Method for Identification of Structural Parameters of Electric Power Consumption of Industrial Enterprises Based on Phase Space Reconstruction

[1]Shevchenko S., [2]Bederak Ya., [3]Kozlovskyi O.
[1]National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine
[2]PrJSC "AZOT", Cherkasy, Ukraine
[3]Central Ukrainian National Technical University, Kropyvnytskyi, Ukraine

Abstract. The main objectives of the study are to develop a new adaptive geometric method for real-time monitoring of industrial electricity consumption schedules to ensure timely detection of structural imbalances and minimization of financial risks for energy-intensive enterprises in competitive electricity markets. To achieve these objectives, the following tasks were accomplished: transition from conventional one-dimensional statistical analysis to multidimensional topology using the method of phase space reconstruction of discrete load time series with consideration of commercial metering intervals; development of an algorithm for spatial limitation of reference phase trajectories through the construction of bounding rectangles; calculation of geometric parameters of the formed daily quasi-cycles, in particular centroid coordinates and semi-perimeters, for accurate identification of the current state of the system; typologization of characteristic behavioral patterns of electricity consumption based on a system of algebraic inequalities. The most significant result is the mathematical confirmation of the regularity of the drift of the centers of mass of daily quasi-cycles within a narrow neighborhood of the first quadrant of the phase plane, which proves the high inertia of the process and its tendency toward autocorrelation. The significance of the obtained results lies in providing dispatch personnel of industrial enterprises with a transparent and fast tool for real-time monitoring of energy consumption without requiring substantial computational resources. This ensures the possibility of immediate managerial decision-making regarding the operational purchase or sale of corresponding volumes of electricity in the intraday market, thereby effectively avoiding financial penalties and optimizing overall production costs under conditions of unstable equipment operation schedules.

***Keywords*:** industrial electricity consumption, phase space reconstruction, intraday market, bounding rectangle, commercial imbalances, real-time monitoring.

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Metodă de identificare a parametrilor structurali ai consumului de energie electrică al întreprinderilor industriale bazată pe reconstrucția spațiului de fază

[1]Șevcenko S., [2]Bederak Ya., [3]Kozlovskyi O.
[1] Universitatea Tehnică Națională „Institutul Politehnic Harkov”, Harkov, Ucraina
[2] PrJSC "AZOT", Cherkasî, Ucraina
[3] Universitatea Tehnică Națională Centrală din Ucraina, Kropâvnîțkîi, Ucraina

Rezumat. Principalele obiective ale studiului sunt dezvoltarea unei noi metode geometrice adaptive pentru monitorizarea în timp real a programelor de consum de energie electrică industrială, pentru a asigura detectarea la timp a dezechilibrelor structurale și minimizarea riscurilor financiare pentru întreprinderile mari consumatoare de energie pe piețele competitive de energie electrică. Pentru a atinge aceste obiective, au fost îndeplinite următoarele sarcini: trecerea de la analiza statistică unidimensională convențională la topologia multidimensională utilizând metoda reconstrucției spațiului de fază a seriilor temporale discrete de sarcină, luând în considerare intervalele de contorizare comercială; dezvoltarea unui algoritm pentru limitarea spațială a traiectoriilor de fază de referință prin construirea de dreptunghiuri de delimitare; calcularea parametrilor geometrici ai cvasiciclurilor zilnice formate, în special coordonatele centroidice și semiperimetrele, pentru identificarea precisă a stării actuale a sistemului; tipologizarea modelelor comportamentale caracteristice consumului de energie electrică pe baza unui sistem de inegalități algebrice. Cel mai semnificativ rezultat este confirmarea matematică a regularității derivei centrelor de masă ale cvasiciclurilor zilnice într-o vecinătate îngustă a primului cadran al planului de fază, ceea ce dovedește inerția ridicată a procesului și tendința sa spre autocorelație. Importanța rezultatelor obținute constă în furnizarea personalului dispecerat al întreprinderilor industriale a unui instrument transparent și rapid pentru monitorizarea în timp real a consumului de energie, fără a necesita resurse de calcul substanțiale. Acest lucru asigură posibilitatea luării imediate a deciziilor manageriale privind achiziționarea sau vânzarea operațională a volumelor corespunzătoare de energie electrică pe piața intraday, evitând astfel eficient penalitățile financiare și optimizând costurile generale de producție în condiții de programe instabile de funcționare a echipamentelor.

***Cuvinte-cheie*:** consum industrial de energie electrică, reconstrucție a spațiului de fază, piață intraday, dreptunghi delimitant, dezechilibre comerciale, monitorizare în timp real.

Методика идентификации структурных параметров электропотребления промышленных предприятий на основе реконструкции фазового пространства

[1]Шевченко С. Ю., [2]Бедерак Я. С., [3]Козловский А. А.
[1]Национальный технический университет «Харьковский политехнический институт», Харьков, Украина
[2]ПрАТ «АЗОТ», Черкасы, Украина
[3]Центральноукраинский национальный технический университет, Кропивницкий, Украина

Аннотация. Основные цели исследования состоят в разработке нового адаптивного геометрического метода оперативного мониторинга графиков промышленного электропотребления для своевременного выявления структурных небалансов и минимизации финансовых рисков энергоемких предприятий на конкурентных рынках электрической энергии. Для достижения поставленных целей были решены следующие задачи: осуществлен переход от обычного одномерного статистического анализа к многомерной топологии методом реконструкции фазового пространства дискретных временных рядов нагрузки с учетом интервалов коммерческого учета; разработан алгоритм пространственного ограничения эталонных фазовых траекторий посредством построения габаритных прямоугольников; вычислены геометрические параметры сформированных суточных квазициклов, в частности, координаты центроидов и полупериметры для точной идентификации текущего состояния системы; выполнена типологизация характерных поведенческих паттернов потребления электроэнергии на основе системы алгебраических неравенств. Наиболее важными результатами является математическое подтверждение закономерности дрейфа центров тяжести суточных квазициклов в узкой окрестности биссектрисы первого квадранта фазовой плоскости, что доказывает высокую инерционность процесса и склонность к автокорреляции. Предложена классификация геометрических аномалий нагрузки, которая включает локальный пробой границы, структурное смещение, волатильное расширение, базовую стабилизацию и глобальный макроразворот. Установлено, что любое пространственное отклонение от эталонного прямоугольника служит надежным сигналом появления коммерческого небаланса, а использование статической геометрической маски радикально снижает вычислительную сложность алгоритма по сравнению с искусственными нейронными сетями. Значение полученных результатов заключается в предоставлении диспетчерскому персоналу промышленных предприятий прозрачного и быстрого инструмента для контроля энергопотребления в реальном времени без привлечения значительных вычислительных мощностей. Это обеспечивает возможность мгновенного принятия управленческих решений по оперативной покупке или продаже соответствующих объемов электроэнергии на внутрисуточном рынке, что позволяет эффективно избегать финансовых санкций и оптимизировать общие производственные затраты в условиях нестабильного графика работы оборудования.

***Ключевые слова*:** промышленное электропотребление, реконструкция фазового пространства, внутрисуточный рынок, габаритный прямоугольник, коммерческие небалансы, оперативный контроль.

I. INTRODUCTION

The structural transformation of European energy markets and the large-scale integration of renewable energy sources have led to a significant increase in price volatility [1]. This trend is further intensified by geopolitical crises and military conflicts, which fundamentally alter the established operating regimes of power systems in Eastern European countries [2]. Under the conditions of the unified European market, including the Day-Ahead Market (DAM) and the Intraday Market (IDM), industrial enterprises bear strict financial responsibility for deviations of actual electricity consumption from the declared schedules. As evidenced by recent studies, particularly using the example of the Romanian market, there is a continuous strengthening of the interdependence between the results of trading on the DAM and the need for оперативной correction of positions on the IDM to minimize imbalance penalties [2, 3].

The high cost of imbalance settlement forces industrial enterprises to seek ways to compensate financial risks. Existing studies often propose solving this problem through hardware-based approaches, such as the use of battery energy storage systems (BESS) for price arbitrage [4], or the application of complex simulation models and optimization algorithms for BESS trading strategies in DAM and IDM markets [5]. However, the implementation of such solutions requires significant capital investments or the involvement of specialized analytical platforms. At the same time, the transition of modern industries toward the concept of active consumers (prosumers) with their own local generation and smart grid elements makes their electricity consumption profile from the external grid extremely dynamic and nonlinear [6].

Under such conditions, the primary task of energy management is not merely the physical saving of electricity, but ensuring real-time dispatch monitoring of compliance with power limits. Traditionally, optimization of the consumption profile is achieved through Demand Response systems, which enable industrial facilities to integrate into smart grid architectures and efficiently manage their own energy resources [6]. The implementation of Demand Response programs, in combination with automated dispatch control algorithms, allows industrial complexes to respond to dynamic market pricing and redistribute their production loads [7].

However, the effectiveness of any Demand Response system directly depends on the speed and accuracy of recognizing the current state of energy consumption. Even with advanced Demand Response algorithms in place, enterprise energy management requires reliable tools for the instantaneous identification of load trends (drift) in real time. If responses are based solely on already realized imbalances, financial losses are unavoidable. For this reason, methods of short-term load forecasting (Short-Term Load Forecasting, STLF) traditionally serve as the foundation that provides Demand Response with the necessary activation time reserve [8].

In order to achieve high accuracy in short-term forecasting of electrical load schedules of industrial consumers, modern studies predominantly employ machine learning and deep learning models. In particular, recurrent neural networks (LSTM, GRU), as well as ensemble algorithms (e.g., XGBoost), demonstrate high mathematical accuracy when analyzing stable sets of historical data [8, 9]. Due to their ability to process multidimensional datasets and identify hidden temporal dependencies, such hybrid architectures make it possible to reduce standard error metrics (MAPE, RMSE) to minimal values [9].

However, despite their high statistical efficiency, these approaches have significant functional limitations in the context of real-time industrial control. The main issue is that complex neural network models operate as algorithmic "black boxes" [10–12]. The complete lack of algorithmic transparency and physical interpretability of results deprives energy management of an understanding of the nature of sudden deviations. Studies show that this lack of transparency constitutes a serious barrier to urgent decision-making under financial risk conditions in the energy market [12]. Moreover, the effectiveness of deep learning depends on the extent to which future operational states correspond to the distributions in the training dataset. In the case of abrupt and unpredictable changes in technological operating modes (for example, emergency shutdown of a production line or the commissioning of an energy-intensive electro-technological installation), these models exhibit significant inertia and error [12]. Restoring their adequacy requires resource-intensive retraining using complex attention mechanisms and optimization algorithms.

Under the conditions of strict time constraints of the IDM, where position adjustments occur continuously, the effectiveness of deep learning faces significant challenges. Although for professional participants in the energy market high computational load may be compensated by trading profits [13], for automated demand response systems at the level of an individual industrial enterprise, such delays and resource intensity represent a substantial drawback.

An alternative approach that allows for an adequate description of the complex and nonlinear nature of industrial electricity consumption is the application of methods from chaos theory, nonlinear dynamics, and fractal geometry. Modern studies demonstrate that electrical load time series are not purely random; they are characterized by the presence of long-term correlations and pronounced fractal properties [14]. Changes in these topological characteristics, in particular the approach of the Hurst exponent to critical boundaries, serve as an indicator that the electricity consumption process is losing quasi-stationarity and acquiring a strongly stochastic nature [15]. Due to such sensitivity to structural changes, methods of fractal interpolation and analysis of strange attractors are increasingly used for modeling electricity consumption profiles [16].

A fundamental mathematical tool for revealing the hidden dynamics of such nonlinear systems is the method of phase space reconstruction, which transforms a scalar time series into a multidimensional geometric structure [17–19]. Such spatial transformation makes it possible to identify the internal topology of a dynamic system and to visualize its behavior, which remains undetectable under standard statistical analysis [17–19].

Despite their strong analytical potential, the practical implementation of nonlinear dynamics algorithms in real-time control systems faces significant technical obstacles. Classical metric tests used to confirm the presence of chaotic dynamics, such as the determination of the correlation dimension of an attractor or the calculation of the Lyapunov spectrum, are extremely computationally intensive tasks [20]. In addition, topological invariants exhibit high sensitivity to hardware noise from measuring instruments, which forces researchers to apply complex and time-consuming nonlinear filtering procedures [17, 21]. The correct computation of these indicators requires the accumulation of large volumes of historical data and substantial processing power [22]. Thus, these mathematical tools prove to be too inertial and cumbersome for the instantaneous identification of local anomalies or load drift "here and now," which is a key requirement for real-time monitoring in the IDM.

The contributions of this work are as follows:

– a spatial analysis approach is proposed, based on the targeted decomposition of a complex phase trajectory into daily quasi-cycles, which is fully consistent with the regulations for bid formation in the European Day-Ahead Market;

– an algorithm for rapid parameterization of the topological properties of these quasi-cycles using the bounding box method is proposed, which significantly reduces computational complexity compared to metric tests and avoids the lack of transparency inherent in machine learning methods;

– a system for visual identification of behavioral patterns of electricity consumption is developed, providing enterprise energy management with an effective tool for instantaneous detection of load drift and timely operational control of positions in the IDM.

The objective of this work is to develop and substantiate a method for real-time monitoring and identification of electricity consumption drift in industrial enterprises based on the spatial geometric parameterization of reconstructed phase trajectories.

II. RESEARCH METHODOLOGY

A. Formalization of the Time Series and Monitoring Task.

The operation of an industrial enterprise in the European DAM requires the preliminary formation of an hourly schedule of expected electricity consumption $P_{DAM}(t)$ for the next trading period. In real time, the actual electricity consumption of the production complex is a non-stationary process [23, 24], which is described by a discrete power time series $P(t_i)$, where t_i – represents the current time with a discretization step determined by commercial metering systems or SCADA.

A critical aspect of this formalization involves the selection of the sampling interval Δt. From an algorithmic perspective, reducing the measurement interval (e.g., to 1 or 5 minutes) leads to excessive detailing of the phase trajectory. In this case, the method captures high-frequency process fluctuations, inrush currents, and hardware noise, resulting in an unwarranted expansion of the base rectangle area B_k^{norm}. Regarding hardware implementation, most modern electronic meters support sampling intervals of 5, 15, or 30 minutes. Although a shorter period improves monitoring responsiveness, an excessively high data acquisition frequency overloads communication channels and database servers of Automated Systems for Commercial Accounting of Power Consumption (ASCAPC/AMR).

Conversely, increasing the interval Δt to 60 minutes results in the mathematical smoothing of extrema. The phase attractor becomes maximally compact; however, the algorithm loses its responsiveness, and structural load shifts are identified with a significant delay. Given these mathematical and hardware factors, the sampling interval must be strictly synchronized with the market's commercial integration window (e.g., 15 or 30 minutes). This approach filters out process noise, prevents infrastructure overload within the AMR system, and ensures a timely response to financially significant deviations. For the visual validation of the proposed method, a sampling interval Δt of 30 minutes was selected in this study.

The main condition for avoiding financial penalties is the minimization of the current imbalance

$$\Delta P(t_i) = P(t_i) - P_{DAM}(t_i) \qquad (1)$$

through timely operational control on the IDM. For rapid identification of load drift, it is proposed to move away from classical one-dimensional statistical analysis in favor of a multidimensional topological approach.

B. Phase Space Reconstruction.

The fundamental method for revealing the hidden dynamics of the studied process is the construction of a phase trajectory. According to Takens' theorem [17], a multidimensional phase space is formed based on the scalar time series of consumed power $P(t_i)$. Considering the need to ensure maximum algorithmic responsiveness for energy management purposes, it is reasonable to limit the analysis to a two-dimensional phase space. In this case, the current state of the system's electricity consumption at time t_i is represented by a point M_i with coordinates (x_i, y_i), where $x_i = P(t_i)$, and $y_i = P(t_i + \tau)$ is the power value with a given time delay τ. In the classical theory of nonlinear dynamics, the selection of the delay parameter is typically performed by calculating the autocorrelation function or identifying the local minimum of mutual information to ensure the topological independence of coordinates. However, in the context of operational dispatch management in the Day-Ahead (DAM) and Intraday (IDM) markets, the primary objective is not the reconstruction of a global mathematical attractor, but rather the strict monitoring of the load gradient between adjacent settlement intervals. Accordingly, in this study, the time delay parameter is set to one minimum sampling step of the measurement system ($\tau = 1$, defining the transition from P_i to P_{i+1}). This choice enables the visualization of power dynamics between two consecutive integration periods. The physical significance of the phase trajectory constructed under such conditions lies in the instantaneous capture of the load's rate of change (ramp rate), which serves as a key indicator of a commercial imbalance within the enterprise.

The sequence of points M_i forms the phase trajectory, visualizing the dynamic properties of the load as a spatial attractor.

C. Decomposition of the Trajectory into Daily Quasi-Cycles.

Since the operation of the European DAM market is strictly regulated by a daily planning cycle (24 hours), it is reasonable to fragment the global phase trajectory of the enterprise's electricity consumption. To this end, the global attractor is divided into separate closed subsets – daily quasi-cycles C_k, where k is the sequential day number. Each quasi-cycle C_k contains a set of points M_i recorded over 24 hours with a given discretization step. This approach allows isolation of normal daily load patterns and comparison of the current operational state exclusively with relevant system behavior in previous periods, completely excluding the influence of long-term seasonal trends on real-time monitoring algorithms.

D. Algorithm for Spatial Parameterization.

To ensure high responsiveness in detecting load drift on the IDM, it is proposed to abandon computationally intensive metric calculations (such as the computation of the Lyapunov exponent). For the base (DAM-scheduled) k-th quasi-cycle C_k^{norm}, a two-dimensional bounding box B_k^{norm} is constructed. The geometric boundaries of this spatial constraint are strictly determined by the extreme values of the coordinates of points $M_i(x_i, y_i)$, belonging to the reference set C_{norm}:

$$X_{\min}^{(k)} = \min\left\{x_i \middle| M_i \in C_k^{norm}\right\}, \quad (2)$$

$$X_{\max}^{(k)} = \max\left\{x_i \middle| M_i \in C_k^{norm}\right\}, \quad (3)$$

$$Y_{\min}^{(k)} = \min\left\{y_i \middle| M_i \in C_k^{norm}\right\}, \quad (4)$$

$$Y_{\min}^{(k)} = \max\left\{y_i \middle| M_i \in C_k^{norm}\right\}. \quad (5)$$

The normal operating region of the enterprise is described by the matrix of geometric boundaries

$$B_k^{norm} = \left[X_{\min}^{(k)}, X_{\max}^{(k)}; Y_{\min}^{(k)}, Y_{\max}^{(k)}\right]. \quad (6)$$

The main spatial indicators, subject to continuous automated calculation, are the coordinates of the centroid $O^{(k)}$

$$X_c^{(k)} = \frac{X_{\min}^{(k)} + X_{\max}^{(k)}}{2}, \quad (7)$$

$$Y_c^{(k)} = \frac{Y_{\min}^{(k)} + Y_{\max}^{(k)}}{2} \quad (8)$$

and the semi-perimeter of the bounding box

$$L^{(k)} = \left(X_{\max}^{(k)} - X_{\min}^{(k)}\right) + \left(Y_{\max}^{(k)} - Y_{\min}^{(k)}\right). \quad (9)$$

In real time, the operational monitoring system overlays the coordinates of new points of actual consumption $M_{current}$ onto the pre-established geometric region B_k^{norm}. If the current phase trajectory breaches the boundaries of the rectangle (e.g., $x_{current} > X_{\max}^{(k)}$) or a sudden structural shift of the centroid $O^{(k)}$ is detected, this serves as a direct mathematical indicator of load drift.

Detection of such a spatial anomaly immediately generates a trigger for energy management regarding the need for urgent adjustment of trading positions on the IDM to avoid imbalance penalties.

III. RESULTS AND DISCUSSION

A. Experimental Validation and Construction of Reference Quasi-Cycles.

To verify the effectiveness of the proposed algorithm, retrospective electricity consumption datasets of an industrial enterprise were used. Figure 1 shows the output discrete time series of active power $P(t_i)$ over a typical 5-day workweek with a discretization step of 30 minutes.

As evident from the presented data, the load profile is characterized by high volatility and the presence of significant local extrema.

Such non-stationarity complicates the task of real-time monitoring using classical linear methods.

Therefore, at the first stage, in accordance with the DAM declared daily schedule (base scenario C_k^{norm}), a reference two-dimensional phase portrait of the load was reconstructed.

Figures 2(a–e) show the obtained phase trajectories for the five working days of the studied week.

Visual analysis of their topology indicates that all daily attractors have a similar structure, and the predominant direction of trajectory traversal is counterclockwise.

This is a defining property of the system, indicating the deterministic nature of transient processes at the enterprise.

B. Spatial Parameterization and Visualization of Drift.

A bounding box B_k^{norm} was automatically constructed around the formed reference quasi-cycle, the limits of which mathematically defined the permissible zones of operational fluctuations.

During the real-time monitoring simulation, by overlaying actual consumption data, the algorithm demonstrated high sensitivity to structural changes.

The results of automated calculation of spatial indicators (centroid coordinates $O^{(k)}$ и and semi-perimeters $L^{(k)}$ for the five studied daily quasi-cycles) are summarized in Table 1.

Analysis of these numerical arrays confirms the high sensitivity of the method to changes in operational states. In particular, the change in the metric parameters of the bounding box on the 5th day mathematically reflects the enterprise’s transition to a reduced operating mode on the eve of the weekend.

It was demonstrated that the occurrence of a commercial imbalance is visually manifested not merely as an amplitude change on a standard linear graph, but as a significant geometric deformation of the phase trajectory.

The system instantaneously recorded moments when the current point $M_{current}$ breached the calculated limits (e.g., $X_{\max}^{(k)}$).

C. Typology of Electricity Consumption Behavioral Patterns.

Analysis of the results of spatial modeling made it possible to identify several characteristic geometric patterns that unambiguously indicate different types of technological deviations in production (Table 2).

It was established that changes in the centroid coordinates $O^{(k)}$ or in the metric parameters of the bounding box directly correlate with the physical nature of deviations:

1. Local Boundary Breach (Spike Anomaly) (No. 1, 2). A sudden, short-term excursion of the trajectory beyond the spatial constraint, followed by a rapid return to the permissible zone B_k^{norm}.

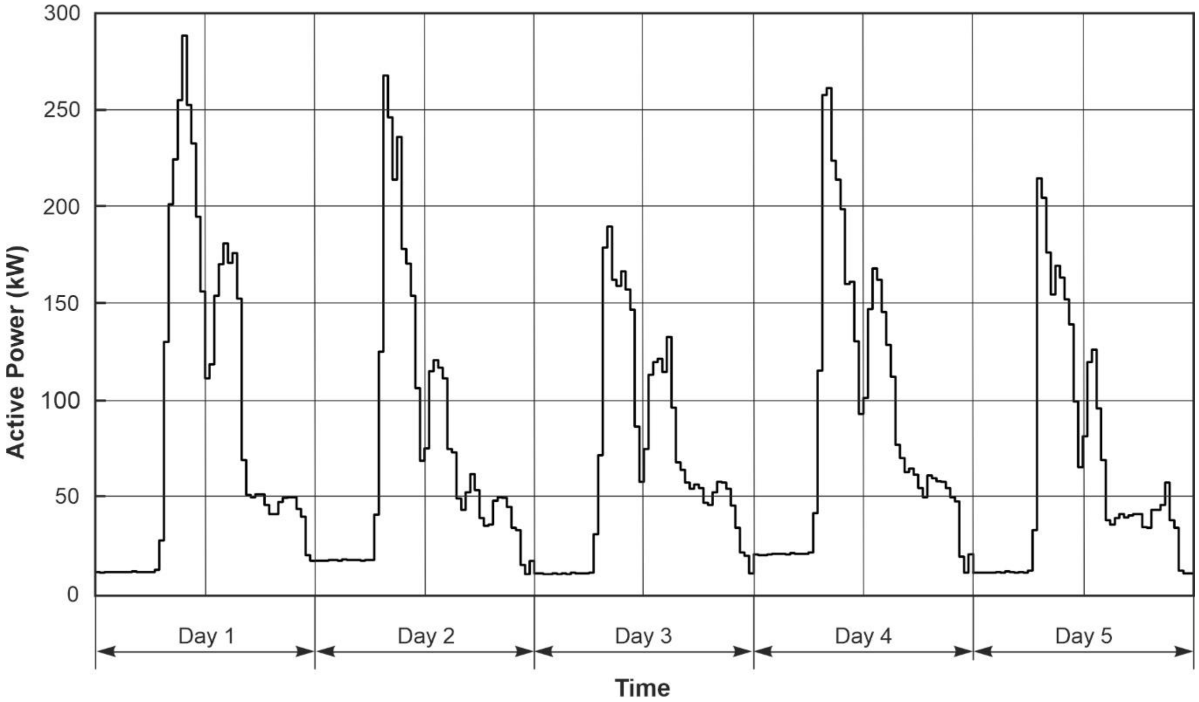
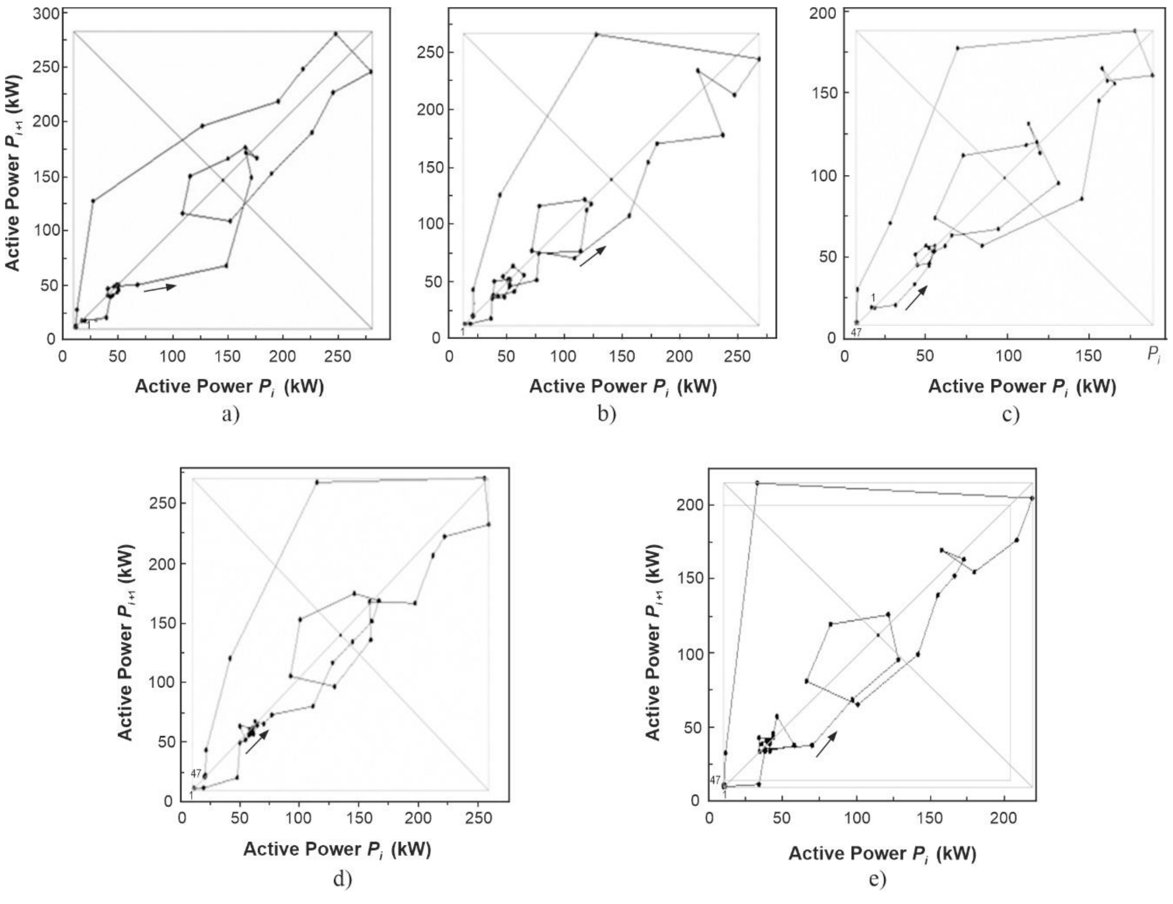


Fig. 1. Daily half-hour active power profiles for a 5-day workweek of a mechanical engineering enterprise.



a) Day 1; b) Day 2; c) Day 3; d) Day 4; e) Day 5

Fig. 2. Phase portraits.

In production terms, this indicates the synchronous startup of high-power equipment (high inrush currents). This pattern usually does not require immediate market intervention on the IDM if the duration of the spike is shorter than the commercial integration window of the metering system.

2. Centroid Structural Shift (Load Drift) (No. 3, 4). A gradual or abrupt displacement of the entire operational trajectory, forming a new local center of attraction outside the base rectangle. This is an unequivocal algorithmic signal of a sustained change in the technological regime (e.g., the startup of an additional production line or overtime operation of a workshop). This pattern generates the highest priority trigger for urgent adjustment of trading bids on the Intraday Market to avoid imbalance penalties.

3. Volatility Expansion (No. 7). An increase in the total area of the bounding box $(L^{(k)} \to \max)$ without a significant shift of its centroid $O^{(k)}$. Geometrically, this indicates the chaotization of electricity consumption, which may result from unstable equipment operation or rhythmic disturbances in the production line.

4. Base Stabilization (Base Plateau / Saturation) (No. 5, 6). Compression of the phase trajectory into a dense local cluster with a sharp reduction in the bounding box area to minimal values. From an engineering perspective, this pattern corresponds to the transition of equipment to a stable nominal operating mode or to a state of technological downtime (idle mode or nighttime dip). If the algorithm detects stabilization during periods when high and unstable load was expected according to the DAM schedule, the system immediately signals the “release” of reserved power volumes. Energy management gains the opportunity to profitably sell this surplus on the IDM.

5. Global Macro-Reversal (No. 8, 9). Formation of elongated U-shaped or dome-shaped phase trajectories that rapidly cross the boundaries of the base bounding box B_k^{norm}. This geometric primitive identifies the large-scale synchronous startup or shutdown of entire production lines (e.g., scheduled morning ramp-up of load or evening system ramp-down). Since such transient processes are accompanied by enormous gradients in power consumption, any temporal shift of this pattern relative to the declared schedule (for example, a 30-minute delay in the start of a work shift) is automatically interpreted by the algorithm as a critical commercial imbalance. Detection of a macro-reversal shift requires immediate purchase or sale of the corresponding volume of electricity to avoid penalties.

A comparative analysis of the proposed geometric method against statistical models (ARIMA) and deep learning algorithms (LSTM, GRU) confirms its superior operational efficiency for real-time monitoring tasks. Unlike stochastic models such as ARIMA, which are mathematically prone to smoothing short-term power spikes by treating them as statistical noise, the topological approach ensures the detection of any trajectory deviation beyond the reference boundaries. While recurrent neural network (RNN) architectures, such as LSTM, provide minimal forecasting error on stable datasets, they possess a fundamental drawback in the context of the IDM market. To mathematically validate a structural load shift, these models require the accumulation of a fresh historical dataset, which generates a critical algorithmic latency. Furthermore, in the event of an abrupt change in the enterprise’s operational mode, machine learning models necessitate resource-intensive retraining involving Graphics Processing Units (GPUs).

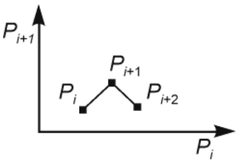
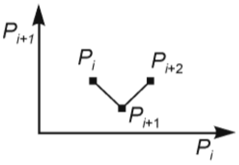
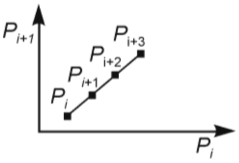
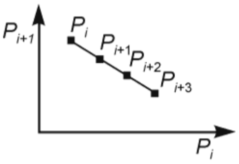
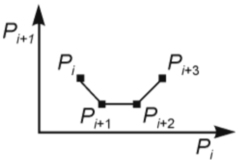
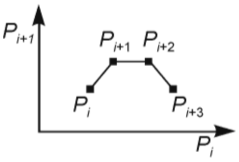
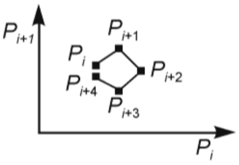
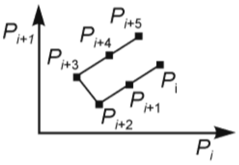
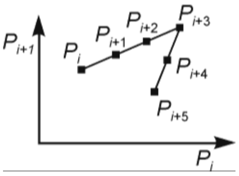
Table 1.

Metric Parameters of Bounding Boxes of Daily Quasi-Cycles

Bounding Box Parameters, kW	Day 1	Day 2	Day 3	Day 4	Day 5
$P_{\min}$	11.5	10.8	10.8	11.5	11.2
$P_{\max}$	288.0	267.5	189.7	261.0	214.6
ΔP	276.5	256.7	178.9	249.5	203.4
$L^{(k)}$	552.9	513.4	357.8	498.9	406.8
$O^{(k)}(X_c, Y_c)$	(149.7; 149.7)	(139.1;139.1)	(100.3; 100.3)	(136.2; 136.2)	(112.9; 112.9)

Table 2

Classification of Graphical Patterns of Phase Trajectories of Electricity Consumption

No.	Pattern Position in Phase Space	Algebraic Description of Phase Space Point Coordinates	Linguistic Description of Time Series Pattern
1		$P_i < P_{i+1} > P_{i+2}$	**Local maximum.** Point of sign change in load increment (trend reversal from increase to decrease).
2		$P_i > P_{i+1} > P_{i+2}$	**Local minimum.** Point of sign change in increment (trend reversal from decrease to increase).
3		$P_i < P_{i+1} < P_{i+2} < P_{i+3}$	**Stable upward trend.** Load increase, activation of technological equipment.
4		$P_i > P_{i+1} > P_{i+2} > P_{i+3}$	**Stable downward trend.** System unloading, disconnection of consumers.
5		$P_i > P_{i+1} = P_{i+2} < P_{i+3}$	**Minimum plateau (saturation).** Load stabilization at a minimal level (idle mode or nighttime dip).
6		$P_i < P_{i+1} = P_{i+2} > P_{i+3}$	**Maximum plateau.** Stabilization of overload at the peak level (operation at nominal power).
7		$P_i \approx P_{i+1} \approx \ldots \approx P_{i+n}$	**Quasi-stationary regime.** Absence of a pronounced trend, random fluctuations around the mean value.
8		$P_i > \ldots > P_{\mathrm{min}} < \ldots < P_n$	**Global reversal (U-shaped).** Smooth transition from decrease to increase (morning load rise).
9		$P_i < \ldots < P_{\mathrm{max}} > \ldots > P_{i+n}$	**Global reversal (dome-shaped).** Smooth transition from increase to decrease (evening load decline).

In contrast, the proposed method of bounding rectangles B_k^{norm} completely mitigates these disadvantages of existing predictive models. The system requires neither data accumulation nor retraining, as it identifies a commercial imbalance immediately upon the arrival of the first anomalous measurement from the metering system. Additionally, as opposed to abstract statistical error metrics (e.g., MAPE), spatial parameterization yields a physically intuitive result: the dispatcher observes the spatial deviation directly in kilowatts. This facilitates the instantaneous calculation of the required volume of compensatory electricity for IDM trading.

Analysis of the geometric evolution of daily quasi-cycles revealed a fundamental regularity: the drift of centroids $O^{(k)}$ occurs within a very narrow neighborhood of the bisector of the first quadrant of the phase plane ($y = x$), as clearly demonstrated in Fig. 3. This mathematically confirms the high level of autocorrelation and inertia of the industrial electricity consumption process. For energy management, this property is decisive. Since the system naturally tends toward a stationary state along the bisector, any sudden displacement of centroid coordinates serves as a reliable indicator of the emergence of a structural imbalance. Such a change in geometry generates a high-priority signal for real-time control of positions on the IDM.

In addition to trend identification, the proposed spatial method is a highly effective tool for strict control of contractual power limits. The algorithm involves overlaying a special "mask" of permissible operating conditions onto the current phase portrait, the boundaries of which are determined by the bids previously submitted to the DAM. In phase space, these constraints (for example, a base declared limit of 200 kW) form a static geometric region. The intersection of the phase trajectory with the boundaries of this mask visualizes the exact moment of physical schedule violation and the transition of the enterprise into the zone of financial penalties. Such geometric monitoring enables full automation of decision-making regarding the urgent purchase or sale of compensating volumes of energy.

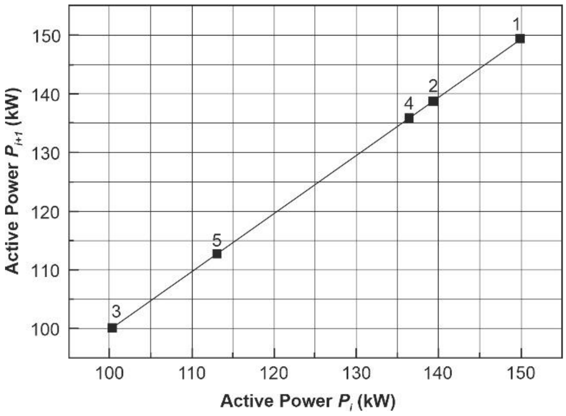


Fig. 3. Drift of the centers of bounding boxes constructed based on the electrical load profiles of mechanical engineering enterprises.

The main efficiency criterion on the IDM is the algorithm delay t_{delay} in recognizing a structural load shift. Classical statistical methods (moving average, exponential smoothing) and predictive models require the accumulation of an array of at least three to five consecutive points for the confident confirmation of a trend change and the filtering of random outliers. With a sampling step of $\Delta t = 30$ minutes, this generates an unavoidable algorithmic delay of 1.5 to 2.5 hours.

The proposed topological method of bounding rectangles captures a spatial anomaly instantaneously at the moment the trajectory crosses the boundary of the reference "mask" B_k^{norm}. In this case, the delay is reduced to the duration of one measurement cycle: $t_{delay} = 1 \cdot \Delta t$ (30 minutes). This provides the energy management service with a net algorithmic time gain of 1 to 2 hours, which is critically important for the timely submission of corrective bids for IDM trading.

An additional quantitative indicator is a radical reduction in computational complexity. To verify the current state of the algorithm, it is necessary to perform only four logical operations comparing coordinates with the rectangle boundaries per cycle, which corresponds to O(1) complexity. For comparison, the classical calculation of attractor metric invariants (e.g., correlation dimension) has $O\left(N^2\right)$ complexity, which requires significantly more microprocessor time and makes the geometric "mask" the only alternative solution for local operational control systems.

IV. CONCLUSION

In this study, a topical scientific and practical problem has been solved – providing energy management of industrial enterprises with an effective toolkit for real-time monitoring of electricity consumption under conditions of volatility in the European DAM and IDM markets.

The proposed geometric method of spatial parameterization of phase trajectories made it possible to formulate the following conclusions:

1. It has been demonstrated that existing “black box” machine learning approaches and computationally intensive metric algorithms of chaos theory are too inertial for the instantaneous identification of commercial imbalances. The proposed transition to two-dimensional topological analysis with decomposition into daily quasi-cycles (in accordance with DAM market regulations) enables fast and adequate recognition of the non-stationary nature of industrial load.
2. The use of the bounding box method for spatial limitation of reference phase trajectories ensures a radical reduction in computational complexity. This makes it possible to perform real-time monitoring while preserving algorithmic transparency and physical interpretability of results for dispatch personnel.
3. Characteristic geometric patterns (local boundary breaches, centroid shifts, volatility expansion) have been classified, allowing visual observations to be transformed into a system of strict algebraic rules. It has been established that centroid drift occurs within a narrow neighborhood of the bisector of the phase plane. This mathematically proves the high inertia of electricity consumption and makes any deviation from this axis a reliable indicator of structural change.
4. A concept for using a static geometric "mask" for automated control of power limits has been proposed. Detection of the current trajectory leaving the spatial domain generates an immediate objective signal for operational control of trading positions on the IDM. This forms a reliable mechanism for avoiding financial penalties for commercial imbalances.

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Information about authors.

Serhii Shevchenko
Doctor of Technical Sciences, Professor, Head of Department of Power Engineering, National Technical University "Kharkiv Polytechnic Institute".
Research interests: surge protection, surge arresters, grounding, and power line design.
E-mail: sergii.shevchenko@khpi.edu.ua
ORCID: 0000-0002-9658-7787

Oleksandr Kozlovskyi
PhD, Associate Professor, Department of Electrical Engineering Systems and Energy Management, Central Ukrainian National Technical University.
Research interests: mathematical modeling, smart grids, grid resource management, bio-inspired power systems.
E-mail: kozlovskyioa@gmail.com
ORCID: 0000-0001-6885-5994

Bederak Yaroslav
PhD, Head of the Electrical Laboratory, PJSC "AZOT".
Research interests: mathematical modeling, diagnostics of electrical equipment.
yaroslav0768@gmail.com
ORCID: 0000-0002-2669-0965