Improvement of the Accuracy of Transient Process Analysis in Complex Power Systems Using the Z-Transform Method

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Abstract. The main objectives of this study were to develop and comparatively analyze methods for digital modeling of dynamic processes in electrical power transmission lines with distributed parameters, as well as to evaluate the accuracy of simplified lumped parameter models against full models. The research aims to overcome limitations of simplified equivalent circuits that may lead to significant errors when analyzing transient conditions, system stability, and emergency situations. To achieve these objectives, the following tasks were accomplished: mathematical models of long lines were developed using operational methods based on Laplace and z-transforms; transfer functions in z-form were obtained for homogeneous lines without distortions and with distortions under different load types (matched and mismatched); for comparison, a simplified lumped parameter power line model was developed; computer simulation of transient responses was performed; and comparative analysis of results was conducted with calculation of absolute and relative errors. The most important results include the identification of significant discrepancies in time delays and amplitudes of transient processes between models; detection of maximum absolute errors reaching 0.45 for matched load and 0.03 for mismatched load; and determination of conditions where the simplified model adequately describes the system. The significance of the obtained results lies in their ability to improve the accuracy of transient process analysis in power systems, provide selection of optimal modeling methods for specific engineering tasks, prevent errors in the design of relay protection and automation devices caused by inadequacy of simplified models, and establish foundations for developing more reliable algorithms for digital protection of power equipment.
***Keywords*:** smart grid, transient processes, transfer function, wave impedance, equivalent circuit, recurrent algorithms, relay protection, system stability, z-transform, simulation model, modeling error.

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Îmbunătățirea preciziei analizei proceselor tranzitorii în sisteme energetice complexe utilizând metoda de transformare Z

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Rezumat. Principalele obiective ale acestui studiu au fost dezvoltarea și analiza comparativă a metodelor de simulare digitală a proceselor dinamice în liniile electrice de transport al energiei electrice cu parametri distribuiți, precum și evaluarea preciziei modelelor simplificate cu parametri agregați în raport cu modelele complete. Cercetarea își propune să depășească limitările circuitelor echivalente simplificate care pot duce la erori semnificative la analiza condițiilor tranzitorii, a stabilității sistemului și a situațiilor de urgență. Pentru a atinge aceste obiective, au fost îndeplinite următoarele sarcini: au fost dezvoltate modele matematice ale liniilor lungi folosind metode operaționale bazate pe transformări Laplace și z; au fost obținute funcții de transfer în formă z pentru linii omogene fără distorsiuni și cu distorsiuni sub diferite tipuri de sarcină (adaptate și neadaptate); pentru comparație, a fost dezvoltat un model simplificat al liniei electrice cu parametri agregați; a fost efectuată o simulare pe calculator a răspunsurilor tranzitorii; a fost efectuată o analiză comparativă a rezultatelor cu calcularea erorilor absolute și relative. Cele mai importante rezultate includ: identificarea discrepanțelor semnificative în întârzierile de timp și amplitudinile proceselor tranzitorii între modele; detectarea erorilor absolute maxime care ating 0.45 pentru sarcina adaptată și 0.03 pentru sarcina neadaptată; determinarea condițiilor în care modelul simplificat descrie în mod adecvat sistemul. Semnificația rezultatelor obținute constă în capacitatea lor de a îmbunătăți precizia analizei proceselor tranzitorii în sistemele energetice, de a oferi selecția metodelor optime de modelare pentru sarcini inginerești specifice, de a preveni erorile în proiectarea dispozitivelor de protecție și automatizare cu relee cauzate de inadecvarea modelelor simplificate și de a stabili bazele pentru dezvoltarea de algoritmi mai fiabili pentru protecția digitală a echipamentelor energetice.
***Cuvinte-cheie*:** rețea inteligentă, procese tranzitorii, funcție de transfer, impedanță de undă, circuit echivalent, algoritmi recurenți, protecție cu relee, stabilitatea sistemului, transformată z, model de simulare.

Повышение точности анализа переходных процессов в сложных энергосистемах с использованием метода z-преобразования

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Аннотация. Основные цели исследования заключались в разработке и сравнительном анализе методов цифрового моделирования динамических процессов в линиях электропередачи с распределенными параметрами, а также в оценке точности упрощенных моделей с сосредоточенными параметрами по сравнению с полными моделями. Исследование направлено на преодоление ограничений упрощенных схем замещения, которые могут приводить к значительным погрешностям при анализе переходных режимов, устойчивости системы и аварийных ситуаций. Для достижения поставленных целей были решены следующие задачи: разработаны математические модели длинной линии на основе операторного метода с использованием преобразований Лапласа и z-преобразования, что позволило перейти от непрерывных передаточных функций к дискретным; получены передаточные функции в z-форме для однородных линий без искажений и с искажениями при различных типах нагрузки (согласованной и несогласованной); для сравнения была разработана модель упрощенной линии электропередачи с сосредоточенными параметрами; проведено комплексное компьютерное моделирование переходных характеристик с использованием специализированного программного обеспечения; выполнен детальный сравнительный анализ результатов с расчетом абсолютных и относительных погрешностей между разными типами моделей. Наиболее важными результатами являются: установление существенных расхождений во временных задержках и амплитудах переходных процессов между моделями, что особенно выражено в режимах несогласованной нагрузки; выявление максимальной абсолютной погрешности до 0.45 для согласованной нагрузки и до 0.03 для несогласованной; определение конкретных условий, при которых упрощенная модель адекватно описывает систему. Значимость полученных результатов состоит в том, что они позволяют существенно повысить точность анализа переходных процессов в сложных энергосистемах, обоснованно выбирать оптимальный метод моделирования для конкретных инженерных задач, избегать критических ошибок при проектировании устройств релейной защиты и автоматики, обусловленных неадекватностью упрощенных моделей, а также закладывают основу для создания более надежных алгоритмов цифровой защиты энергооборудования.

***Ключевые слова*:** интеллектуальная сеть, переходные процессы, передаточная функция, волновое сопротивление, эквивалентная схема, рекуррентные алгоритмы, релейная защита, устойчивость системы, z-преобразование, ошибка моделирования.

INTRODUCTION

Digital modeling of dynamic modes in power transmission lines (PTL) is a critically important tool for the design, analysis, and stability maintenance of modern electric power systems.

With the increasing complexity of power grids, the integration of distributed generation, and the growing requirements for power quality, accurate forecasting of transient processes becomes an integral part of control and relays protection tasks. A particular challenge is posed by the modeling of long power transmission lines (LPTL), where disregarding the distributed nature of parameters (resistance, inductance, capacitance, and shunt conductance) can lead to significant errors when analyzing fast-acting electromagnetic processes such as insulation breakdown, switching overvoltages, and wave propagation [1, 2, 3].

Traditionally, to simplify calculations, lumped-parameter equivalent circuits (П- or T-type models) are used, which adequately describe the system in steady-state modes. However, as shown in a number of studies, including works [4-7], such models demonstrate insufficient accuracy when modeling high-frequency transient processes, which can lead to incorrect assessments of system stability and the response time of protective devices. These high-frequency transients, often filtered out as undesirable noise in traditional analysis, are fundamental to travelling-wave-based phenomena and can be positively utilized for precise fault location, as shown in [8]. In this regard, a relevant task is the development and comparison of digital modeling methods based on rigorous mathematical approaches that account for the distributed parameters of PTL.

In modern research, the z-transform and discretization methods [9, 10] are widely used to transition from continuous models in the frequency domain (based on the Laplace transform) to discrete models suitable for implementation on digital computing devices. This approach allows for the effective use of the computational power of modern systems to analyze a circuit's response to arbitrary excitations. The development of such digital models, capable of accurately propagating voltage and current profiles along the line [8] and accounting for complex frequency-dependent effects [3], is key to advancing real-time simulation and protection algorithms.

The task of digitally modeling the dynamic processes that occur in lines with distributed parameters is one of the most difficult undertakings. Often, to simplify the understanding of the processes occurring in a power system, electrical equivalent circuits of power transmission lines are used, which are applicable for the study of steady-state or quasi-steady-state modes. Depending on the desired level of accuracy, these circuits may be configured in various forms. This simplification, while greatly facilitating the modeling process, carries an inherent trade-off: the introduction of inaccuracies into the dynamic model of the system [11, 12].

Consequently, these approximations can lead to incorrect conclusions regarding system stability and the analysis of emergency modes in real time. Therefore, for the correct digital modeling of transient processes, it is critically important to take into account the fundamental information of PTL with distributed parameters [13, 14].

A recommended methodology for the digital modeling of power transmission lines involves applying the Laplace or z-transform. The practical utility of the z-transform, in particular, stems from the fact that the original function can be reconstructed by expressing its fractional-rational image as a Laurent series arranged in decreasing powers of z. The implementation of this method is typically carried out using existing computer mathematics programs, which provides an effective solution to the given problem [15].

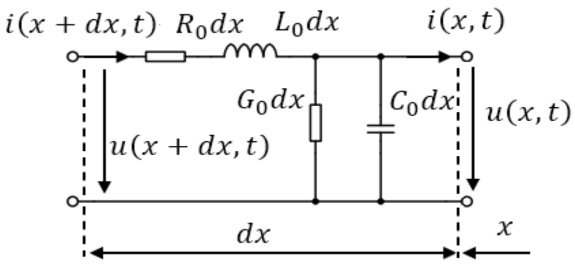

Fig.1. **Equivalent circuit of an elementary section of a power transmission line.**

where C_0 –shunt capacitance between the forward and return conductors; L_0 –loop inductance of the conductor pair; R_0 – series resistance of the forward and return conductors; G_0 –dielectric shunt conductance due to insulation leakage; dx – infinitesimally small section of the long line; x – distance from the end of the line.

The digital modeling of transients in a homogeneous two-wire long line with distributed parameters is based on the standard equivalent circuit representing an infinitesimally small segment, depicted in Fig. 1 [13, 16].

For a homogeneous long power transmission line with distributed parameters, its circuit can be proposed as a four-terminal network, which in turn is replaced by an equivalent circuit scheme (equivalent network), as shown in Fig. 2.

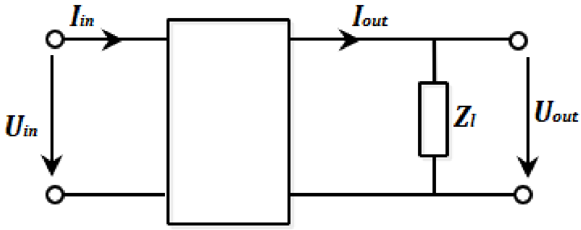

Fig.2. **Equivalent network for a long power transmission line.**

In accordance with reference [13], the expression for the voltage transfer function of a uniform ladder network was derived.

$$W(p)=\frac{e^{\gamma x}[Z_l(p)+Z_w(p)]+e^{-\gamma x}[Z_l(p)-Z_w(p)]}{e^{\gamma l}[Z_l(p)+Z_w(p)]+e^{-\gamma l}[Z_l(p)-Z_w(p)]} \quad (1)$$

where x - distance from the end of the line; l - length of the long line; $Z_l(p)$ - line load impedance; $Z_w(p)$ - wave impedance; $\gamma(p)$ - propagation coefficient.

$$Z_w(p)=\sqrt{\frac{R_0}{G_0}}+\frac{1}{2}\sqrt{\frac{R_0}{G_0}}\cdot\frac{L_0G_0-R_0C_0}{G_0R_0}\cdot p=A^{"}p+B^{"} \quad (2)$$

$$\gamma(p)=\sqrt{G_0R_0}+\frac{1}{2}\sqrt{G_0R_0}\cdot\frac{R_0C_0+L_0G_0}{G_0R_0}=Cp+D \quad (3)$$

By applying the inverse difference method formula, $p=(z-1)/zT$, an expression for the transfer function in the z-variable in a discrete form was obtained, which allows for determining the system's response to an external excitation at the end of the line (x=0):

$$W(z)=\frac{2Z_l(z)e^{Dl}z^m}{(Z_l(z)+Z_w(z))e^{2Dl}z^{2m}+Z_l(z)-Z_w(z)} \quad (4)$$

where $m=Cl/T$;

$$Z_w(z)=\frac{1}{2}\sqrt{\frac{R_0}{G_0}}\cdot\frac{(2G_0R_0T+L_0G_0-R_0C_0)z-L_0G_0+R_0C_0}{G_0R_0Tz}.$$

For practical engineering calculations, a simplified power transmission line (SPTL) with lumped parameters was examined, where each phase of this line is represented as a four-terminal network, which is replaced by a simple electric circuit. Fig. 3 shows the simplified and operational equivalent circuit of the line for calculating transient processes in circuits with voltages above 35 kV [13, 17, 18]. For lines shorter than 300 km in length, the equivalent circuit parameters are defined as follows: $R = R_0 l$, $X = X_0 l$, $B = B_0 l$, or $R = R_0 l$, $L = L_0 l$, $C = C_0 l$.

In digital modeling, the excitation of a unit step impulse at the input of the power transmission line is assumed, and an active-inductive load of the long and simplified line with lumped parameters, which is equal to $Z_l(p)=R_l+pL_l$, is also taken into account.

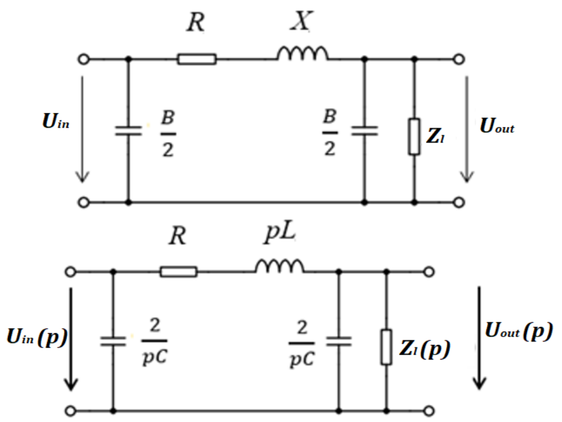

Fig.3. Equivalent circuit of a power transmission line with lumped parameters: a) simplified; b) operational.

A long power transmission line (LPTL) without distortions and with a matched load was considered. In a line without distortions, the wave impedance $Z_w(z)$ is a real number. In expression (4), the matched load at $Z_l(z) = Z_w(z)$, which means the load of the line circuit $Z_l(p) = Z_w(p)$, for matching the load, requires that $L_l=0$, then $Z_l(p) = R_l = Z_w(p)$.

The following parameters were utilized as the subject of the investigation: $R_0=38.4\cdot10^{-3}$ ohm/m; $L_0=3.93\cdot10^{-3}$ H/m; $C_0=5.12\cdot10^{-12}$ F/m; $G_0=5\cdot10^{-11}$ S/m; $x=0$; $l=140955$ m; $T=0.001$ s; number of iterations $N=1000$; $0<\omega<1000$ rad/s [13].

The following values were obtained: $C=1.4185\cdot10^{-7}$; $D=1.3856\cdot10^{-6}$; $m=20$; $Z_w(p)=27712.8129$ Ohm. Based on expression (4), the transfer function of the LPTL without distortions with a matched load has the form:

$$W_{10}(z)=0.8226\cdot z^{-20} \tag{5}$$

The SPTL with a matched load was also considered. From the equivalent circuits in Fig. 3, the transfer function was obtained using the nodal potential method [19, 20]:

$$W_{11}(z)=\frac{a_0}{b_0+b_1z^{-1}+b_2z^{-2}} \tag{6}$$

where $a_0= 2T^2Z_l$; $b_0 = CZ_lL+(2L+RCZ_l)T+(2R+2Z_l)T^2$; $b_1 = -[2CZ_lL+(2L+RCZ_l)T]$; $b_2 = CZ_lL$; $R = R_0l$; $L = L_0l$; $C = C_0l$; $Z_l = Z_w(p) =$ 27712,8129 Ohm.

Based on the aforementioned parameters and utilizing expression (6), the line's transfer function is formulated as follows:

$$W_{11}(z)=\frac{0.055426}{12.3615-23.37434z^{-1}+11.0791z^{-2}} \tag{7}$$

Fig. 4 displays the transient response characteristics of the lines, which were derived using the z-forms method.

The discrepancy between the models was quantified by comparing the LPTL without distortions and the SPTL under matched load conditions. The resulting error profiles (Fig. 5) show a peak absolute error of $\Delta h = 0.45079$ at $n=19$.

Similarly, the digital modeling of the LPTL with distributed parameters, including its version without distortions, and the SPTL with lumped parameters was performed under the condition of their mismatched loads.

A formula for the transfer function is proposed for the LPTL without distortions in the presence of a mismatched load:

$$W_{20}(z)=\frac{-135.3z^{-11}+338.25z^{-20}}{28708-68.649z^{-1}-27546z^{-20}-66.667z^{-21}} \tag{8}$$

For the SPTL with a mismatched load, the transfer function in the p-variable was obtained from the simplified equivalent circuits in Fig. 3 [20]:

$$W_{21}(z)=\frac{Z_2(p)}{Z_1(p)+Z_2(p)} \tag{9}$$

where $Z_1(p)=R+pL$;

$$Z_2(p)=\frac{\frac{2}{pC}\cdot Z_l(p)}{\frac{2}{pC}+Z_l(p)}=\frac{\frac{2}{pC}\cdot(R_l+pL_l)}{\frac{2}{pC}+R_l+pL_l}=\frac{2R_l+2L_lp}{2+R_lCp+L_lCp^2}$$

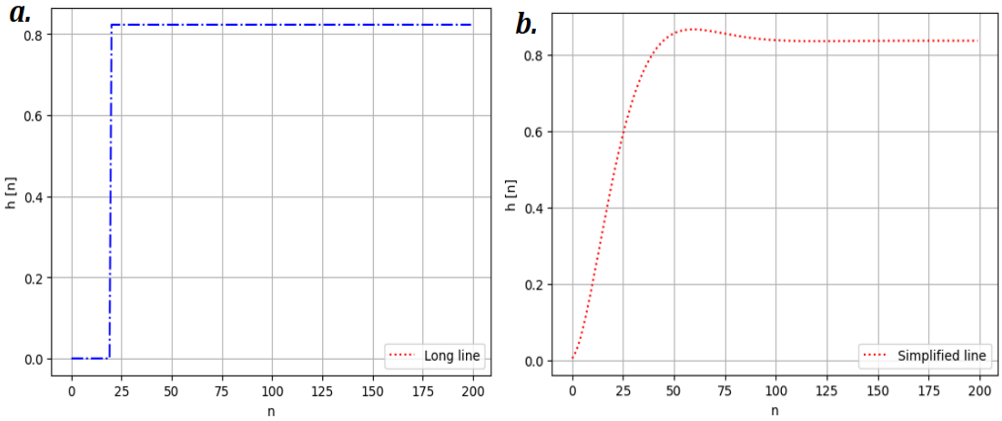

Fig.4. Transient characteristics in power transmission lines: a) LPTL without distortions with a matched load; b) SPTL with a matched load.

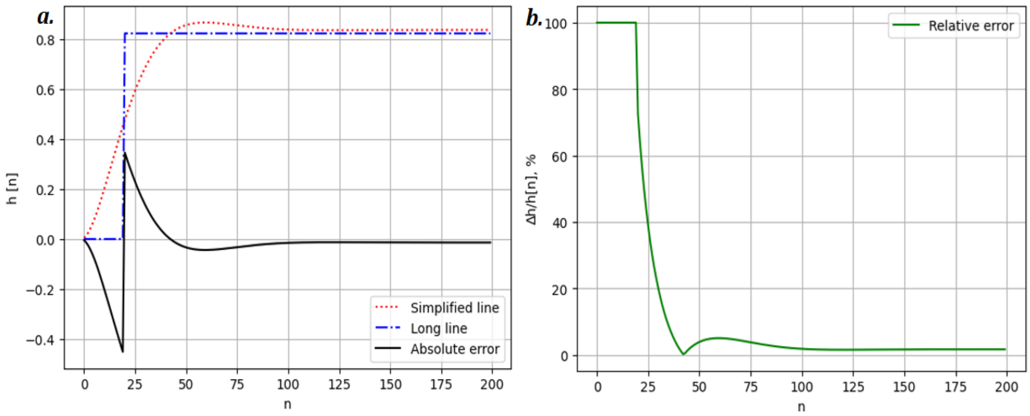

Fig.5. Graphs with a matched load: a) Absolute error; b) Relative error.

$$W_{21}(p)=\frac{2R_l+2L_lp}{2(R+R_l)+(RR_lC+2L+2L_l)p+(RL_lC+LR_lC)p^2+LL_lCp^3} \tag{10}$$

where $a_0= 2L_lT^2+2R_lT^3$; $a_1 = -2L_lT^2$; $b_0 = LL_lC+(RL_lC+LR_lC)T+(RR_lC+2L+2L_l)T^2+(2R+2R_l)T$; $b_1 = -[3LL_lC+(2RL_lC+LR_lC)T+RR_lC+2L+2L_l)T^2]$. $b_2 = 3LL_lC+(RL_lC+LR_lC)T$; $b_3 = -LL_lC$.

The following parameters were utilized for this case: R_l=100 Ohm; L_l=0.01 H; R_0=38.4·10^{-3} Ohm/m; L_0=3.93·10^{-3} H/m; C_0=5.12·10^{-12} F/m; G_0=5·10-11 S/m; x=0; l=10574 m; T=0.0001 s; number of iterations N=6666; $0<\omega<1000$ rad/s [13].

Then, the transfer function of the SPTL with a mismatched load termination is given by:

The transfer function in the z-variable:

$$W_{21}(p)=\frac{a_0+a_1z^{-1}}{b_0+b_1z^{-1}+b_2z^{-2}+b_3z^{-3}} \tag{11}$$

Fig. 6 displays the transient characteristics for the LDTL and SPTL models. Figure 7 shows the error plots from the model comparison, revealing a maximum absolute error of $\Delta h\approx 0.02909$ at n=681.

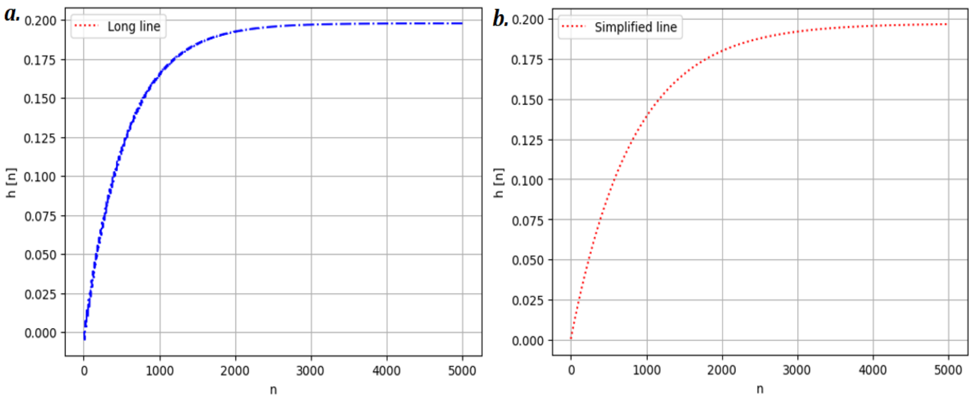

Fig.6. Transient characteristics in power transmission lines: a) LPTL without distortions with a mismatched load; b) L with a mismatched load.

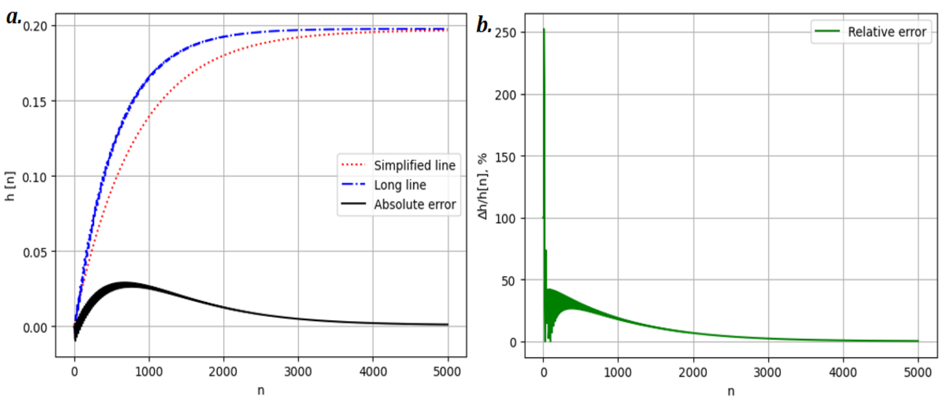

Fig.7. Graphs with a mismatched load: a) Absolute error; b) Relative error.

Analogous studies for the LPTL with distortion and the SPTL utilizing simplified equivalent models were carried out under conditions of mismatched load.

$$W_{30}(z) = \frac{248.11407z^{-20} - 22.5558z^{-21}}{2287900.7 - 2252525.2z^{-1} - 1798567z^{-40} + 1770954.2z^{-41}} \tag{13}$$

and SPTL with mismatched load:

$$W_{31}(z) = \frac{2.2\cdot 10^{-7} - 2\cdot 10^{-8}z^{-1}}{15.7902\cdot 10^{-4} - 16.1308\cdot 10^{-4}z^{-1} + 4.434\cdot 10^{-5}z^{-2} - 3.41\cdot 10^{-6}z^{-3}} \tag{14}$$

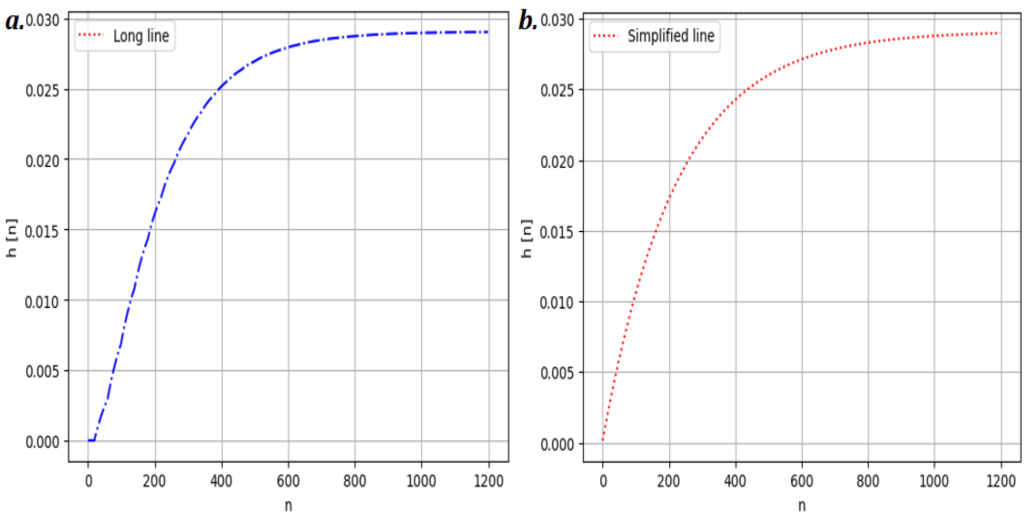

Fig.8. Transient characteristics in power transmission lines: a - LPTL with distortion and mismatched load; b - SPTL with mismatched load.

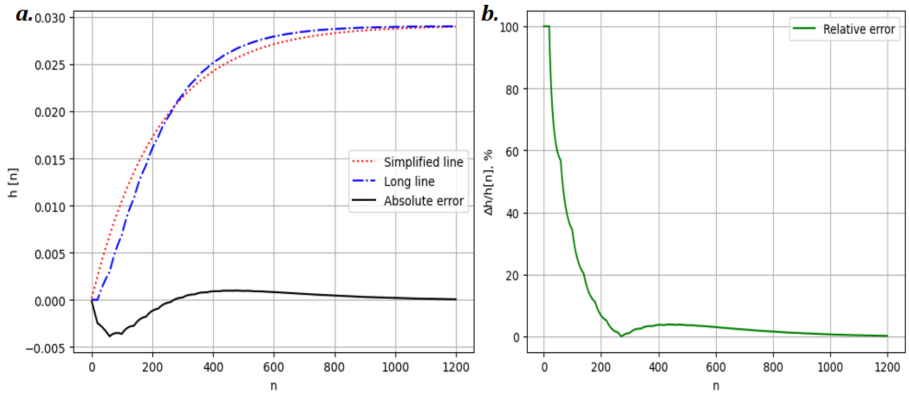

Fig.9. Graphs: a - Absolute error; b - Relative error.

The computations were executed using the following parameters: R_l = 100 Ohm; L_l = 0.01 H; R_0 = 38.4·10^{-3} Ohm/m; L_0 = 8.84·10^{-3} H/m; C_0 = 5.12·10^{-12} F/m; G_0 = 5·10^{-11} S/m; x = 0; l =

The transient responses derived via the z-transform and the ensuing comparative error analysis are depicted in Figures 8 and 9. The modeling error reaches its maximum absolute value of Δh = 0.00389 at the discrete step n=59.

86791 m; T = 0.001 s; number of iterations N = 6666; 0<ω<1000 rad/s [13, 21, 22]. The transfer function for the LPTL with distortion and mismatched load was obtained:

For the power system (PS) with an LPTL without distortion and a matched load and SPTL with a matched load, the output signal of the linear blocks when an excitation of unit impulse is applied at the PS input is shown in Fig. 10.

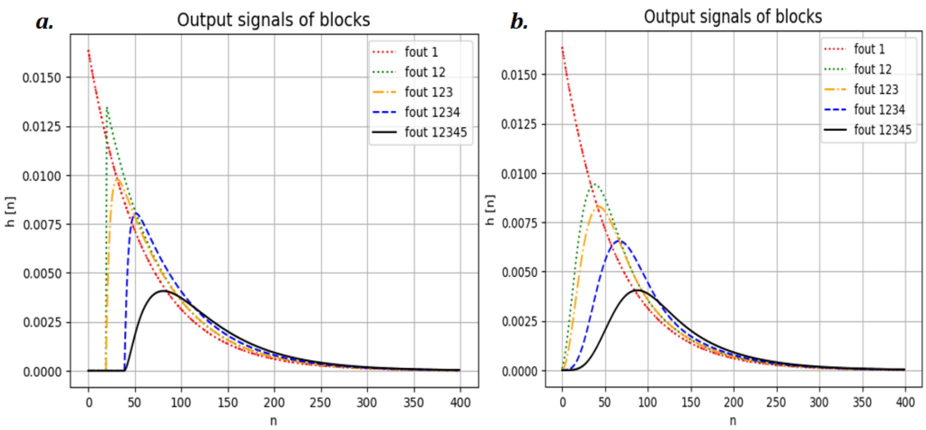

Fig.10. Transient characteristics of the series connection of PS blocks: a - LPTL without distortion with a matched load; b - SPTL with a matched load.

Table 1.

Serial connections of blocks	Clock moments in time *[n]* at *h[n]* = *max*	Maximum value *h[n]*
Long power transmission line without distortion with matched load		
$f_{out\ 1}$	0	0.01639344262295082
$f_{out\ 12}$	20	0.013485245901639345
$f_{out\ 123}$	31	0.009774094593772301
$f_{out\ 1234}$	51	0.008040170212837094
$f_{out\ 12345}$	81	0.004073840510567478
Simplified power transmission line with matched load		
$f_{out\ 1}$	0	0.01639344262295082
$f_{out\ 12}$	38	0.009421078830160846
$f_{out\ 123}$	43	0.008288407024191974
$f_{out\ 1234}$	67	0.006554152596865126
$f_{out\ 12345}$	88	0.004054762818590622

Based on the plots in Figure 10, the calculation algorithm within the computer software was employed to determine the time steps at which the output signal *h[n]* of the functions attains its maximum value, and these values are presented in Table 1.

Based on Table 1, it can be concluded that the transient process in the power system connected with a LPTL without distortion and a matched load occurs with a smaller time delay than with the connection involving a SPTL with a matched load.

The transient response of the output signal $f_{out12345}$ and the corresponding absolute error for the PS are shown in Figure 11. A peak absolute error of Δh=0.001067 is reached at n=40.

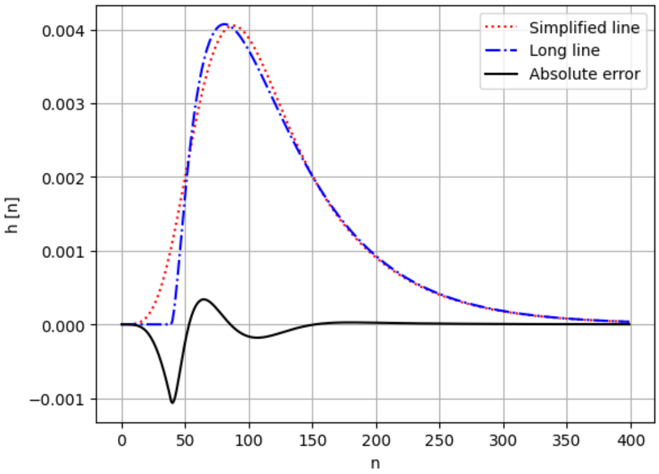

Fig.11. **Graphs of the transient characteristics of the output signals $f_{out12345}$ of the line with a matched load of PS, and the absolute error.**

Upon the excitation of a unit step impulse at the PS input, the transient characteristics of the output signals $f_{out12345}$ and the error plots from their comparative analysis are presented in Figure 12.

The maximum absolute modeling error, $\Delta h \approx$ 0.01804, occurs at the discrete time step n =152.

Similarly, the same digital modeling of the PS connected with the remaining lines was conducted.

The transient characteristics for the LPTL without distortion with a mismatched load and the SPTL with a mismatched load, upon the excitation of a unit impulse at the PS input, were obtained in Fig. 13.

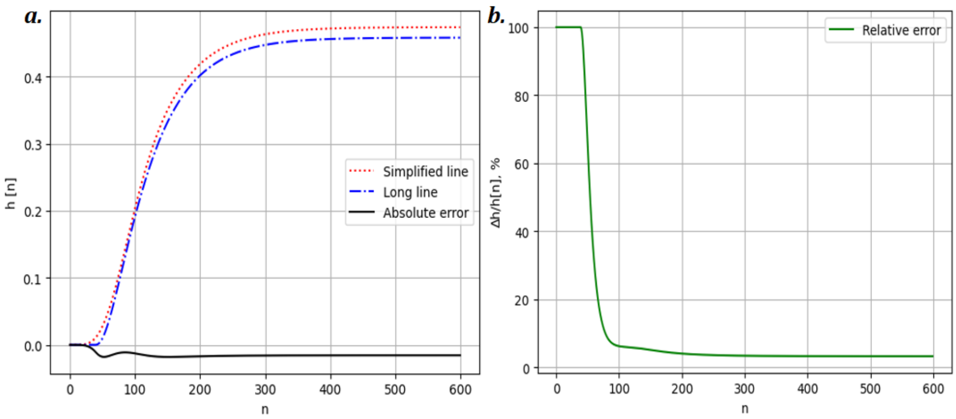

Fig.12. **Plots: a) Transient characteristics and absolute error; b) Relative error.**

Based on the plots in Figure 10, the calculation algorithm within the computer software was employed to determine the time steps at which the output signal *h[n]* of the functions attains its maximum value, and these values are presented in Table 1.

The calculation algorithm within the computer software was employed to determine the maximum values of the output signal *h[n]* of the functions at the corresponding time steps, and these values are presented in Table 2.

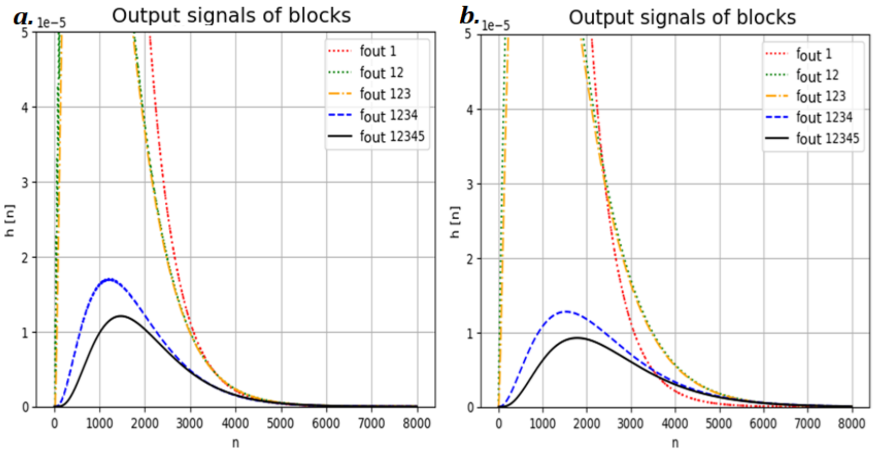

Fig.13. Transient characteristics of the series connection of PS blocks: a – LPTL without distortion with a mismatched load; b - SPTL with a mismatched load.

Table 2.

Serial connections of blocks	Clock moments in time *[n]* at *h[n] = max*	Maximum value *h[n]*
Long power transmission line without distortion with mismatched load		
$f_{out\ 1}$	0	0,001663893510815307
$f_{out\ 12}$	581	0,000129731203905735
$f_{out\ 123}$	650	0,000114170163889094
$f_{out\ 1234}$	1210	$1{,}70259675292878 1\cdot 10^{-5}$
$f_{out\ 12345}$	1473	$1{,}206423043709771\cdot 10^{-5}$
Simplified power transmission line with mismatched load		
$f_{out\ 1}$	0	0,001663893510815307
$f_{out\ 12}$	697	0,000102503537336695
$f_{out\ 123}$	754	$9{,}255240696919497\cdot 10^{-5}$
$f_{out\ 1234}$	1528	$1{,}278059489788136\cdot 10^{-5}$
$f_{out\ 12345}$	1782	$0{,}925851035305011\cdot 10^{-5}$

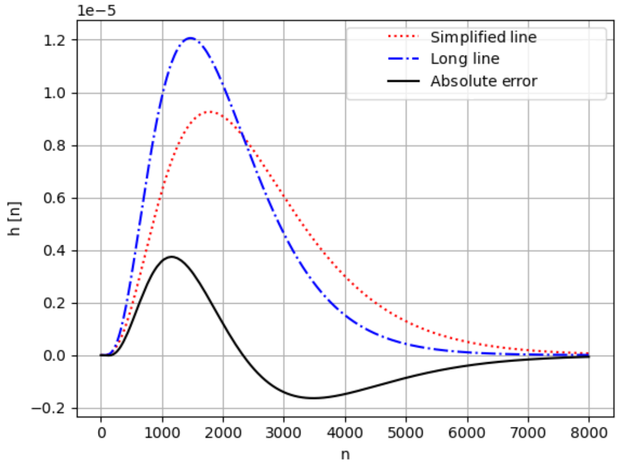

Fig.14. Plots of the transient characteristics of the output signals fout12345 of the line with a mismatched load of PS, and the absolute error.

Following excitation by a unit step function at the PS input, the transient responses of the output signals $f_{out12345}$ and the error plots are presented in Figure 15.

The peak absolute discrepancy between the models, Δh = 0.004514, occurs at n =2329. The time moment at which the output signal *h[n]* of the functions reaches its maximum value were obtained in Table 3.

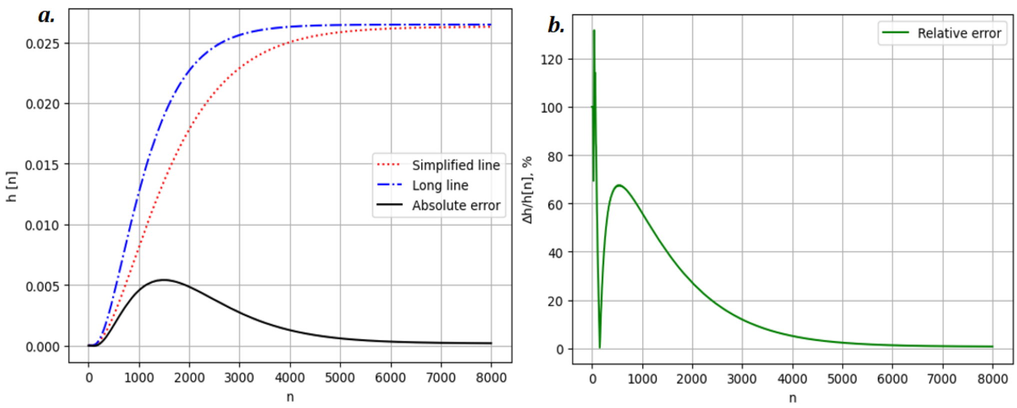

Fig.15. Plots: a - Transient characteristics and absolute error; b - Relative error.

Regarding the LTL with distortion and a mismatched load and the SPTL with a mismatched load, Figure 16 presents the plots of the transient responses of the output signals following unit impulse excitation at the PS input.

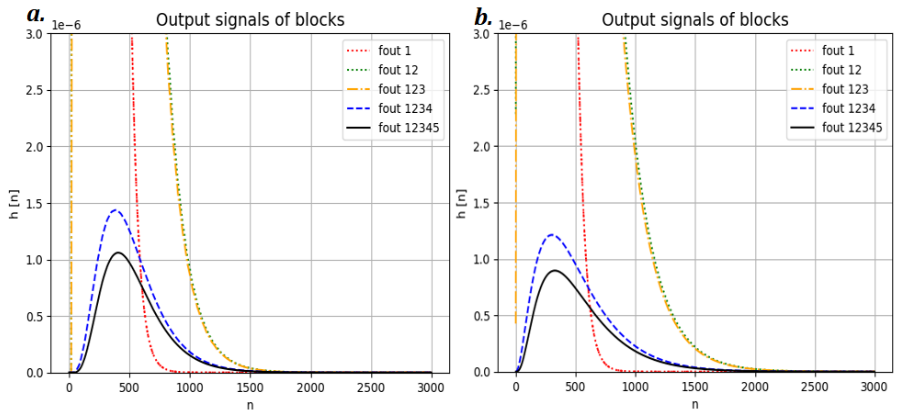

Fig.16. Transient characteristics of the series connection of PS blocks: a - LPTL with distortion and a mismatched load; b - SPTL with a mismatched load.

Table 3.

Serial connections of blocks	Clock moments in time *[n]* at *h[n] = max*	Maximum value *h[n]*
Long power transmission line with distortion with mismatched load		
$f_{out\ 1}$	0	0.01639344262295082
$f_{out\ 12}$	160	$8.473397585189009 \cdot 10^{-5}$
$f_{out\ 123}$	165	$7.676123656076127 \cdot 10^{-5}$
$f_{out\ 1234}$	387	$1.436921269589052 \cdot 10^{-6}$
$f_{out\ 12345}$	406	$1.059695152348151 \cdot 10^{-6}$
Simplified power transmission line with mismatched load		
$f_{out\ 1}$	0	0.01639344262295082
$f_{out\ 12}$	107	$8.010574072461381 \cdot 10^{-5}$
$f_{out\ 123}$	113	$7.273231000285308 \cdot 10^{-5}$
$f_{out\ 1234}$	302	$1.212114132706986 \cdot 10^{-6}$
$f_{out\ 12345}$	321	$0.895978017319045 \cdot 10^{-6}$

Table 3 clearly shows that the transient process in the PS, which includes a LPTL with distortion and a mismatched load, exhibits a longer time delay compared to the PS configured with a SPTL under a mismatched load.

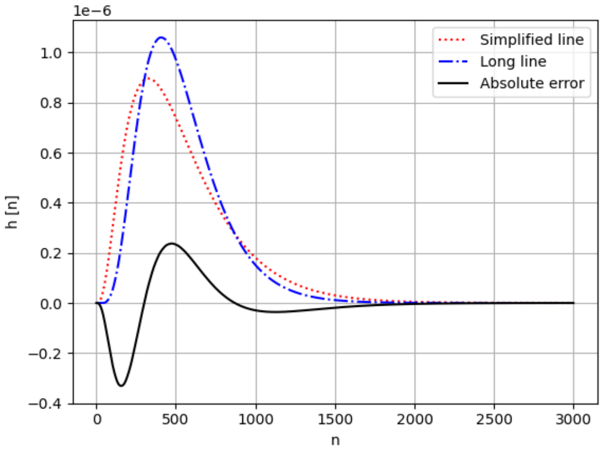

Fig.17. **Plots of the transient characteristics of the output signals $f_{out12345}$ for the PS, and the absolute error.**

Following excitation by a unit step function at the PS input, the transient characteristics of the output signals $f_{out12345}$ and the error plots are presented in Figure 18. At iteration n=300, the error reaches its maximum value Δh=5.433·10^{-5}.

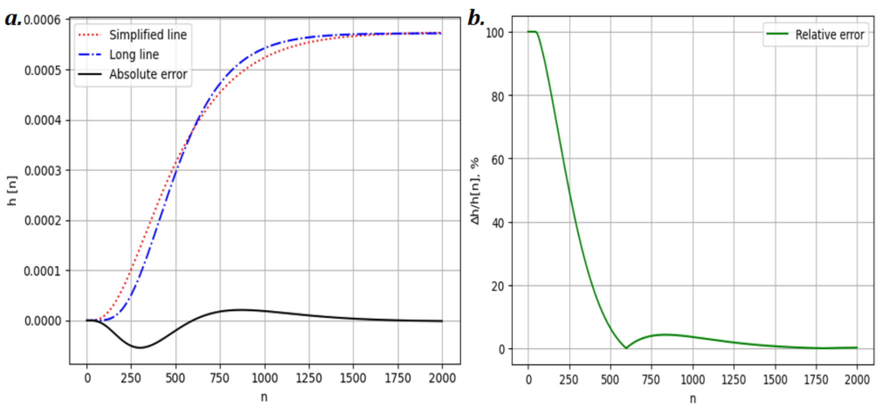

Fig.18. **Graphs: a - Absolute error; b - Relative error.**

Figure 17 illustrates the transient characteristics of the output signals $f_{out12345}$ for the PS, along with the absolute error from their comparative analysis. At n=156, the maximum absolute error is Δh=3.308·10^{-7}.

CONCLUSION

The conducted research demonstrates the high efficiency of the method based on the z-transform for the digital modeling of transient processes in power transmission lines with distributed parameters. To provide a deeper justification of the obtained results, it is worthwhile to conduct a comparative analysis of the proposed method with other widely known approaches.

Unlike Dommel's method (EMTP), based on numerical integration and the replacement of distributed parameters with lumped parameters for small line segments, the proposed z-transform method allows for accurate modeling of wave properties and the phenomenon of time delay without the need to discretize the line along its length. This eliminates the error accumulation associated with approximating infinitesimal segments with finite ones and ensures more accurate reproduction of wave fronts during switching events, which is critically important for analyzing high-frequency transients. As calculations have shown, the maximum absolute error of the proposed method for the mismatched load condition was only 0.03, whereas the use of simplified lumped parameter models (similar to those used in EMTP for long lines) leads to errors reaching 0.45 in the matched load condition.

Compared to frequency-domain methods based on the Fourier transform, the proposed approach has a significant advantage when modeling non-stationary transients of complex shapes. The z-transform makes it possible to obtain impulse transfer functions that can be directly implemented as recurrent algorithms for digital relay protection and automation devices. While frequency-domain methods require laborious inverse Fourier transforms and can face spectral discretization problems, the z-transform method provides a direct transition from an analytical description to a discrete model ready for software implementation. This is confirmed by the modeling results, which established that the transient process in a system with a distortion-free LPTL and matched load occurs with a smaller time delay when using the proposed method.

The results of modeling dynamic processes in overhead power transmission lines using the z-transform framework, presented in the article, confirm the promise of this method. This is because the method allows for adequate and highly accurate modeling of transient processes in the line. The main errors of digital modeling are associated with the use of approximate formulas for the transition from the analog operator p to the discrete operator $z=e^{pT}$. In the formula of the analog transfer function of the line (1), derived from the equations rather than from equivalent circuits, all exponential functions are replaced by the variable z precisely. These exponential functions model the time delay process. Furthermore, the obtained impulse transfer functions are comparatively simple to implement technically without complex computations. Currently, with the introduction of digital technologies and Smart Grid into the power industry, the task of digital modeling of all elements of the power system arises. Solving this task will bring their models to a unified digital form. Thus, the digital modeling of an overhead power transmission line is a relevant scientific and technical task, and the proposed method based on the z-transform represents an effective solution for creating accurate and practically implementable power line models.

The obtained results create the basis for improving the accuracy of electromagnetic transient processes analysis in complex power systems and for developing more reliable algorithms for digital relay protection.

The prospects for further research are related to extending the developed methodical approach to more complex power system schemes, including multi-circuit transmission lines and complex network configurations, along with formulating actionable guidelines for selecting the appropriate model fidelity based on the specific problem to be addressed.

The practical contribution of this study is the generation of guidelines for determining the suitable complexity of the power transmission line model for engineering calculations of transient processes in actual power systems, which has a direct bearing on the dependability and operating speed of digital relay protection and automation equipment.

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