in production technology [2]. They are given by intermediate deliveries for the total output of the industry:

\[
a_j = a_q \cdot x_q^n \cdot \frac{x_j^n}{x_j^n + x_j^m} + a_q \cdot x_j^m \frac{x_j^n}{x_j^n + x_j^m}, \tag{5}
\]

where \( a^0 \) and \( x^0 \) represent the old process, and \( x^0 \) represents the process after technological changes.

In [3] the model of IIB of Canada illustrates the equilibrium of the total supply of products to the general demand:

\[
q + m_p + m_q + b + v + s = A g + e + x_p + x_q, \tag{6}
\]

where \((n \times 1)\) means vectors of commodity flows that are defined as (in monetary units):

- \( q \) is total number of all domestically produced goods and services;
- \( m_p \) is imported goods and services which are used in the country;
- \( m_q \) is imported or reimported goods and services;
- \( b \) is goods and services produced by state-owned enterprises;
- \( v \) is cost of withdrawals from stocks;
- \( s \) is other flows from other organizations (income), sales of vehicles and spare parts, services of educational institutions, etc.;
- \( A g \) is intermediate demand for goods and services associated with the \((m \times 1)\) vector of production \( g \), and technological coefficients in the \((n \times m)\) matrix \( A \);
- \( e \) is domestic final demand (the sum of personal spending, fixed capital accumulation, government spending and restocking);
- \( x_P \) is export of domestic production;
- \( x_q \) is re-export.

A model of the equilibrium of the interregional economy has been developed for the EU [4]. Indices \( r = 1, ..., R \) denote regions, \( i \) or \( j = 1, ..., I \) denote industries, \( k = 1, ..., K \) denote factors of production. The following designations were introduced:

- \( \mathbf{A}' (I \times I) \) is the matrix of intermediate cost coefficients, where \( a_{ij}' \) is the input of goods from sector \( i \) per unit of output of sector \( j \);
- \( \mathbf{B}' (K \times I) \) is the matrix of primary data of
input coefficients \( b_{ij}' \) specifying the demand for goods for industrial purposes \( k \) per unit of output of sector \( j \);
- \( x' (I \times 1) \) is the vector of regional releases with notation \( x'_i \);
- \( D' (I \times 1) \) is the vector of regional demand for products with designation \( D'_r \);
- \( F' (I \times 1) \) is the vector of final demand for regional products;
- \( f' (K \times 1) \) is the vector of regional nutritional factor with designation \( f'_r \);
- \( Y' (K \times 1) \) is the vector of income with the designation \( Y'_r \);
- \( S' (K \times 1) \) is the supply vector of regional factor with designation \( s'_r \);
- \( w' \) is the price vector labeled \( w'_r \);
- \( N' \) means transfers of net income from other regions to region \( r \).

The balance of the region’s economy has the following form (taking into account the change in the coefficients \( a'_i \) and \( b_{ij}' \) and the minimum costs and demand for its goods):

\[
D' = A' x' + F'. \tag{7}
\]
\[
f' = B' x'. \tag{8}
\]
\[
Y' = S'^T w' + N'. \tag{9}
\]
\[
S' = f'. \tag{10}
\]

In [5], an interindustry model for Austria is given, and the supply of a part of products to the banking sector is investigated. In the presented work it is interpreted as service consumption. Let \( q \) be the vector of the total output of goods, then

\[
q = (I - A)^{-1} \cdot f, \tag{11}
\]

where \( I \) is the unit matrix, \( A \) is the matrix of technological coefficients; and \( f \) is the vector of final demand for consumer goods. The vector of weighted multiplier release for export has the following form:

\[
mq = (I - A)^{-1} \cdot fs_{xi}. \tag{12}
\]

where \( fs_{xi} \) is the vector of export shares of goods \( i \), whose elements are defined as:

\[
fs_{xi} = f_{xi} / \sum_{i=1}^{n} f_{xi}. \tag{13}
\]

The weighted vector of the multiplier of added export value \( mva_e \) equals:

\[
mva_e = VA \cdot (I - A)^{-1} \cdot fs_{xi}. \tag{14}
\]

where \( VA \) is the coefficients matrix of added value (per unit of production).

The vector of the employment multiplier for exports \( me_e \) with the diagonal matrix \( \hat{E} \) of employment rates \( e \), is:

\[
me_e = \hat{E} \cdot (I - A)^{-1} \cdot fs_{xi}. \tag{15}
\]

The individual elements in the multiplier vectors are grouped into product groups, and the sum over all the elements of the multiplier vector provides overall export multipliers for output, value added, or employment. Multiplicative analysis of the impact of the banking services sector as an imbalance on the nature of value added in sectors is carried out for these goods.

In order to determine the level of economic development activities on an integrated and coordinated basis by region and sector, a multi-regional model "input-output" (MROI) is developed [6]. The model is based on the assumption that for each region \( r \) and sector \( i \), intermediate and final demand \( i \) in \( r \) is satisfied in fixed proportions of products of sector \( i \) from different regions of the system (including region \( r \)), without distinguishing between final and intermediate consumption, among different sectors of use. The basic equations of the MROI model are as follows:

\[
X = (I - T \cdot A)^{-1} \cdot [T \cdot (C + F + DS + E) - P_T - \text{Imp}], \tag{16}
\]

where \( X \) is the vector (340 × 1) of output (17 sectors in 20 areas); \( A \) is the diagonal square matrix with 20 blocks (17 × 17) of technological coefficients: \( a (i, j, r) = x (i, j, r) / x (j, r) \); \( T \) is a square matrix (17×20) × (17×20), divided into blocks of diagonal vectors (20×20) of dimension (17 × 17) of interregional exchange coefficients of the shares of demand for products sold to the region \( r \). Further, for 17 sectors in 20 regions: \( C \) is the vector of expenditures on domestic consumption (both personal and state one); \( E \) is the export vector; \( F \) is the investment vector; \( DS \) is the vector of stock changes; \( \text{Imp} \) means import; \( PT \) means product transfer vector.

The so-called rectangular accounting system based on the principles of national accounting of the UN and Eurostat is usually used as data for the IIB model [7]. The main components of this system are Supply and Use Tables for region \( r \), which are analogous to the input and output tables, respectively. The Production Table for
regional industries, \((v_{ie} \in V')\) shows the supply of goods from industries \(i\). The Supply Table reflects goods/services from main and auxiliary activities. It contains the line for importing goods from abroad \((m')\). The upper part of the Use Table contains the supply of goods to industries \(i\), \((u_{ie} \in U')\), and the costs of goods for final demand of type \(f\), \((e_{ij} \in E')\), including the column for exports abroad, \((x')\). The usage of goods/services is grouped into goods in the same way as in the Supply Table. The lower part of the Use Table describes the use of primary resources \(v\) by industry \(i\), that is, the creation of added value \((Y_{ij} \in Y')\). It includes compensation of production factors (wages, social benefits, depreciation, etc.), indirect taxes and subsidies, and operating surplus.

There are two links between the Supply and Use Tables. The first link is the output equation, where the output of industry \(i\), read line by line from the regional Supply Table for region \(r\), \((q')\), is equal to the total use of goods plus the value added of industry \(i\) from the columns in the Use Table.

\[
g^r = V^r i = [1' U^r + i' Y^r]' \quad (17)
\]

where \(i\) is a unit vector. The second link is the equation of balance of goods, where the supply of goods \(c\) from the columns of the supply matrix \((q')\) is equal to the total demand for goods \(c\) from the lines of the usage matrix. It includes industries, population, imports from other regions, \((r^r)\) inter-regional exports to other regions \((t^r)\), etc.

\[
g' = i' V^r + m^r + r^r = [U^r i + E^r i + x^r + t^r]' \quad (18)
\]

In practice, there is a mechanism for how Intersectoral Tables are processed in the IIB model [8]. Typically, Input and Output Tables are used to obtain the so-called “impact matrices”. The main derivation equations for such a matrix is the expression:

\[
[U - D(I - \mu - \beta B)]^{-1} D,
\]

where \(D\) is the matrix of coefficients obtained from the Output Table for each good in the Output Table, where each of the 300 cells is divided by the volume of the product output; \(B\) is the matrix of technological coefficients obtained from the Table of Input - for each industry in the matrix of input, each of 727 cells is divided by the total output; \(I\) is a unit matrix; \(\mu\) is a diagonal matrix of coefficients obtained from imports as a ratio of use - for each commodity, the total volume of imports is divided by the total use of the commodities such as \((i)\) intermediate ones spent on resources output; and \((ii)\) final demand, which is the ratio of imports and consumption of goods; \(\beta\) is a diagonal matrix of coefficients obtained from stock withdrawals as a utilization rate - for each commodity the total stock withdrawals are divided by the total use of the commodities such as \(i\), which is intermediate output resources, \(ii\), which is final consumer demand, i.e. the ratio of stock withdrawals to total product use.

Using the input and output matrices \(V\) and \(U\) in the IIB model should give it market flexibility [9]. The \(u_{ji}\) element is the volume of product \(j\) used in the output of products in industry \(i\). Streams \(u_{ji}\) include their own and imported products of the \(j\)-th industry; \(v_{ij}\) denotes domestic output of product \(j\) by industry \(i\). Final demand \(y\) is private and public consumption, investment and export. It is covered by its own and imported goods of industry \(j\). The output \(p\) of each product is a commodity balance:

\[
p_j = \sum_i v_{ij} + m_j = \sum_i u_{ji} + y_j.
\]

There is a similar balance for industry \(i\) with the vector \(w_i\) of its added value:

\[
g_i = \sum_j v_{ij} = \sum_j u_{ji} + w_i.
\]

Technological coefficient \(q_{ji} = u_{ji} / g_i\) in the equation (1) gives the following:

\[
p = \sum_i q_{ji} s_i + y_j.
\]

In matrix form it looks like following:

\[
p = Q g + y.\]

where \(Q\) is the matrix of coefficients. The share of the branch \(i\) in supply of goods \(j\) looks like \(s_{ij} = v_{ij} / p_j\) or \(v_{ij} = s_{ij} p_j\), then it follows from (21):

\[
g_i = \sum_i s_{ij} p_j
\]

or in the matrix form it is:

\[
g = Sp
\]

After a series of transformations, we obtain:
This equation estimates the impact on the output of industry products when the final demand for products changes, regardless of geography.

The IIB model is applicable to a specific industry, such as tourism [10]. The products of sectors are their production and purchases from other regions and abroad. Demand, on the other hand, has intermediate and final components. Final demand includes consumer and investment demand, government purchases, supplies to regions and abroad. For sector $i$ we have the following equation:

$$X_i + M_i^{TR} + M_i^D = \sum_{j=1}^{N} X_{ij} + C_i + Z_i + G_i + E_i^{TR} + E_i^D,$$  \hspace{1cm} (27)

where $X_i$ is output for the production of industry $i$; $M_i^{TR}$ is purchases in other regions; $M_i^D$ is import; $X_{ij}$ means intermediate sales in industry $j$; $C_i$ is demand; $Z_i$ is infrastructure investment; $G_i$ is government procurement; $E_i^{TR}$ is sale of trip vouchers; and $E_i^D$ is export. We derive the following from (27):

$$X_i = \sum_{j=1}^{N} X_{ij} + C_i + Z_i + G_i + E_i^{TR} - M_i^{TR} - M_i^D,$$  \hspace{1cm} (28)

$$NE_i^{TR} = E_i^{TR} - M_i^{TR}.$$  \hspace{1cm} (29)

Introducing $X_{ij} = a_{ij} X_j$, where $a_{ij}$ means technological coefficients, (28) can be rewritten as:

$$X_i = \sum_{j=1}^{N} a_{ij} X_{ij} + C_i + Z_i + G_i + E_i^{TR} - M_i^{TR} - M_i^D + NE_i^{TR},$$  \hspace{1cm} (30)

where subscript $0$ represents an exogenous variable. This system looks as follows in matrix form:

$$X = AX + C_0 + Z_0 +$$

$$+ G_0 + E_0^{TR} - M_0^D + NE_0^{TR}.$$  \hspace{1cm} (31)

The solution is:

$$X^* = (I - A)^{-1} (C_0 + Z_0 +$$

$$+ G_0 + E_0^{TR} - M_0^D + NE_0^{TR}).$$  \hspace{1cm} (32)

Equation (31) is used to analyze the influence of exogenous factors on the region.

The widespread use of renewable energy sources (RES) requires investment, which is not necessarily related to the energy industry itself, and creation of photovoltaic devices, etc. Since renewable and non-renewable energy sources are unevenly distributed, there is a cross-country distribution of investment and a solution to the problems of energy transmission over long distances, which brings the infrastructure closer to oil and gas. Country data are contained in the AMADEUS database with information on 5 million firms from 27 EU countries. The IIB model NEMESIS works with it, and is used to calculate direct and indirect impact on the economy and employment. [11].

It reflects the performance of sectors of the economy, and the price of electricity is taken from the price equations of the model of the economy and households sectors. The NEMESIS model has the following form for the $s$-th sector of country $c$:

$$PUSH_{c,s} = INVNATRES_{c,s} - INVNATavoid_{c,s} + FUEDEM -$$

$$-FUEDEMAV - EXPRES_{c,s} - IMPRES_{c,s} +$$

$$+ OPMAINRES_{c,s} - OPMAINavoid_{c,s} + AGRIRES,$$  \hspace{1cm} (33)

where

- $PUSH_{c,s}$ is governing input;

- $INVNATRES_{c,s}$ is investment in research;

- $INVNATavoid_{c,s}$ is unnecessary investment due to the deployment of RES;

- $EXPRES_{c,s}$ is RES production import;

- $IMPRES_{c,s}$ is RES production export;

- $OPMAINRES_{c,s}$ is operation and maintenance (O&M) costs due to RES deployment;

- $OPMAINavoid_{c,s}$ is unnecessary O&M costs due to RES deployment;

- $FUEDEM - FUEDEMAV$ is fuel demand minus demand for fuel not needed due to the deployment of RES;

- $AGRIRES$ is additional needs in agriculture and forestry.

The goal of such energy development is to conserve non-renewable resources and solve the problems of climate change due to emissions. Emissions-saving states sell surplus emissions quotas to other states. This is a negative price item. When its output falls, a positive (not negative, as usual) economic effect arises in the form of payments. This is reflected in the model of the IIB “carbon tax” [12]. To apply tax rates to the consumption table on which the IIB model is based, the tax matrix is built on the basis of zero matrix of goods in industries. The coal tax rate $t_0$ is displayed in each column of the coal line. The oil tax rate $t_o$ applies to nearly every...
column of the oil and gas production lines. The natural gas tax rate $t_g$ appears in the oil and gas production line, but only for the natural gas column of the distribution and electricity industries. The matrix has the form:

$$
T = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
t_e & t_e & t_e & t_e & t_e & t_e & \ldots & t_e \\
t_o & t_o & t_e & t_e & t_o & t_o & \ldots & t_o \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0
\end{bmatrix}
$$

(34)

This matrix for fuel users is converted into a vector of indirect value added tax coefficients:

$$
t = (U \otimes T) \hat{g}^{-1} D / i,
$$

(35)

where $U$ is the goods matrix of intermediate costs; $M$ is the matrix of industries receiving intermediate goods as raw materials; $g$ is the amount of output; and $D$ is the matrix od market share:

$$
D = M\hat{q}^{-1},
$$

(36)

where $q$ is output of good, $q = Ui + e$, ($Ui$ is industries output, $e$ is the final demand), $\hat{q}$ is the notation of the diagonal matrix, $i$ is summring for all the goods. Tax vector coefficient $t$ adjusts value added in the IIB model by estimating changes in raw material prices:

$$
p = (I - A)^{-1} (v + nci + t),
$$

(37)

where $v$ is added value, $nci$ is the share of goods import, which are not produced in the region.

In this regard, it becomes necessary to analyze the interregional effects of pollution using IIB models which include pollution indicators. Calculations have been made based on the TELAS methodology [13] using the UK example of full consumption accounting for the main greenhouse gas, CO$_2$, but using analytical methods that can be applied to any greenhouse gas or other pollutant.

Due to globalization of economy, the indirect import of greenhouse gases becomes important, when the release of goods destined for another country leads to large emissions of greenhouse gases in the producing country. The MRIO model which is described above in the review takes this phenomenon into account [14].

Calculations were made for 11 European countries and 37 types of products. Calculations show that, for example, Spain is a net importer of greenhouse gases, which comprise 29% of their domestic production. When using the Kyoto quota, this should be taken into account.

A model for reducing carbon dioxide emissions in the UK by 60% by 2050 is considered in [15]. Estimates of carbon dioxide production by different sources and industries are applied. For example, the emission of carbon dioxide per one monetary unit (for example £) of output in the $i$-th industry is equal to:

$$
u_i = \frac{1}{x_i} \sum_{n=1}^{m} f_n c_n,
$$

(38)

where $f_n$ is the amount of $n$-type fuel, used in $i$-th industry; $c_n$ is the share of carbon dioxide, emitted by $n$-type fuel; $x_i$ is gross output of the $i$-th industry.

Naturally, this indicator is included in the usual IIB model.

The tasks listed above are designed to solve the following set of questions [16]:

1) How much of a country’s industry-specific emissions is for its own use, and how much for consumption by other countries and industries?

2) How does a country’s release of a particular end product cause emissions in other industries and between countries in the global production network?

3) Who makes emissions, for whom and along which route of the global production network in the production of gross exports?

4) How much emission is produced to create one unit of GDP at each stage of production and along different routes of the global production network?

Based on the combination of Leontief’s interregional IIB model and the MRIO model already mentioned in the review, the authors partially answered these questions by analyzing CO$_2$ emissions in the global production and trade network between 41 economies in 35 sectors from 1995 to 2009 based on the World Input Output Database (WIOD), which showed the impact of cross-country split of output on environment.

In the global economy, both energy consumption and energy conservation are important. They are done by traditional technologies increasing the "cosine phi" parameter and technologies to reduce energy consumption by devices. Efficient energy storage
infrastructure is necessary. In [17], these problems of the Austrian energy system were solved by the regional energy model of the IIB. It defines the energy requirements of 57 industries for 23 energy sources. Technological coefficients $A$ and energy intensity $e$ are introduced, which are calculated as:

$$ e_j^t = \frac{FE_j^t}{X_j}, $$(39)

where $e_j^t$ is intensity of energy carrier $i$ in sector $j$ [kWh/€]; $FE_j^t$ is application of energy carrier $i$ from sector $j$ [kWh/y]; and $X_j$ is sector $j$ output [€/y]. The matrix of energy carrier consumption rates for the unit of production $(RE)$ is applied. The columns show the energy consumption in the supply chain. The sum of the columns is equal to the sum of the costs of one energy carrier for the output of a unit of the industry output to cover final demand.

$$ RE = \hat{e} \cdot (I - A)^{-1}. $$ (40)

Energy demand is expressed as a function of final demand in the economy:

$$ E = \hat{e} \cdot (I - A)^{-1} \cdot Y. $$ (41)

Multiplying the diagonal demand matrix gives the vector of the sum of energy consumption:

$$ E_{p-g} = \hat{e} \cdot (I - A)^{-1} \cdot \hat{Y}. $$ (42)

Application of the model showed that Austria is a net importer of energy.

In [18], it is proposed to combine the Solow growth model with the IIB model to analyze the impact of economic growth in energy consumption. Control feedback arises because energy itself is a condition for economic growth. The structure of energy consumption in the IIB matrix is modeled. Growth factors are investigated for the analysis of model parameters in terms of the rate of consumption of renewable and non-renewable energy, while the consumption has the form:

$$ Z_i = P_i X_i, E_i = P_m X_i. $$ (43)

where $Z$ and $E$ are vectors $(m \times 1)$ of industry energy consumption, and $P_i$ and $P_m$ are matrices $(m \times m)$ of energy consumption coefficients with elements $z_{it} = Z_{it} = X_{it}$ and $e_{it} = E_{it} = X_{it}$. Let us assume that $Y_t$ is sector production $i$ in period $t$. Its amount equals to the amount of demand in balance, that is:

$$ Y_t = \sum_{i=1}^{m} Y_{it} = \sum_{i=1}^{m} (C_i + D_i + G_i + NE_{it}), $$ (44)

where $C_i$ is consumption, $D_i$ is investment, $G_i$ is public expanses, and $NE_{it}$ is export. Energy consumption like final demand is as follows:

$$ TE_t = P_t \cdot (I - A)^{-1} \cdot Y_t, $$ (45)

where $TE_t$ is the vector of demand for renewable and non-renewable energy resources during period $t$, and $P_t$ is coefficient matrix of energy consumption. Energy consumption during period $t + 1$ is equal to:

$$ Z_{t+1} = P_{t+1} \cdot (I - A)^{-1} \cdot Y_{t+1}, \quad E_{t+1} = P_{t+1} \cdot (I - A)^{-1} \cdot Y_{t+1}, $$ (46)

where $Y_{t+1}$ is set through $Y_{t+1} = \sum_{i=1}^{m} Y_{it}$ and growth $g_{it}$, defined by power industry, is as follows:

$$ g_{it} = \frac{1}{\beta_i + \kappa_i + \varepsilon_i} g_{Ai} + \frac{\kappa_i}{\beta_i + \kappa_i + \varepsilon_i} (z_{it} - \hat{s}^Z) - \frac{e_i}{\beta_i + \kappa_i + \varepsilon_i} s^F - \frac{\kappa_i + e_i}{\beta_i + \kappa_i + e_i} g_{Li}. $$ (47)

where $\beta_i$, $\kappa_i$ and $\varepsilon_i$ are fractions of labour, renewable and non-renewable power, $g_{Ai}$ is productivity growth, $g_{Li}$ is the growth of those employed, $z_{it}$ and $s^Z$ are the speed of regeneration of renewable resources and a fraction of already available ones, $s^F$ is the fraction of non-renewable resources in production.

The FEC management tool is investment in construction of power plants serving several regions and joint analysis of their efficiency and environmental friendliness [19]. Here the vector of the volume of emissions of industries in each region $p$ (i.e. pollutants / mln. RS$) is determined and has the following form:

$$ p_L^t = \frac{TP_L^t}{X_i^L}, $$ (48)

where $TP_L^t$ is the vector of emissions released per year, and $X_i^L$ is the vector of output for every $i$ industry in region $L$. Secondly, the energy
intensity vector \( (e) \) is introduced, it defines the energy consumption for production output of 1 mln. R$ for sector \( i \) in region \( L \) (MWh):

\[
e_i^L = \frac{CTE_i^L}{X_i^L},
\]

where \( CTE_i^L \) is the vector \((n \times 1)\) of energy consumption in region \( L \) per year by industry \( i \).

Then we can find output \( X \), direct and indirect demand impacts on the construction of power plants \((\dot{Y})\). These values are converted to emissions in order to find emissions \( P_{\text{construction}} \). Then energy demand \( E_{\text{construction}} \) is estimated by multiplying the diagonal matrix of output \((\dot{X})\) and the vector of energy intensity \((e)\). This vector is a parameter in the energy model based on Leontieff’s interregional IIB model.

For the intercountry level, a similar GINFORS model is proposed in [20]. The model-based forecast points to problems in sustainability of the consumption of natural resources and in solving environmental problems.

The core of the model consists of the IIB model and the Energy Emissions Model (EEM).

In the IIB model, the final demand for product \( i \) at constant prices is:

\[
f_i[t] = c_i[t] \cdot C[t] + b_i[t] \cdot B[t] + d_i[t] \cdot G[t] + X_i[t], \quad i \in \{1, \ldots, 41\}
\]

where \( f_i \) is the final demand; \( c_i, b_i, d_i \) are exogenous variables; \( C \) is private consumption, \( B \) is the amount of investment; \( G \) is the amount of government procurement; and \( X \) is export. Prices for import \( q_i[t] \) in local currency lead to prices for import in the USA $ \( q_i^*[t] \) in exchange rate \( \text{EXRA}[t] \):

\[
q_i[t] = q_i(q_i[t-1], \dot{q}_i[t] \cdot \text{EXRA}[t]).
\]

Import in constant prices \( m_i[t] \) is the function of relative price on the basis of import and output prices \( q_i[t] / p_i[t] \) in local currency and final demand \( f_i[t] \) of production \( i \):

\[
m_i[t] = m_i\left(\frac{q_i[t]}{p_i[t]} \cdot f_i[t]\right).
\]

The gross production vector \( Y \) has the following form:

\[
y[t] = [I - AR[t]]^{-1} \cdot \{fd[t] - m[t]\},
\]

where \( AR \) is the matrix of technological coefficients of non-power industries, for power industries \((2, 7 \text{ и } 25)\) it is defined by energy model (EEM).

This is the model of relationship of economic processes, energy consumption and emission. Final energy consumption \( fe \) of sector \( j \) is defined by output \( y \) and ratio of energy price \( pe \) to price \( p \) for the sector production:

\[
fe_j[t] = fe_j(y_j[t], pe[t]/p_j[t], t).
\]

Final energy carrier demand \( i \) is calculated multiplying carrier’s fraction \( i \) in energy consumption \( cf \) of sector \( j \) and the final energy demand in all \( n \) sectors:

\[
fe_j[t] = \sum_{i=1}^{n} cf_i[j] \cdot fe_j[t].
\]

The amount of fossil fuel supply and fraction of CO\(_2\) in this amount define emissions.

III. THE STRUCTURE OF THE PRODUCTION BALANCE UNIT

In the developed ATMO model for the \( r \)-th region with \( n \) industries interregional production balance of the region in matrix form is as following: \( \{t \text{ is the number of the month}\} \):

\[
\begin{align*}
&Y^{(pr)} + A^{(pr)} \cdot X^{(pr)} + G^{(pr)} \cdot Y^{(pr)} = \\
&= Y^{(op)} + Z^{(op)} + U^{(op)},
\end{align*}
\]

or in natural units:

\[
\begin{align*}
&Y' + P' + A^{(pr)} \cdot X' + P' + G^{(pr)} \cdot Y' = \\
&= Y^{(op)} + Z' + U',
\end{align*}
\]

where:

\( \cdot P' \) is the diagonal matrix of selling prices of manufactured products;

\( Y' \) is the stocks of products in warehouses of the industry enterprises;

\( X' \) is the production output on the industry enterprises of the region;

\( Z' \) is import of production to the region;

\( Y^{(op)} \) is the consumption of industries production in the region;

\( Z' \) is export of production from the region;

\( U' \) is service consumption in the sector of personal services within the framework of small business;

\( A^{(pr)} \) is technological coefficients of industry production fractions, spent on the output of a unit of goods by other industries;
• \( G[v] \) is logistics coefficients of industry production fractions, spent on import of a unit of goods by other industries.

Given coefficients are determined by the following equations:

\[
a^{(0)}_{ij} = (\lambda^{(0)}_{ij} + \lambda^{(n)}_{ij}) \cdot p^{(0)}_j = (\lambda^{(q)p}_{ij} + \lambda^{(n)p}_{ij}) \cdot p^{(0)}_j, \quad (58)
\]

This basic technological coefficient comprises the average amount of deliveries of released production \((\lambda^{(x)}_{ij})\) and imported products \((\lambda^{(x')}_{ij})\) from industry \(i\) to \(j\) for a number of years and average production capacity \((\lambda^{(x')}_j)\) by manufacturing enterprises of industry \(j\) in average prices \(p^{(0)}_j\).

Basic logistic coefficient is the following:

\[
g^{(0)}_{ij} = (\lambda^{(x)}_{ij} + \lambda^{(n)}_{ij}) \cdot p^{(0)}_j = (\lambda^{(q)p}_{ij} + \lambda^{(n)p}_{ij}) \cdot p^{(0)}_j, \quad (59)
\]

It includes the average amount of deliveries of released production \((\lambda^{(x)}_{ij})\) and imported products \((\lambda^{(x')}_{ij})\) from industry \(i\) to \(j\) for a number of years in the average amount of import \((\lambda^{(x')}_{ij})\) by trade enterprises of industry \(j\) in average prices \(p^{(0)}_j\).

Coefficients \(a^{(0)}_{ij}\) and \(g^{(0)}_{ij}\) together with the with indicators of savings (over expenditure) of production costs by industries \(da^{(0)}_{ij}\) and \(dg^{(0)}_{ij}\) form annual coefficients in total by months (\(f\) is the index of the year) using the following formulas:

\[
a^{(0)}_{ij} = a^{(0)}_{ij} + f \cdot a^{(f-1)}_{ij} + \sum_{t=1}^{12} t \cdot da^{(0)}_{ij}, \quad (60)
\]

\[
g^{(0)}_{ij} = g^{(0)}_{ij} + f \cdot g^{(f-1)}_{ij} + \sum_{t=1}^{12} t \cdot dg^{(0)}_{ij}, \quad (61)
\]

where

• \(a^{(0)}_{ij}\) is a technological coefficient at the beginning of the planning period;
• \(a^{(0)}_{ij}\) is logistics coefficients at the beginning of the planning period;
• \(da^{(0)}_{ij}\) are technological coefficients for the previous year \((f-1)\);
• \(dg^{(0)}_{ij}\) are logistics coefficients for the previous year \((f-1)\).

These coefficients are in the expanded form of a block which has a construction of \(n\) equations (here \(I\) is a unity matrix):

\[
\sum_{t=1}^{t} Y^{(r)} \cdot p^{(0)}_j + (I - t \cdot B^{(r)}_{ij}) \cdot t X^{(r)} \cdot p^{(0)}_j +
\]

\[
\sum_{t=1}^{t} (1 - C^{(r)}_{ij}) + A^{(r)} \cdot \sum_{t=1}^{t} (1 - C^{(r)}_{ij}) \cdot t X^{(r)} \cdot p^{(0)}_j +
\]

\[
(1 - t \cdot C^{(r)}_{ij}) \cdot t X^{(r)} \cdot p^{(0)}_j = \frac{t}{A^{(r)}} G^{(r)}_{ij}
\]

where

• \(B^{(r)}_{ij}\) are fractions of supplies of own products within the framework of government procurement;
• \(C^{(r)}_{ij}\) are fractions of supplies of imported products within the framework of government procurement;
• \(Y^{(r)}_{ij} = t^{1}W^{(r)}_{ij} \cdot t^{1}Y^{(r)}_{ij}\) is the demand as the product of the number of consumers by the specific consumption per month \((t-1)\) due to demand forecast lags.

Equation for industry \(i\), region \(r\) and month \(t\) has the following component-wise form \(p^{(r)}\):

\[
t \cdot v^{(r)} - t \cdot p^{(r)} + (1 - b^{(r)}_{ij}) \cdot s^{(r)}_{ij}, \quad (63)
\]

There is a number of parameters: independent indicators, variables for the previous period which is month \(b\) \((t-1)\):

\[
t \cdot b^{(r)}_{ij} = (1 + t \cdot d^{(r)}_{ij}) \cdot \sum_{t=1}^{t} \cdot d^{(r)}_{ij}, \quad (64)
\]

There is the fraction of governmental procurement in the output as the increment \(t \cdot d^{(r)}_{ij}\) of the ratio of household consumption \(t \cdot Y^{(r)}_{ij}\) to output \(t^{1}X^{(r)}\), providing its increment:

\[
t \cdot c^{(r)}_{ij} = (1 + t \cdot d^{(r)}_{ij}) \cdot \sum_{t=1}^{t} \cdot d^{(r)}_{ij}, \quad (65)
\]

There is the fraction of government procurement in import as an increment \(t \cdot d^{(r)}_{ij}\) of service consumption ratio \(t \cdot u^{(r)}_{ij}\) to import \(t^{1}X^{(r)}\), providing its increment:

\[
t \cdot d^{(r)}_{ij} = (1 + t \cdot d^{(r)}_{ij}) \cdot \sum_{t=1}^{t} \cdot d^{(r)}_{ij}, \quad (66)
\]
There is a fraction of technological savings as fraction of distribution of unit costs for R&D \( p_i^{(a)} \) in proportion to fractions of costs for goods of other industries \( \alpha_i \) and the effectiveness of costs for their reduction \( b_i^{(a)} \) and inversely proportional to selling prices \( p_i \):

\[
t_i^{(a)} r = \frac{t-1 \cdot (u_i r)^{t-1} - p_i r}{t^{-1} \cdot (p_i^r + t^{-1} p_i^{(c)} r) - t^{-1} x_i r}.
\]

There are also basic unit costs for R&D as a fraction of R&D volume \( t^{-1} p_i^{(a)} \), proportional to the fraction of selling prices \( p_i^{(r)} \) to the sum of sale and purchase prices \( t^{-1} p_i^{(r)} + t^{-1} p_i^{(c)} \) per good unit with the size of \( t^{-1} x_i \) (all these indicators are taken for the previous period):

\[
 t \cdot x_i r = (1 + d x_i r)^t \cdot t^{-1} x_i r
\]

There is current production output as its growth \( 'dx' \) to it for the previous month (t-1):

\[
 t \cdot x_i (v) r = (1 + d x_i (v) r)^t \cdot t^{-1} x_i (v) r
\]

There is current import of products as its growth \( 'dx^{(v)}' \) to it for the previous month (t-1):

\[
 t \cdot x_i (v) r = (1 + d x_i (v) r)^t \cdot t^{-1} x_i (v) r
\]

There are stocks as fraction \( 'd(x)' \) service consumption for the previous month \( t^{-1} u_i \):

\[
 t \cdot d(x) i r = \frac{t \cdot v_i r}{t^{-1} u_i r}
\]

The fraction of stocks in service consumption \( 'u' \) defines stock \( 'v' \) on the basis of the previous experience.

The fraction of technological over expenditure \( l \) saving \( da_i^l \) is the fraction of production reflecting the value of positive or negative change in production supply if industry \( i \) to industry \( j \) due to R&D.

In supplies of industry \( i \) notation \( da_i^l \) means growth, decrease or stabilization of its products supply to industry \( j \) depending of goods quality of industry \( i \).

In demand of industry \( i \) notation \( da_i^l > 0 \) means a quality good supplied in large quantity from the industry \( j \) (growth of material consumption), \( da_i^l < 0 \) means a cheaper but lower quality good produced using materials saving. When \( da_i^l = 0 \), the quality does not change. This indicator is the sum of the actual unit costs for R&D \( p_i^{(a)} r \) of industry \( i \).

The change of \( 'da_i^l \) for output \( 'x' \) of production of other industries (partially compensating each other) is provided by the proceeds:

\[
 j \cdot x_i r - p_i r \cdot \sum_{j=1}^{n} \frac{t \cdot p_j r}{t \cdot v_i (d) r} \leq t \cdot v_i r - p_i r
\]

hence

\[
 \sum_{j=1}^{n} t \cdot da_{ji} r \leq t \cdot v_i (d) r
\]

where \( v_i (d) r \) is per capita GRP growth rate as the reproduction rate.

The fraction of these costs for the reduction \( l \) growth of material consumption of goods of \( j \)-th industry as a part of a unit of industry product \( i \) is obtained by multiplying R&D \( p_i^{(a)} r \) by the goods fraction for of industry \( j \) for output \( (\alpha_i) \) of industry \( i \). This value of expenditures in changing material consumption is multiplied by \( b_i^{(a)} \) which means the change the gross specific consumption of raw materials \( l \) components of industry \( i \) for goods of industry \( j \) to the unit of costs. Parameter \( (p_i)^{-1} \) means the fraction of decrease \( l \) growth of cost for the material consumption \( p_i^{(r)} r \). \( p_i^{(a)} r \cdot b_i^{(a)} r \) in the price of product \( x_i r \). R&D costs change the material intensity of the industry:

\[
 t \cdot da_{ji} r = t \cdot p_i^{(a)} r \cdot \alpha_i \cdot b_i^{(a)} r \cdot p_i^{-1}
\]

Let us use (74) in the condition of limiting changes in technological coefficients by the growth rate of per capita GRP from above and express the unit costs for R&D:

\[
 \sum_{j=1}^{n} t \cdot da_{ji} r \leq t \cdot v_i r
\]

hence

\[
 p_i^{(a)} r \leq t \cdot v_i r \cdot p_i^{-1}
\]

Let us equate both parts (76), adding \( 'dp_i^{(a)} r \) to its right part, its positiveness sets the growth of costs that reduce material consumption:

\[
 t \cdot p_i r = (1 + t \cdot dp_i r) \cdot t \cdot v_i r
\]

\[
 \cdot p_i^{-1} \cdot \sum_{j=1}^{n} (t \cdot da_{ji} r \cdot b_i^{(a)} r) r^{-1}
\]

Capital unit costs \( t \cdot p_i^{(a)} r \) provide the addition \( 'dp_i^{(a)} r \) for R&D, hence we have the following condition:
\[
\frac{d p^a}{i} \leq \frac{p^a}{i}.
\] (78)

For industries where at least one \(d a_j'\) or \(d g_j'\) is different from zero (in interindustry financial balance of the model ATMO) the following equation is true:

\[
\begin{align*}
\frac{d v_i'} &= \frac{p_i'} - \sum_{j=1}^{n} \frac{d a_{ij}'}{t x_j'} + \\
&+ \frac{p_i'} r \sum_{j=1}^{n} \frac{d g_{ij}'}{t x_j'}.
\end{align*}
\] (79)

Investment growth of regional production \(d p^{(\text{reg})}_i\) is the cost of production of industry \(i\) for growth of the output in other industries and that is reflected in coefficients \(d a_j'\) and \(d g_j'\).

The block reflects the flows of products among the subjects of the region and both production and trade enterprises, respectively, producing and importing products.

Most of the components of the model are divided accordingly to this feature, however, a number of variables have industry-wide significance, and are disaggregated into components with the following indices corresponding to production and trade:

- \(i - n\) is the index defined by the production of the region;
- \(i + n\) is the index showing trade enterprises activity.

For example, production stocks are marked \(v_{i'}\) for industry and \(v_{i+n}'\) for trade and the following equation is true:

\[
\frac{t v_i'} = \frac{t v_{i-n}'} + \frac{t v_{i+n}'}.
\] (80)

Also, the distribution by sources of the arrival of goods in the service sector, expressed by the indicator of service consumption, is important for industry \(u_{i,n}'\) and for trade \(u_{i+n}'\) and provides the following final industry parameter:

\[
\frac{t u_i'} = \frac{t u_{i-n}'} + \frac{t u_{i+n}'}.
\] (81)

It does not have any sense for a number of variables, for example, for the output indicator \(x_{i}'\), however, we can formally write:

\[
\frac{t x_i'} = \frac{t x_{i-n}'} + \frac{t x_{i+n}'} = 0.
\] (82)

Additional variables form:

- additional conditions specifying planning according to the model;

- additional tasks within the practice of working with the model.

### IV. GENERAL RESTRICTIONS ON THE VALUES OF PRODUCTION BALANCE VARIABLES

The production balance as an element of the ATMO model performs the function of finding the optimal values of the output variables \(x_i'\) and \(x_{i+n}'\) as well as technological and logistics coefficients \(u_i'\) and \(g_i'\) together with the parameters of governmental procurement of produced and imported production \(b_i'^{(\text{opt})}\) and \(c_i'^{(\text{opt})}\).

Let us introduce the indicator \(f d_{i,k}^{(x)}\) which is the fraction of \(k\)-th corporation in production output of the industry \(i\) in region \(r\). It makes it possible to decompose the sectoral characteristics of production / trade into corporate ones.

Optimal indicators are defined by maximizing indicators under restrictions for industries, regions and new enterprises with their fractions \(f d_{i,k}^{(x)}\) of corporation \(k = 1, ..., s\) in a region \(b\) during the year \(f\); increasing the fraction:

\[
\begin{align*}
& f c_{i,k}^{(x)} r \left( i, b \right) = f c_{i,k}^{(x)} r \left( i, b \right) + \sum_{b=1}^{f} \frac{f d_{i,k}^{(x)}}{x_i'^b} = \\
&= f c_{i,k}^{(x)} r \left( i, b \right) + \sum_{b=1}^{f} \frac{f d_{i,k}^{(x)}}{x_i'^b}.
\end{align*}
\] (83)

The fraction of new enterprises in the industry is as follows:

\[
\frac{f c_{i,k}^{(x)}}{i} = \sum_{k=1}^{s} \frac{f c_{i,k}^{(x)}}{i}.
\] (84)

Indicator by month based on annual one is as following:

\[
\frac{f d_{i,k}^{(x)}}{t} i = \frac{d c_{i,k}^{(x)}}{t} i = \frac{d c_{i,k}^{(x)}}{t} / 12,
\]

\(t = 12; f + 1, 12; f + 2, ..., f = 200(1), 200(2), ...\) (85)

Let us single out \(s = 400\) of largest corporations of the Russian Federation, covering all sectors of its economy and capable of operating in every region of any area. Let there be no more than \(c = 50\) regions of any area. At the same time, a corporation working in the agricultural sector, which is the main one for any region, receives index \(k = 1\). If there is no corporation in the region \(k\) or there is no district \(b\) in the region, then corresponding indicator \(f d_{i,k}^{(x)} i r = 0\) is equal to 0.
In production balance, restrictions are aimed at service consumption as an indicator of growth in the service sector (with fraction of new enterprises \( \bar{d}_i^{(v)} \)):

\[
t^r_{\bar{x}_i} \cdot p^r_{\bar{x}_i} = (1 - t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i}) \cdot p^r_{\bar{x}_i} = \sum_{j=1}^{n} a^{(v)}_{ji} \cdot x^{(v)}_{ji} \cdot p^r_{ji} +
\]

\[+(1 - t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i}) \cdot x^{(v)}_{\bar{x}_i} \cdot p^r_{\bar{x}_i} - t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i} \cdot p^r_{\bar{x}_i} = t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i} (x^{(v)}_{\bar{x}_i} - x^{(v)}_{\bar{x}_i}) \] (86)

The indicator reflects the goal of consumer development which is the growth of service consumption while meeting the needs of industries and population:

\[
t^r_{\bar{x}_i} (x^{(v)}_{\bar{x}_i}, x^{(v)}_{\bar{x}_i}) \rightarrow \text{max},
\] (87)

(that is done on the basis of the output and import).

1. Output is limited by average output growth \( \bar{d}_i^{(v)} \) and growth of the enterprises in industry \( \bar{d}_i^{(v)} \), which increase the output of production:

\[
(1 + t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i}) \cdot x^{(\text{mid})}_{\bar{x}_i} \leq x^{(v)}_{\bar{x}_i},
\] (88)

where \( t^{(\text{mid})}_{\bar{x}_i} \) is the average output of production, and its following value is calculated by this way:

\[
t^{(x)}_{\bar{x}_i} = t^{(-1)}_{\bar{x}_i} + (t^{(-1)}_{p_i} - 1),
\]

\[
\cdot \sum_{j=1}^{n} d^{(v)}_{ji} \cdot t^{(-1)}_{p_j} \cdot x^{(v)}_{ji} \cdot x^{(v)}_{ji},
\] (89)

where \( n^{(v)} \) is the number of the industries producing equipment.

2. Imports are limited by the growth / fall rate of average output \( t^{(d^{(v)})}_{\bar{x}_i} \) and growth of the enterprises in industry \( t^{(d^{(v)})}_{\bar{x}_i} \), that stimulate the import into the region:

\[
(1 + t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i}) \cdot x^{(\text{mid})}_{\bar{x}_i} \leq x^{(v)}_{\bar{x}_i},
\]

\[
\cdot \sum_{j=1}^{n} d^{(v)}_{ji} \cdot t^{(-1)}_{p_j} \cdot x^{(v)}_{ji} \cdot x^{(v)}_{ji},
\] (90)

where \( x^{(\text{mid})}_{\bar{x}_i} \) is the average amount of output equal to:

\[
t^{(v)}_{\bar{x}_i} = t^{(-1)}_{\bar{x}_i} + (t^{(-1)}_{p_i} - 1).
\]

5. The investment growth of the industry is provided by the growth of population, its income and growth of enterprises \( \bar{d}_i^{(v)} \):

\[
t^{d_{ji}}_{\bar{x}_i} \leq (1 + t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i}) \cdot t^{(d^{(v)})}_{ji} \cdot t^{(-1)}_{y_j} \cdot t^{(-1)}_{y_j},
\] (93)

where

\[
\begin{align*}
\cdot & \text{t}^{(d^{(v)})}_{ji} \text{ is the population growth rate; } \\
\cdot & \text{t}^{(d^{(v)})}_{ji} \text{ is GDP growth per capita which is considered as GRP for the region; } \\
\cdot & t^{(d^{(v)})}_{ji} \text{ is the average number of customers for the previous month; } \\
\cdot & t^{(d^{(v)})}_{ji} \text{ is the unit demand of population for the previous month. }
\end{align*}
\]

6. The forecast of demand helps the growth of government procurement, taking into account new enterprises \( \bar{d}_i^{(v)} \):

\[
\sum_{j=1}^{n} t^{(-1)}_{d_j} \cdot t^{(x)}_{\bar{x}_i} \cdot t^{(-1)}_{p_j} \cdot x^{(v)}_{ji} \geq
\]

\[
(1 + t^{(v)}_{\bar{x}_i} \cdot x^{(v)}_{\bar{x}_i}) \cdot t^{(d^{(v)})}_{ji} \cdot t^{(-1)}_{w_j} \cdot t^{(-1)}_{y_j} \cdot t^{(-1)}_{y_j},
\] (95)

where

\[
\begin{align*}
\cdot & \text{t}^{(d^{(v)})}_{ji} \text{ is the growth of output of the fraction of governmental procurement; } \\
\cdot & \text{t}^{(d^{(v)})}_{ji} \text{ is the growth of the import section of governmental procurement. }
\end{align*}
\]

V. FRAMEWORK ENERGY PRODUCTION BALANCE OPERATING CONDITIONS

The energy factor, which determines the
The development of the economy of the Russian Federation, in particular, reflected by the ATMO model of the real sector of the economy, consisting, as already indicated earlier, of production and trade, plays an important role in modeling territorial processes.

For this, the ATMO model formulates a number of principles for the connection between the fuel and energy complex and the economy in terms of the nature and direction of commodity flows between them. These macroeconomic requirements are tried to be implemented in practice by both enterprises within the framework of their production policy, and by the state through the macroeconomic instruments available to it.

The model uses the designation \( n \approx 457 \) showing the number of sectors, which include types of economic activities from the standard list of the All Russia Classifier of Economic Activity Types (OKVED) of the RF (1600 items) related to the production of any material product, including the production of agricultural products, electricity and construction. Let us single out the industries (types of economic activities) that are part of the fuel and energy complex - they correspond to numbers (30 sectors):

- 64-71: mining coal, lignite and peat;
- 72-75: mining oil and natural gas;
- 76-78: mining uranium ore;
- 443-457: generation, transmission and distribution of electricity, gas, steam and hot water.

Then let us write out the following relations in the order of numbering 7-11:

7. The principle of reducing the energy intensity of the output is reflected in fraction of technological savings \( da_i \). Its negative value indicates decrease in technological coefficient \( a_i \) of supplies of industry \( i \) by FEC to non-power industries \( j \) where \( 'da_i \) is included:

\[ i \text{ } da_i r < 0, \quad i \in I^{(i)}, \quad j \in I^{(i)}, \tag{96} \]

where

- \( I^{(i)} = \{64-71, \ 72-75, \ 76-78, \ 443-457\} \) and means a number of FEC.

8. The principle of energy sales promotion is also expressed by the share of technological savings \( 'da_i \). Its positiveness indicates increase in supplies of non-power industry \( i \) to a number of FEC industries \( j \), having influence on technological coefficient \( a_i \) where \( 'da_i \) is included:

\[ i \text{ } da_i r > 0, \quad i \in I^{(i)}, \quad j \in I^{(i)}, \tag{97} \]

where

- \( I^{(i)} = \{64-71, \ 72-75, \ 76-78, \ 443-457\} \) is a number of FEC industries.

9. The principle of growth in the FEC exports is implemented through indicator of growth of reserves \( 'dv_i \) for energy industries in two versions - for production and trade. In the first option, the increase in the FEC production for the implementation of the investment policy is exhausted with the sales policy of the FEC enterprises, and in the second option, the reserves are accumulated, compensating for the departure of their own energy resources abroad at the expense of import:

\[ i \text{ } dv_i r \leq 0, \quad i \in I^{(i)}, \tag{98} \]

where

- \( 'dv_i \) is the increase in production inventories;
- \( 'dv_i \) is the increase in stocks in trade.

10. The principle of household consumption at the expense of the FEC is observed using increase in service consumption \( 'dv_i \), provided by trade enterprises, where index \( i+n \) means that import production purchased from FEC export earnings requires services from the service sector, which is where the growth of service consumption comes from:

\[ \sum_{i \in I^{(i)}} i \text{ } dv_i r \leq \sum_{i \in I^{(i)}} i \text{ } dv_i r \leq \sum_{i \in I^{(i)}} i \text{ } dv_i r \], \tag{99} \]

where

- \( i \text{ } dv_i \) is the export of production which is the constant from another block of the model;
- \( i \text{ } dv_i \) are the selling prices of manufactured products.

11. The principle of GRP support at the expense of the FEC is displayed by the service consumption indicator \( 'u_i \) with industrial production with index \( i \) as the basis of real GRP from commodity-producing industries; the income from their activities complements the sale of products of the fuel and energy sector - if they exist in the region, a maximized target function is introduced, which is "pulled" by the FEC, and other industries exceed their average output level for the projected GDP growth:

\[ \sum_{i \in I} i \text{ } du_i r \leq p_i r \to \max, \tag{100} \]
Taking into account the general restrictions above, this system of conditions sets the process of optimizing the strategy of the regional economy.

VI. DISTINCTIVE FEATURES OF THE OFFERED MODEL

The block of IIB has a number of features:

• taking into account only of industries engaged in material production in the model;
• linking the enterprises that make up the industry to service companies importing and selling products from outside the region which are of the same type in relation to those produced by these enterprises;
• taking into account the use in the region of products manufactured not only by the enterprises of each industry, but also imported by trading companies;
• detailed control of the location of the supply of products of an enterprise of a specific industry in its region and import by a trading company to:
  - industrial enterprises of other industries;
  - firms selling goods of other industries;
  - to state institutions: schools, hospitals, military units, etc. (supplies of both own and imported products);
  - households;
  - other regions;
  - for export;
• small business with the functions of service enterprises, minus the volume of products reserved within the balance;
• definition of technological coefficients as the ratio of mutual deliveries not only of region’s own production, but also of imported products to the output of other industries;
• introduction of logistic coefficients as the ratio of mutual deliveries of non-own production and imported products in the region to the volume of imports of goods from other industries by trading companies;
• calculation of technological and logistic coefficients by step-by-step accumulation of their monthly changes in the total annual value;
• modeling of the amounts allocated for scientific and technological development as the cost of changing technological coefficients by means of unit costs for R&D;
• introducing imbalances in the intersectoral equations, for example, other consumption in small business, as targets for industries which is an increase in the consumption of their goods after meeting other obligations;
• application of special parameters for the decomposition of sectoral indicators into corporate, and regional - into district, where the corporation is correlated with the region where a particular enterprise is located;
• imposing constraints on the variables of the balance equation based on the parameters of the macroeconomic situation in the regions;
• clarification of solutions obtained using balance equations, through the framework conditions of influence of FEC on the rest of the region's economy.

Conclusion

These features will enable the user of the model to simulate the following processes in the system of regional industries:

• determination of the optimal amounts of output and import of industrial enterprises in the conditions of competition for resources using the criterion of equilibrium;
• search for the best operating parameters of industrial and commercial enterprises in terms of sustainability of economic activity, mainly the coefficients of change in the unit costs of technology and logistics;
• development of possible strategies of the state, contributing to the activities of sectoral regional manufacturing and trading enterprises, consisting mainly in establishing the most appropriate size of public procurement of manufactured and imported products.

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