The Improvement of Energy Harvesting Efficiency of Constant Current Source

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Abstract. This study aims at determination the maximum power point parameters for the constant current source with nonlinear parasitic elements. The aim has been achieved by analyzing the differential resistance and equivalent parameters of a circuit with a constant current source. As a result, the buck-boost converter circuit is considered with the equivalent current source, which is formed with a photovoltaic module. The problem of the maximum photovoltaic module of energy harvesting is related to the research of its nonlinearity, which determines operating points at the current-voltage curves under different irradiances and temperatures. Thus, the differential resistance of photovoltaic module is examined to determine the parameters of the maximum power point mode.
The main result of the research is the model, which differs from the known models by the description of the dependence between the buck-boost converter duty cycle and input equivalent current source parameters in the maximum power point mode. The results of modelling are supported by experimental research of the laboratory layout. The presented circuit ensures the operating point close to the maximum power point of the solar panel equivalent current source. The duty cycle of the buck-boost converter is determined directly from the equivalent current source model with the parameters estimated analytically from the irradiance and temperature of the solar cells. The presented approach allows developing the maximum power point tracking algorithms based on the estimation of the equivalent current source parameters that provide improvement of the energy harvesting efficiency.
***Keywords*:** current source, energy losses, nonlinearity, solar panel, DC-DC converter, differential resistance.
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Îmbunătățirea eficienței energetice a sursei DC

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Rezumat. Problema extragerii maxime a energiei este importantă în multe domenii ale ingineriei electrice moderne, cum ar fi în special energia solară și eoliană. Dezvoltarea energiei regenerabile moderne necesită dezvoltarea unor noi surse de energie, convertoare si algoritmi de control care asigură o eficiență energetică ridicată de selecție a surselor de curent continuu. Scopul principal al lucrării propuse este de a determina punctul de putere maximă a unei surse de curent continuu cu elemente parazitare neliniare. Scopul este realizat prin analiza parametrilor rezistenței diferențiale și a parametrilor echivalenți ai circuitului cu o sursă de curent constantă. Ca rezultat, pulsul este considerat circuit de convertizor cu o sursă de curent echivalent, care este constituit dintr-un modul fotoelectric. Problema de a cantităţii maxime de energie de la modulul fotovoltaic ţine de cercetarea neliniarităţii acestuia, care determină punctul de lucru pe caracteristicilor curent-tensiune pentru diferite valori de luminozitate și temperatură. Neliniaritatea sursei de energie echivalentă afectează în mod semnificativ pierderea de energie în circuitul exterior și, prin urmare, ar trebui să fie luate în considerare în dezvoltarea convertoarelor de impulsuri care controlează punctele de operare. Astfel, rezistența diferențială a modulului fotovoltaic este investigată pentru a determina parametrii modulului de putere maximă. Rezultatul principal al cercetării este un model, care este diferit de cele cunoscute prin descrierea dependenţei dintre coeficientul de umplere a impulsurilor de dirijare cu cheia electronică şi parametrii echivalenţi ai sursei de curent de intrare în punctul de putere maximă. Rezultatele simulării sunt confirmate de un studiu experimental pe un stand de laborator.
***Cuvinte-cheie*:** sursă de curent, pierdere de energie, neliniaritate, panou solar, convertor DC-DC, rezistență diferențială.

Повышение эффективности отбора энергии источника постоянного тока

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Аннотация. Проблема максимального отбора энергии важна во многих областях современной электротехники, таких как, в частности, солнечная и ветровая энергетика. Развитие современной возобновляемой энергетики требует разработки новых источников энергии, преобразователей и алгоритмов управления, которые обеспечивают высокую эффективность отбора энергии источников постоянного тока. Главной целью предложенной работы является определение точки максимальной мощности источника постоянного тока с нелинейными паразитными элементами. Цель достигается путем анализа дифференциального сопротивления и эквивалентных параметров цепи с источником постоянного тока. Как результат, рассматривается схема импульсного преобразователя с эквивалентным источником тока, который образован фотоэлектрическим модулем. Проблема максимального отбора энергии фотоэлектрического модуля связана с исследованием его нелинейности, которая определяет рабочие точки на вольт-амперных характеристиках при различных значениях освещенности и температуры. Нелинейность эквивалентного источника тока существенно влияет на потери энергии во внешних цепях, и поэтому должна учитываться в процессе разработки импульсных преобразователей, управляющих рабочими точками. Таким образом, дифференциальное сопротивление фотоэлектрического модуля исследовано с целью определения параметров режима максимальной мощности. Главным результатом исследований является модель, которая отличается от известных описанием зависимости между коэффициентом заполнения импульсов управления ключом преобразователя и параметрами эквивалентного входного источника тока в режиме точки максимальной мощности. Результаты моделирования подтверждены экспериментальным исследованием лабораторного макета. Представленная схема обеспечивает рабочую точку, близкую к точке максимальной мощности эквивалентного источника тока солнечной панели. Коэффициент заполенния импульсного сигнала управления преобразователя определяется непосредственно из модели эквивалентного источника тока с параметрами, аналитически оцененными исходя из освещенности и температуры солнечных элементов. Предложенный подход позволяет разрабатывать алгоритмы отслеживания точек максимальной мощности, основанные на оценке параметров источника эквивалентного тока, которые обеспечивают повышение эффективности отбора энергии.

Ключевые слова: источник тока, потери энергии, нелинейность, солнечная панель, DC-DC преобразователь, дифференциальное сопротивление.

INTRODUCTION

The improvement of efficiency of energy harvesting from constant current source is required in many areas of modern electrical engineering, especially, in solar [1] and wind [2] energetics, low power devices [3] and different power supplies [4]. When the requirements for energy losses are strict, the electrical circuits and systems should be designed taking into account many parasitic parameters and exact characteristics of operation mode [5], which impact substantially the energy losses. Work [5] describes the effect of parasitic elements on the modes and transient characteristics of the DC-DC converters. It is shown that small changes in the equivalent circuit of the switching device can cause significant changes in energy losses.

At present, the renewable energy sources are the most important issues of modern energetics that is described in [6]. Paper [6] shows that maximum power point tracking allows to significantly increase the value of harvested energy. Thus, the development of renewable energetics impacts substantially the research and design of new power converters [7], which should have highly flexible and controllable dynamics [8] for integration into the Smart Grids [9]. Thus, the circuits of switching converters should make it possible to control increasing and decreasing output current and voltage, relatively to the input values.

This requirement leads to the application of the buck-boost converters [10]. Paper [11] describes the modes and main problems of modeling the buck-boost converter circuit.

For the purposes of design of the solar energy devices, the DC-DC converters can be modeled numerically [12], considering the exact ripple values. But the envelopes of switching circuit transient processes can be obtained using simpler analytical methods based on the state-space averaging [13].

A significant part of the described energy-generating devices are characterized by the constant current source behavior [14].

The main problem considered is the maximum power point (MPP) analysis and control of the current sources with nonlinear parasitic elements [6]. The problem occurs because of the implicit equations and nonlinear dynamics, which characterize the dependences between currents and voltages of such sources.

Thus, the purpose of this study is determination of the MPP mode parameters for the constant current source with nonlinear parasitic elements. For the research, the solar module is selected as one of the most useful devices characterized by the constant current source behavior under constant irradiance and temperature.

The aim has been achieved by analyzing the current source MPP and control problem, with the following main tasks being set in the research:

1) the analysis of the techniques, which ensure the capabilities for the increase in the energy harvested from a current source with nonlinear parasitic elements.

2) the development of a mathematical model which describes the mode parameters of the maximum power point for the current source formed by a solar module with nonlinear parasitic elements.

3) the experimental research of the maximum power point mode determined by the developed model, which is realized with the DC/DC converter circuit.

I. THE ANALYSIS OF TECHNIQUES FOR INCREASING SOURCE ENERGY HARVESTING EFFICIENCY

First, let us consider operation of a source [7] with constant internal resistance R_0. It is known that the MPP mode is the matched mode for the direct source-to-load connection (Fig. 1):

$$R_{ld} = R_0; \quad U_{ld0} = \frac{E}{2}; \quad P_{ld0} = \frac{E^2}{4R_0}, \tag{1}$$

where E is the electromotive force, R_{ld} is the load resistance, U_{ld0} and P_{ld0} are the direct connection load voltage and power, respectively.

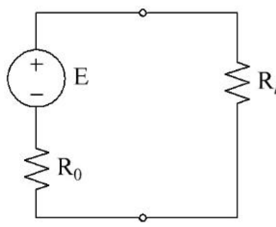

Fig. 1. Source to load connection circuit.

At arbitrary load resistance $R_{ld} = kR_0$, the load voltage U_{ldk} and load power P_{ldk} are as follows

$$U_{ldk} = E\frac{k}{1+k}; \quad P_{ldk} = \frac{U_{ld}^2}{kR_0}, \tag{2}$$

where $k = R_{ld} / R_0$.

To emit maximum power P_{ld0} for the arbitrary load R_{ld}, the equality (3) must be satisfied.

$$P_{ld0} = P_{ldk} \tag{3}$$

Thus, to satisfy the equivalent load matching condition (1), the corresponding arbitrary load voltage U_{ldk} should be determined as (4):

$$U_{ldk} = \frac{E}{2}\sqrt{k}, \tag{4}$$

and the ratio between the matched load voltage (4) and load direct connection voltage (2) is as follows:

$$\frac{\frac{E}{2}\sqrt{k}}{E\frac{k}{1+k}} = \frac{1+k}{2\sqrt{k}}. \tag{5}$$

The required load voltage (4) at $0 < k < 1$ is always higher than the direct connection voltage U_{ldk}.

Let us consider some simple ways to obtain load voltage, which is higher than direct load connection voltage. According to [8], the impulse operation mode is analyzed for the connection of constant voltage source and capacitor.

Fig.2 shows the impulse operation mode of a source with parasitic resistance.

Here, the load voltage is applied during the time t_1. According to [7], it is ensured that after completing the envelope transient process, when the load voltage changes periodically with the switch commutation, at $R_{ld} = R_0$, the equation (6) is satisfied:

$$U_{ld} = U_C = \frac{E}{2} + Q\frac{E}{2}e^{-\frac{t}{\tau}}, \tag{6}$$

where t is the time; τ is the constant time of the transient process, $\tau = \frac{R_0 C}{2}$.

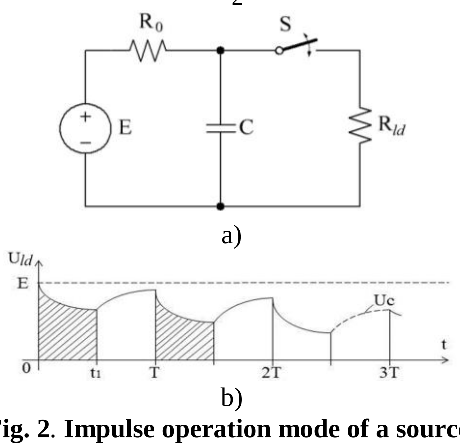

Fig. 2. Impulse operation mode of a source.

The constant $Q \approx 0.5$ is obtained at different values of the switching period T ($T = \frac{\tau}{5}$, $T = \frac{\tau}{10}$, $T = \frac{\tau}{25}$), and $t_1 = \frac{T}{2}$, from the solutions of circuit (Fig. 2) differential equation for the ON and OFF switch states [15].

Then, in a circuit without a capacitor, the energy for the period is as follows:

$$W = 0.25\frac{E^2T}{R_{ld}}, \quad (7)$$

and in a circuit with a capacitor:

$$W = 0.278\frac{E^2T}{R_{ld}}. \quad (8)$$

The increase in the energy is about 11%. It is shown that at $R_{ld} = 0.5R_0$, the increase in the energy will be 22%. Thus, the impulse operation mode of the source of constant voltage with a capacitor is more effective at $R_{ld} \le R_0$, but when $R_{ld} > R_0$, a “pause” in the load voltage leads to a decrease in the energy compared to the circuit in Fig. 1.

The analyzed circuit (Fig. 2) shows that the switching transient processes allow obtaining the increased average voltage $U_{ldAV} > U_{ld0}$ with $R_{ld} \le R_0$. Such an increase in the power transmitted from source to load, is obtained regarding to the capacitor, which acts as a voltage source in the transient processes.

A further improvement of the circuit with a capacitor is discussed in [16]. Such an approach is also used in different circuits of the buck converters [17]. The results presented in [18], show that it is possible to obtain a higher average value of the load voltage U_{ld} during the period T and approximate it to $U_{ldAV} \approx E$, but without an excessive increase. The above mentioned switching converter circuits are useful in the cases of small load resistance values [18].

A similar approach can be used to obtain the average load voltage higher than the input source voltage. In Fig. 3, the circuit allows obtaining $U_{ldAV} > E$, according to [7].

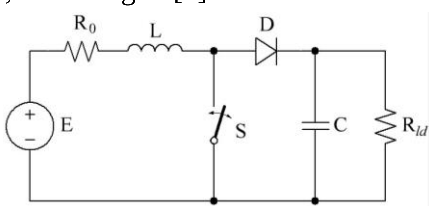

Fig. 3. The boost converter circuit.

The inductor current $i_L(t)$ is determined using expression (9):

$$i_L(t) = \frac{1}{L}\int_0^t u_L(\tau)\,d\tau. \quad (9)$$

If the switch S is closed, then inductor voltage is positive $u_L(t)$, and the current $i_L(t)$ increases. When the switch is open, then the inductor charges the capacitor C, acting as a current source, according to (9). The capacitor voltage $u_C(t)$ is given by (10):

$$u_C(t) = \frac{1}{C}\int_0^t i_C(\tau)\,d\tau \approx$$

$$\approx \frac{1-D}{C}\int_0^t i_L(\tau)\,d\tau - \frac{1}{R_{ld}C}\int_0^t u_C(\tau)\,d\tau, \quad (10)$$

where D is the switching duty cycle.

Thus, the capacitor voltage increases during the time range $t \in [T_1; T_2]$, if the inequality (11) is satisfied:

$$\frac{1-D}{C}\int_{T_1}^{T_2} i_L(\tau)\,d\tau > \frac{1}{R_{ld}C}\int_{T_1}^{T_2} u_C(\tau)\,d\tau, \quad (11)$$

such as

$$u_C(T_2) = u_C(T_1) + \frac{1-D}{C}\int_{T_1}^{T_2} i_L(\tau)\,d\tau - \frac{1}{R_{ld}C}\int_{T_1}^{T_2} u_C(\tau)\,d\tau.$$

The expressions (10) and (11) show that the maximum load voltage value is not limited by the source electromotive force value *E*, regarding to the inductor *L*, which acts as a current source that charges the capacitor *C*.

The development of switching converter circuits based on the described approach is shown in [4]. Boost converter circuits [19] allow significant increases of load voltage relatively to the input source voltage ($U_{ld} > E$). It is shown that the most efficient applications of boost converter circuits can be obtained for high values of load resistance $R_{ld} >> R_0$.

Now, let’s consider operation of the circuit with a source of constant voltage, which contains nonlinear internal resistance (Fig. 4) [5].

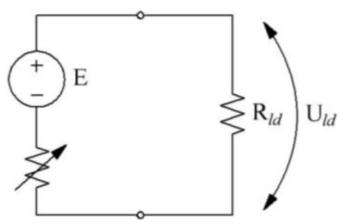

Fig. 4. The circuit with the source and nonlinear internal resistance.

It is noteworthy, that the presence of nonlinear elements can significantly increase the complexity of the DC-DC converter circuit dynamics [20].

The load power is given by (12):

$$P_{ld} = U_{ld} I = (E - U_{NE}) I , \qquad (12)$$

where I is the current, and U_{NE} is the nonlinear element voltage. According to [21], the DC-DC converter load power can be described by the differential equation:

$$\frac{dP_{ld}}{dI} = E - U_{NE} - I \frac{dU_{NE}}{dI} = 0 .$$

Thus, taking into account that $E - U_{NE} = IR_{ld}$, we have come to:

$$IR_{ld} - I \frac{dU_{NE}}{dI} = 0 ; \quad R_{ld} = R_{dif} . \qquad (13)$$

Therefore, the current value in the circuit is determined by the value of static resistance of the nonlinear element R_{st} and the load resistance value R_{ld}, which in its turn is equal to the differential resistance R_{dif} of the nonlinear element for obtaining $P_{ld} = \max$.

The performed analysis of the energy harvesting improvement techniques shows that the current source maximum power point tracking requires accurate control of switching converter equivalent resistance.

In case of nonlinear parasitic elements, the differential resistance should be used to characterize the source and converter operating point.

For the described sources, the DC/DC converters should combine the capabilities for increasing and decreasing the output voltages relatively to the input voltage with control of equivalent differential resistance.

II. ESTIMATION OF SOLAR MODULE LOAD MATCHING CONDITIONS

The equivalent circuit of considered solar battery is shown in Fig. 5. The circuit (Fig. 5) includes the current source and diode with nonlinear equivalent resistance.

Thus, the differential resistance R_{dif} estimation is required for the analysis of the load matching for such a source.

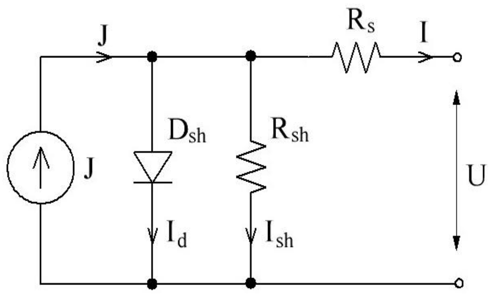

Fig. 5. Solar battery equivalent circuit.

Let us analyze the effect of the solar battery parameters on the equivalent differential resistance R_{dif} that occurs when the solar panel is connected into the circuit with current I and voltage U, which depend on the load characteristics.

$$I = J - I_d - I_{sh} = J - I_0 \left(e^{\frac{U+IR_s}{U_T}} - 1 \right) - \frac{U + IR_s}{R_{sh}} , \quad (14)$$

where J is the internal source of current, which depends on the solar panel irradiance, I_0 is the reverse saturation current, U_T is the thermal voltage, R_s and R_{sh} are equivalent series and parallel (shunt) resistances, respectively. The expression (14) can be represented as follows:

$$I \left(\frac{R_{sh} + R_s}{R_{sh}} \right) = J - I_0 \left(e^{\frac{U+IR_s}{U_T}} - 1 \right) - \frac{U}{R_{sh}} ,$$

where the ratio of resistances is obtained from the differentials:

$$dI \left(\frac{R_{sh} + R_s}{R_{sh}} \right) = -I_0 d \left(e^{\frac{U+IR_s}{U_T}} - 1 \right) - \frac{dU}{R_{sh}} ,$$

$$\frac{R_{sh} + R_s}{R_{sh}} = -I_0 \frac{d \left(e^{\frac{U+IR_s}{U_T}} - 1 \right)}{dI} - \frac{1}{R_{sh}} \cdot \frac{dU}{dI} ,$$

or

$$\frac{R_{sh} + R_s}{R_{sh}} = -\frac{I_0}{U_T} \left(\frac{dU}{dI} + R_s \right) e^{\frac{U+IR_s}{U_T}} - \frac{1}{R_{sh}} \cdot \frac{dU}{dI} .$$

Thus:

$$\frac{dU}{dI} \left(\frac{I_0 R_{sh}}{U_T (R_{sh} + R_s)} e^{\frac{U+IR_s}{U_T}} + \frac{1}{R_{sh} + R_s} \right) =$$

$$= -1 - \frac{I_0 R_{sh} R_s}{U_T (R_{sh} + R_s)} e^{\frac{U+IR_s}{U_T}}$$

Finally, expression (15) is obtained as follows:

$$R_{dif} = \frac{dU}{dI} = -\frac{1 + \frac{I_0 R_{sh} R_s}{U_T\left(R_{sh}+R_s\right)} e^{\frac{U+IR_s}{U_T}}}{\frac{1}{R_{sh}+R_s} + \frac{I_0 R_{sh}}{U_T\left(R_{sh}+R_s\right)} e^{\frac{U+IR_s}{U_T}}}. \quad (15)$$

The calculation results have been verified for a Jinko JKM 260PP-60 solar panel with the following characteristics: U_{oc}=38,1 V, I_{sc}=8,98 A, I_L=8,99 A, U_{opt} =31,1 V, I_{opt}=8,37 A, R_s=0,3 Ohm, R_{sh}=162 Ohm, t°=25°C.

Taking into account the internal diode voltage U_d, thermal voltage U_T and the current I_0:

$$U_d = U + IR_s = 31{,}1 + 0{,}3 \cdot 8{,}37 = 33{,}61\,V,$$

$$U_T = \frac{k_B T}{q} n I N_c = \frac{1{,}3806 \cdot 10^{-23} \cdot 298 \cdot 0{,}95 \cdot 60}{1{,}6022 \cdot 10^{-19}} = 1{,}4637\,V,$$

$$I_0 = 4{,}6715 \cdot 10^{-11}\,A,$$

we have obtained $R_{dif} = -3{,}71$ Ohm, whereas the optimal load resistance indicated in the datasheet is $R_{opt} = U_{opt} / I_{opt} = 3{,}71$ Ohm. It means that the results coincide. The calculation for the simplified model $\left(R_{sh} = \infty\right)$ is the following:

$$R_{dif} = -\frac{U_T}{I_0 e^{\frac{U+IR_s}{U_T}}} - R_s = -3{,}64\,\text{Ohm} \quad (16)$$

Here, the result is close to that obtained for the refined model (the difference is ≈2%). It is necessary to mention that the value of resistance R_{dif} calculated according to its external characteristics differs from the internal resistance of the panel in its sign.

In a real-life situation $R_{ld} \neq R_{Lopt}$, that is why the following three cases are possible:

1) increase ($R_{ld} > R_{Lopt}$);
2) decrease ($R_{ld} < R_{Lopt}$);
3) increase/decrease ($R_{ld} \lesseqgtr R_{Lopt}$).

For a buck-boost converter [6] we obtain

$$k = \frac{D}{1-D}, \quad R_{opt} = k^2 R_{ld}$$

If illumination W=const, t°=const, then $D = \frac{k}{1+k}$ (without tracking), but in practice t°=var, illumination W=var, that is why the duty cycle must depend on the solar panel parameters.

III. THE MODEL AND REALIZATION OF SOLAR MODULE MPP MODE

As is shown above, to obtain the maximum power point (MPP) mode, the switching converter should ensure the capabilities for increasing and decreasing the output voltage relatively to the voltage of the solar panel.

Thus, this study is performed for the connection of the solar module and buck-boost converter circuit.

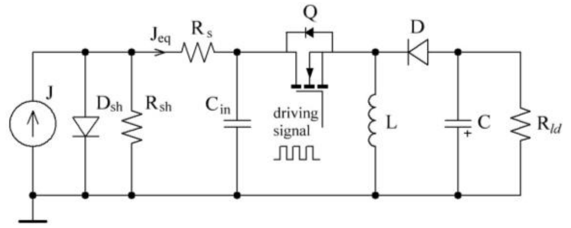

Fig. 6. Buck-boost converter based solar optimizer.

In Fig.6, the solar panel model is included in accordance with Fig. 5. For a clearer representation, the current source J with parallel diode D_{sh} and parallel resistance R_{sh} can be replaced by the equivalent current source J_{eq}, as shown in Fig.7.

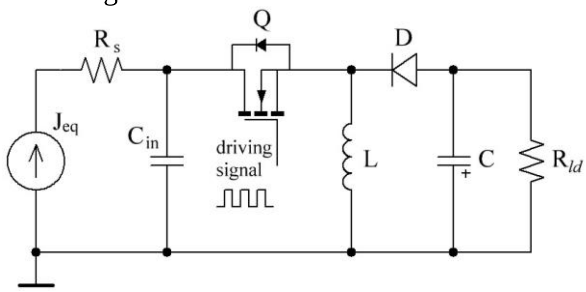

Fig. 7. The circuit with input equivalent current source.

The equivalent current source is determined in accordance with the photovoltaic panel model (14), which can be represented as follows (17):

$$i = J - I_0\left(e^{\frac{U_d}{U_T}} - 1\right) - \frac{U_d}{R_{sh}}, \quad (17)$$

where U_d is the voltage of diode D_{sh}.

The analysis of expression (17) shows that the parameters J, I_0, U_T and R_{sh} do not depend on the voltage U_d.

Thus, the maximum power, which can be produced by the equivalent current source of solar panel, is determined from the characteristic (18):

$$P(U_d) = i \cdot U_d =$$

$$= (J + I_0)U_d - \frac{1}{R_{sh}}U_d^2 - I_0 U_d e^{\frac{U_d}{U_T}} \qquad (18)$$

Taking into account the estimated solar panel model parameters, the maximum internal source power point can be obtained using the analytical or numeric methods.

However, the value of U_d voltage depends both on current i and voltage u of the solar panel:

$$U_d = u + iR_s . \qquad (19)$$

Expressions (18) and (19) show that different load-dependent voltages u and currents i can ensure the same operating point of equivalent internal current source.

The solar panel voltage u and current i can take any values that match the conditions given by (17) and (19). Whereas, the exact values of u and i are determined by the external DC-DC converter circuit which is connected to solar panel.

Thus, the problem of maximum power point tracking (MPPT) can be divided into two parts:

1) tracking the maximum internal power point of equivalent current source in order to obtain the best capabilities of the energy harvesting, using the analytical expression (18);

2) tracking the maximum external power point due to the load and energy transition criteria.

To determine the best possible MPP of the solar panel with buck-boost converter (Fig.7), let us analyze two equivalent circuits, for ON and OFF switch states, respectively.

The ON switch state circuit is shown in Fig.8.

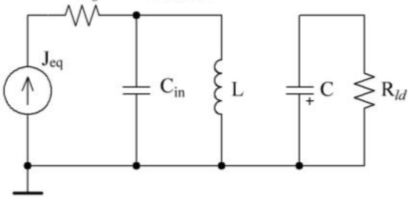

Fig. 8. **The ON switch state circuit.**

Here, the inductor current is increased by the impact of the solar panel and capacitor C_{in}. The output capacitor C discharges to the load resistance.

Fig.9 shows the OFF switch state circuit.

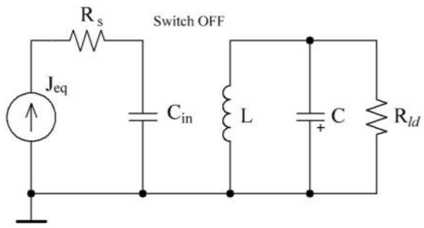

Fig. 9. **The switch OFF state circuit.**

In the OFF switch state, the input capacitor is charged by the solar panel, whereas the inductance L charges the output capacitor and increases the voltage on load resistance R_{ld} .

To analyze the solar panel maximum power point that can be ensured by the buck-boost converter, it is convenient to use the state-space averaging theory [5].

The ON switch state circuit is described by expression (20):

$$\begin{bmatrix} C_{in}\frac{du}{dt} \\ L\frac{di_L}{dt} \\ C\frac{du_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{R_{ld}} \end{bmatrix} \times \begin{bmatrix} u \\ i_L \\ u_C \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} i \\ 0 \\ 0 \end{bmatrix}, \qquad (20)$$

where u is the solar panel voltage, i is the solar panel current generated by equivalent source J_{eq}, i_L is the inductor current, and u_C is the load voltage.

Respectively, expression (21) describes the switch OFF state.

$$\begin{bmatrix} C_{in}\frac{du}{dt} \\ L\frac{di_L}{dt} \\ C\frac{du_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & \frac{1}{R_{ld}} \end{bmatrix} \times \begin{bmatrix} u \\ i_L \\ u_C \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} i \\ 0 \\ 0 \end{bmatrix}, \qquad (21)$$

The averaged model (22) is obtained by addition of the circuit matrices from (20) and (21) with the weight coefficients that correspond to the switch duty cycle D:

$$\begin{bmatrix} C_{in}\frac{du}{dt} \\ L\frac{di_L}{dt} \\ C\frac{du_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -D & 0 \\ D & 0 & (1-D) \\ 0 & (1-D) & \frac{1}{R_{ld}} \end{bmatrix} \times \begin{bmatrix} u \\ i_L \\ u_C \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} i \\ 0 \\ 0 \end{bmatrix}, \qquad (22)$$

The solar panel behavior is described by the above equations (17) and (19). In the steady state, the values of u, i_L, u_C remain unchanged, and the derivatives are zero (23).

$$\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0 & -D & 0\\ D & 0 & (1-D)\\ 0 & (1-D) & \frac{1}{R_{ld}}\end{bmatrix}\times\begin{bmatrix}u\\ i_L\\ u_C\end{bmatrix}+\begin{bmatrix}1\\0\\0\end{bmatrix}\times\begin{bmatrix}i\\0\\0\end{bmatrix}, \quad (23)$$

The expression (23) allows obtaining the voltages and currents of the circuit shown in Fig.7, upon the completion of the transient processes.

Taking into account the maximum power point current J_{MPP} and voltage U_{dMPP} of the internal equivalent current source, we can describe the circuit (Fig.7) by the following system of equations:

$$\begin{cases}-Di_L+J_{MPP}=0,\\ Du+(1-D)\,u_C=0,\\ (1-D)i_L+\dfrac{u_C}{R_{ld}}=0,\\ u+R_s\cdot J_{MPP}=U_{dMPP}.\end{cases} \quad (24)$$

The maximum power point J_{MPP} and voltage U_{dMPP} of the internal equivalent source can be estimated by the parameters of the solar cells with expressions (18) and (19). Thus, the system (24) allows obtaining the duty cycle D, which ensures the MPP of the internal equivalent current source:

$$\begin{cases}i_L=\dfrac{1}{D}J_{MPP},\\ u=U_{dMPP}-R_s\cdot J_{MPP},\\ u_C=-\dfrac{D(U_{dMPP}-R_sJ_{MPP})}{(1-D)},\\ \dfrac{1-D}{D}J_{MPP}-\dfrac{D(U_{dMPP}-R_sJ_{MPP})}{R_{ld}(1-D)}=0.\end{cases} \quad (25)$$

Since J_{MPP} and U_{dMPP} are estimated from the physical parameters of the solar cells, we can determine the buck-boost converter duty cycle which ensures the maximum power point of the internal equivalent current source. For this purpose, let us solve the last equation of system (25), which looks like as (26):

$$\frac{(1-D)^2}{D^2}J_{MPP}-\frac{(U_{dMPP}-R_sJ_{MPP})}{R_{ld}}=0\,, \quad (26)$$

or

$$aD^2+bD+c=0\,, \quad (27)$$

where

$$a=J_{MPP}-\frac{U_{dMPP}-R_sJ_{MPP}}{R_{ld}}\,,$$

$$b=-2J_{MPP}\,,$$

$$c=J_{MPP}\,.$$

Thus, the values of the duty cycle, which ensure the internal equivalent source MPP, are the following:

$$D_1=\frac{-b+\sqrt{b^2-4ac}}{2a}\,, \quad (28)$$

$$D_2=\frac{-b-\sqrt{b^2-4ac}}{2a}\,, \quad (29)$$

The formulae from (25) to (29) describe the mathematical model, which determines the buck-boost converter mode that ensures the maximum possible solar module of the energy harvesting taking into account the internal equivalent current source characteristics.

It should be noted, that in some cases, the load MPP may not match the internal source MPP exactly. Then, the MPP value can be increased additionally, by improving the converter circuit or control algorithm.

If there are limitations to the parameters of the DC-DC converter elements (caused by the PCB size or price conditions), then the load MPP can be tracked with respect to the dependencies between i, u and u_C which can be obtained from (23) and (25).

Table 2

The maximum power point parameters

R, Ohm	t^0C	*S*,	MPP Duty cycle D		
			Experiment	Model (28), (29)	Relative Error (30),
2.65	50	600	0.415	0.408	1.7%
6.25	33	600	0.50	0.505	1%
10.6	46	600	0.586	0.578	1.4%
21.3	42	600	0.66	0.658	0.3%

Table 2 shows the required MPP duty cycles, which can be obtained from the solar panel irradiance S and temperature t°. The modeling accuracy is estimated by the relative error (30):

$$\varepsilon=100\cdot\left|\mathrm{D_{exp.}}-\mathrm{D_{model}}\right|/\,\mathrm{D_{exp.}}, \quad (30)$$

where $\mathrm{D_{exp.}}$ is the experimental MPP duty cycle, $\mathrm{D_{model}}$ is the modeled MPP duty cycle value. The maximum error of modeled MPP duty cycle is 1.7% relatively to the experimental value.

The obtained results mean that the MPPT procedure can be realized based on the solar panel internal equivalent current source characteristics, which depend on the irradiance and temperature values.

CONCLUSIONS

The harvesting of the constant current source energy is one of the most important purposes in the modern energetics. Especially, it appears in renewable energy and smart grids where the requirements to energy losses and load matching are strict.

In this paper, different architectures of switching converters are considered for the purpose of increasing the energy production of the constant current source. The load matching analysis is performed for a solar panel that functions as a constant current source with nonlinear parasitic elements under constant irradiance and temperature.

In accordance with the obtained results, the buck-boost converter circuit is selected for the improvement of the solar panel energy production.

The aim of the MPP mode determination is achieved by estimating the internal equivalent current source parameters for the constant current source with nonlinear parasitic elements.

The main obtained results are described as follows:

1) The analysis is performed for the techniques, which increase the energy harvested from a current source with nonlinear parasitic elements. As a result, the differential resistance estimation is obtained for the solar module as a constant current source with nonlinear parasitic elements.

2) The new mathematical model is proposed for the determination of the maximum power point mode parameters for the nonlinear current source formed by a solar module. The proposed mathematical model differs from the known ones in terms of internal equivalent current source parameters for the solar module. The model allows developing the MPP tracking techniques taking into account the measured solar module temperature and irradiance values.

3) The experimental research is performed for the solar module maximum power point. The MPP mode is controlled by buck-boost converter circuit with the duty cycle determined from the proposed model. The performed experiments show that the proposed model ensures the values of the buck-boost converter duty cycle, which almost match the duty cycle obtained experimentally for the MPP mode. The maximum relative error between theoretical and experimental values of the buck-boost converter duty cycle is less than 1.7%.

Thus, the obtained results can be used for the maximum power point tracking of the current sources formed by the solar modules.

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