The Sufficient Condition for Ensuring the Reliability of Perception of the Steganographic Message in the Walsh-Hadamard Transform Domain

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Abstract. In view of the high compatibility of the Walsh-Hadamard transform to the architecture of modern computing facilities, it is sensible to use it in methods of steganography and steganalysis. Now, there are some efficient steganographic methods that use the Walsh-Hadamard transform domain for information embedding presented in the literature. It is known that to ensure the reliability of the steganographic message perception, information should be embedded in its high-frequency components; nevertheless, the issues of the correspondence of the Walsh-Hadamard transformants to the frequency components of the original matrix of the cover image are poorly researched. The purpose of this paper is to develop a formal sufficient condition for ensuring the reliability of perception of steganographic messages in the Walsh-Hadamard transform domain. This purpose was achieved by establishing the relationship between the Walsh-Hadamard transformants and the discrete cosine transform transformants, which was theoretically substantiated in two ways, and also experimentally confirmed. Based on the established relationship between the Walsh-Hadamard transform and the discrete cosine transform, as well as the components of the singular value decomposition of the corresponding matrices, a sufficient condition has been developed to ensure the reliability of the perception of steganographic messages in the Walsh-Hadamard transform domain. The sufficient condition consists in the fact that after embedding of additional information using any steganographic method, those Walsh-Hadamard transformants that correspond to the singular triples of the matrix corresponding to small (smallest) in value singular numbers, should change.

***Keywords*:** steganographic communication channel, steganographic message perception reliability, digital image, Walsh-Hadamard transform, sequence, discrete cosine transform, singular value decomposition of a matrix.
DOI: https://doi.org/10.52254/1857-0070.2022.2-54.08
UDC: 004.056

Condiție suficientă pentru asigurarea unei percepții fiabile a mesajului stegano în domeniul transformării Walsh-Adamar

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Rezumat. Având în vedere corespunderea înaltă a transformării Walsh-Adamar cu arhitectura instalațiilor de calcul moderne, un interes practic prezintă utilizarea acesteia în metodele moderne de steganografie și steganoanaliză. Actualmente, literatura de specialitate prezintă metode steganografice eficiente care utilizează domeniul transformării Walsh-Adamar pentru încorporarea informațiilor. Este cunoscut faptul că, pentru o percepție fiabilă a unui mesaj de tip stegano, încorporarea informațiilor ar trebui să aibă loc în componentele de înaltă frecvență ale acestuia, dar problemele legate de potrivirea transformărilor Walsh-Adamar cu componentele de frecvență ale matricei originale sunt slab investigate. Scopul acestei lucrări este de a formula o condiție formală suficientă pentru fiabilitatea percepției mesajelor de tip stegano în domeniul transformării Walsh-Adamar. Acest obiectiv a fost atins prin stabilirea relației dintre transformările Walsh-Adamar și transformările discrete ale cosinusului, care a fost argumentată teoretic în două moduri și, de asemenea, verificată experimental. Pe baza relației stabilite între transformata Walsh-Adamar și transformata discretă a cosinusului, precum și a componentelor descompunerii singulare a matricelor corespunzătoare, a fost formată o condiție suficientă pentru fiabilitatea percepției mesajelor steganografice în regiunea transformării Walsh-Adamar, care constă în faptul că, după introducerea de informații suplimentare prin orice metodă steganografică, acele transformări Walsh-Adamar, care corespund triplurilor singulare ale matricei corespunzătoare matricelor mici.

***Cuvinte-cheie*:** canal de comunicare steganografică, fiabilitatea percepției mesajului steganografic, imagine digitală, transformată Walsh-Adamar, frecvență, transformată cosinus discretă, descompunerea matricei singulare.

Достаточное условие обеспечения надежности восприятия стеганосообщения в области преобразования Уолша-Адамара

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Аннотация. В виду высокого соответствия преобразования Уолша-Адамара архитектуре современных вычислительных средств практический интерес представляет его использование в современных методах стеганографии и стеганоанализа. В настоящий момент в литературе представлены эффективные стеганографические методы, использующие область преобразований Уолша-Адамара для внедрения информации. Известно, что для обеспечения надежности восприятия стеганосообщения, внедрение информации должно происходить в его высокочастотные составляющие, тем не менее вопросы соответствия трансформант Уолша-Адамара частотным составляющим исходной матрицы являются малоисследованными. Целью настоящей статьи является формирование формального достаточного условия обеспечения надежности восприятия стеганосообщений в области преобразования Уолша-Адамара. Поставленная цель была достигнута за счет установления взаимосвязи между трансформантами Уолша-Адамара и трансформантами дискретного косинусного преобразования, которая была теоретически обоснована двумя способами, а также подтверждена экспериментально. На основе установленной взаимосвязи преобразования Уолша-Адамара и дискретного косинусного преобразования, а также составляющих сингулярного разложения соответствующих матриц сформировано достаточное условие обеспечения надежности восприятия стеганосообщений в области преобразования Уолша-Адамара, которое состоит в том, что после внедрения дополнительной информации с помощью любого стеганографического метода, изменению должны подвергнуться те трансформанты Уолша-Адамара, которые соответствуют сингулярным тройкам матрицы, отвечающим малым (наименьшим) по значению сингулярным числам. Полученные в работе результаты могут служить основой разработки новых стеганоалагоритмов, а также методов стеганоанализа, основанных на применении свойств области преобразования Уолша-Адамара, при этом стеганопреобразование возможно проводить в любой области контейнера (пространственной, преобразования).

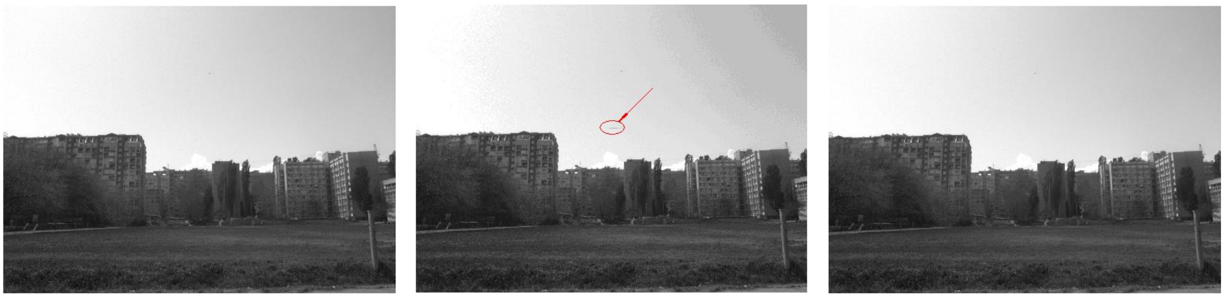
Ключевые слова: **с**теганографический канал связи, надежность восприятия стеганосообщения, цифровое изображение, преобразование Уолша-Адамара, частость, дискретное косинусное преобразование, сингулярное разложение матрицы.

I. INTRODUCTION AND STATEMENT OF THE PROBLEM

In modern complex information security systems, steganographic methods are increasingly used, which can not only protect information from possible reading by intruders, but also hide the very fact of its transfer from unauthenticated users. At present, the theory and practice of steganography are rapidly developing [1], as well as many steganographic methods for protecting information are being developed and improved. These methods are based on a wide variety of mathematical constructions, starting from the classical LSB (Least Significant Bit) method, which ensures the effective embedding of additional information into a container, most often in the spatial / temporal domain [2], ending with methods using the container transform domains [3-7] (discrete cosine transform (DCT), discrete wavelet transform, Fourier transform, domains of various matrix decompositions (singular value, spectral, etc.)), as well as the Walsh-Hadamard transform [8-9]) for the embedding of additional information.

The development of steganography is accompanied by the development of steganalysis methods, the main task of which is to reveal the presence of embedded information in information content [10], which makes it very difficult in modern conditions to meet the requirement of reliable concealment of the fact of organizing a covert communication channel. Because of this, for modern steganographic methods, the relevance of the requirement for the reliability of perception of the steganographic message (ensuring the absence of visual differences between the container and the steganographic message) increases when organizing a covert communication channel [11]. Today, we are talking not only about the appearance of obvious artifacts in the steganographic message but also about the appearance of any differences that may not be perceived as changes in the original image without container availability: a change in brightness, color shades, slight smoothing of contours, etc., however, become definable by direct comparison with the container. It should be noted that exactly the guaranteed reliability of the steganographic message perception makes the LSB method so "tenacious" and widely used, despite its well-known shortcomings, in particular, its vulnerability to attacks against an embedded message.

Quantitative estimation of the reliability of steganographic message perception today is carried out using standard differential distortion indicators [12]: MSE (*Mean Square Error*), SNR (*Signal to Noise Ratio*), PSNR (*Peak Signal to Noise Ratio*), although they are often biased since they cannot fully consider the features of human vision. This possible bias is especially pronounced in cases when changes in digital content occur in its local (small in size) areas. Here, a formal quantitative assessment of distortion can be acceptable in the conditions of artifacts presence (Fig. 1, b)), while the values of the differential indicators in the absence of obvious visual distortions can be low (Fig. 1, c)). As a container in this paper, we consider a digital image (DI) or a digital video frame, which, under the conditions of the considered problem, do not fundamentally differ in anything in the sense that the formal representation of each is one (image in grayscale) or several (color image) two-dimensional matrices, and any changes to the original content, including the steganographic transformation, can be considered as a perturbation of the corresponding matrix or matrices.



a) b) c)

Fig. 1 Illustration of the imperfection of the differential indicators for assessing the visual distortions of the DI: a) initial DI; b) distorted DI (PSNR = 52 dB); c) distorted using Gaussian noise DI (PSNR = 28 dB)

In the existing steganographic methods, the restrictions on the area of their applicability often occur precisely because for some DI containers the reliability of perception of the generated steganographic message can be violated. This most often occurs due to the fact that when developing a steganographic algorithm, sufficient formal conditions for the reliability of perception are not considered (or such formal conditions have not been found in the used domain of the container), and the assessment of the reliability of perception is done “a posteriori”. This situation does not fully allow the use of a random container and that is a drawback of the corresponding methods. It should be noted that the formal mathematical apparatus for ensuring the reliability of perception of steganographic messages have been developed in relation to steganography not so long ago [13], before that everything was limited to considering the peculiarities of human vision: changes are more strongly perceived in areas of digital insight with small differences in brightness values, or background, which corresponds to low-frequency components. In [13], using the mathematical apparatus of the perturbation theory and matrix analysis, sufficient conditions were obtained to ensure the reliability of perception of the generated steganographic message, the use of which made it possible to “a priori” ensure/check the fulfillment of this property in the domain of the singular value (or spectral) decomposition of the corresponding matrix. Considering the possible provision of the uniqueness of such decompositions, the obtained sufficient conditions can be applied regardless of the chosen steganographic transformation domain (spatial, frequency, various decompositions); however, in practice, it is still advisable to obtain such sufficient conditions in each of the domains of the DI, where it is possible to carry out the steganographic transformation which will make it possible to avoid the transformation to the domain of the singular value (spectral) decomposition of the matrix, in order to analyze the degree of ensuring the reliability of perception of the steganographic message if the steganographic transformation is performed in another domain of the container.

In view of the high computational efficiency, as well as compliance with the architectural features of modern processors, steganographic methods based on the use of the Walsh-Hadamard transform domain are promising for modern information security systems. In particular, in [8], it was proposed to embed information into a container by modifying the matrices of the two-dimensional Walsh-Hadamard transform of the original image. In this method, it is proposed to modify all the elements of the resulting matrices of the two-dimensional Walsh-Hadamard transform of the original image, except for the elements of their first column, which in the general case does not guarantee the reliability of perception of the steganographic message, which will be shown below in this paper. Today in the literature there is no rigorous substantiation of the influence of modification of one or another two-dimensional Walsh-Hadamard transformant on the distortions that arise in the container image. This circumstance significantly complicates the development of promising steganographic methods based on the use of the properties of the Walsh-Hadamard transform.

The *purpose* of this paper is to obtain a formal sufficient condition for ensuring the reliability of perception of steganographic messages in the Walsh-Hadamard transform domain.

To achieve this purpose, it is necessary to solve the following tasks:

1. Finding the relationship between the Walsh-Hadamard transform domain and the frequency domain of a DI.
2. Finding the relationship between the Walsh-Hadamard transform and the components of the singular value decomposition of the corresponding matrix.
3. Experimental confirmation of the obtained theoretical results.

II. RELATIONSHIP BETWEEN THE WALSH-HADAMARD TRANSFORM AND THE DISCRETE COSINE TRANSFORM

The basic transform used in many modern graphic compression algorithms, as well as steganographic algorithms, is the discrete cosine transform (DCT), which can be written in matrix form

$$S = C_N X C_N^T, \tag{1}$$

where X is a fragment of the original image of size $N \times N$, C is the $N \times N$ matrix of discrete cosine transform, the elements $C(i, j)$, $i, j = 0,1,...,N-1$ of which are calculated in accordance with the formula

$$C(i,j) = \begin{cases} \dfrac{1}{\sqrt{N}}, & \text{if } i = 0; \\ \sqrt{\dfrac{2}{N}}\cos(2j+1)\cdot i \cdot \pi, & \text{if } i > 0. \end{cases} \tag{2}$$

The resulting DCT matrix S (1) has the following distribution of frequency components, schematically shown in Fig. 2 [14].

It is well known [10] that the modification of the high-frequency components (located in the lower right corner of the DCT transformants matrix) leads to the least visual distortions of the original image, while the modification of the middle frequencies corresponds to larger distortions. The greatest distortions of the original image occur when modifying the low-frequency components (upper left corner) of the DCT transformants matrix.

Another type of transform which is often used in cryptography [15] and signal theory [16] is the discrete Walsh-Hadamard transform, which in matrix form can be written as the following matrix product

$$V = YH_N, \tag{3}$$

where H_N is the Walsh-Hadamard matrix of order $N = 2^k$, which can be constructed in accordance with Sylvester's construction

$$H_{2^k} = \begin{bmatrix} H_{2^{k-1}} & H_{2^{k-1}} \\ H_{2^{k-1}} & -H_{2^{k-1}} \end{bmatrix}, \tag{4}$$

where $H_1 = 1$, and Y is a row vector of length N.

Expression (3) is the one-dimensional Walsh-Hadamard transform, while in graphic information processing applications, in particular, for steganography applications, the two-dimensional discrete Walsh-Hadamard transform is used, which is defined as

$$W = H'_N X H'^T_N, \tag{5}$$

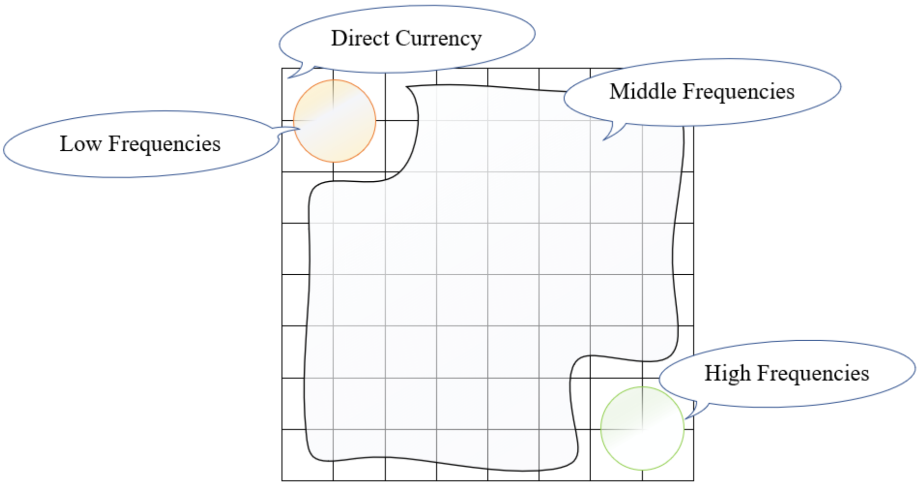


Fig. 2. Distribution of frequency components in DCT transformants

where $H'_N = \frac{1}{2^{\frac{\log_2 N}{2}}} H_N = \frac{1}{\sqrt{N}} H_N$, and X is the matrix of size $N \times N$.

Let us find the relationship between the elements of the of the Walsh-Hadamard transformants matrix and the frequency components of the matrix X. In particular, considering the purpose of the paper, the localization of the high-frequency components of the matrix X of the DI in the Walsh-Hadamard transformants matrix is of greatest interest. Each of the Walsh functions, while not being harmonic, is characterized by a sequency [17], which is analogous to the frequency for harmonic functions, and in the case of the harmonic functions, these two characteristics coincide. In accordance with [17], if the number of sign changes in the time interval of the function f is equal to η, then the sequency $\bar{\eta}$ of the function f is determined as $\eta/2$ for even η, or $(\eta+1)/2$ for odd η, respectively. Moreover, each of the Walsh functions has a natural correspondence with the harmonic in the time interval $t \in [0,1]$ [17], at which the higher sequency of the Walsh function corresponds to the higher frequency of the corresponding harmonic.

Let us denote by $H_N(i,:)$ the i-th row of the matrix H_N. In the accepted notation, considering (4), the correspondence between the initial rows of H_N and harmonic functions will have the form

$$\begin{aligned} H_N(1,:) &\rightarrow \sin(\pi t), \\ H_N(2,:) &\rightarrow \sin(N\pi t), \\ H_N(3,:) &\rightarrow \sin\left(\frac{N}{2}\pi t\right), \\ H_N(4,:) &\rightarrow \cos\left(\frac{N}{2}\pi t\right), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \end{aligned} \qquad (6)$$

etc.

The highest sequency among the rows of matrix (4), which are discrete Walsh functions, ordered according to Hadamard, will always have $H_N(2,:)$, for which $\bar{\eta} = N/2 = 2^{k-1}$; the highest frequency of all the corresponding harmonics (6) has the corresponding to $H_N(2,:)$ harmonic $\sin(N\pi t)$.

For a clearer understanding of the relationship between the frequency components and the components of the Walsh-Hadamard transformants matrix, we assume that $X = E$, where E is the identity matrix of the corresponding size. In this case, the result of relation (5) will not depend in any way on the image matrix, but will be determined only by the coefficients of the Walsh-Hadamard matrix

$$W = H'_N X H'^T_N = H'_N H'_N. \qquad (7)$$

Relation (7) can be rewritten as

$$H'_N H'_N = \frac{1}{N}\begin{pmatrix} H_N(1,:) \\ H_N(2,:) \\ \cdots\cdots\cdots \\ H_N(N,:) \end{pmatrix} \cdot \left(\left(H_N(1,:)\right)^T, \left(H_N(2,:)\right)^T, \dots, \left(H_N(N,:)\right)^T \right) =$$

$$= \frac{1}{N}\begin{pmatrix} H_N(1,:)\left(H_N(1,:)\right)^T, & H_N(1,:)\left(H_N(2,:)\right)^T, & \dots, & H_N(1,:)\left(H_N(N,:)\right)^T \\ H_N(2,:)\left(H_N(1,:)\right)^T, & H_N(2,:)\left(H_N(2,:)\right)^T, & \dots, & H_N(2,:)\left(H_N(N,:)\right)^T \\ \cdots & \cdots & \cdots & \cdots \\ H_N(N,:)\left(H_N(1,:)\right)^T, & H_N(N,:)\left(H_N(2,:)\right)^T, & \dots, & H_N(N,:)\left(H_N(N,:)\right)^T \end{pmatrix}. \quad (8)$$

Considering (8), as well as the relationship (6) between the Walsh functions and the corresponding harmonics, as well as the well-known formulas for transforming the product of trigonometric functions into a sum, we formulate the following hypothesis.

Condition A. In the Walsh-Hadamard transformants matrix (5), element (2,2) will correspond to the highest-frequency component of X, some of the high-frequency components will be localized within the second row and the second column of matrix (5), regardless of its size. There are no low-frequency components within the second row and second column. In general, in matrices that are the result of the Walsh-Hadamard transform, the high-frequency components will correspond to the elements at the intersection of rows and columns corresponding to discrete Walsh functions with the highest sequencies. Thus, to localize the elements corresponding to the high-frequency components of the DI block (matrix), it is sufficient to determine the functions with the highest sequencies among the Walsh functions. So, considering the sequency of discrete Walsh functions, ordered according to Hadamard, used to transform matrices of size $l \times l$, where $l \in \{4,8,16\}$, indicated in Table 1, it can be argued that the high-frequency components of the signal in the matrix of size 4×4 will correspond to the elements in the matrix (5) at the positions (written in decreasing order of sequency): (2,2), (2,4), and (4,2); in the matrix of size 8×8: (2,2), (2,6), and (6,2), (2,8), and (6,8), and (6,6); in the matrix of size 16×16: (2,2), (2,10), and (10,2), (2,14), and (14,2), (2,6), and (6,2), etc. In Table 1 we can write out all the elements of the matrix in the order of frequency decreasing. Note that if the sequencies of the functions are the same, then when identifying compliance with high-frequency components, preference should be given to the one with the greater value of η.

Table 1

Correspondence between values η and $\bar{\eta}$ for Walsh functions ordered according to Hadamard for different sizes of Walsh-Hadamard matrices

l=4	Row number	1				2				3				4			
	$\eta/\bar{\eta}$	0/0				3/2				1/1				2/1			
l=8	Row number	1		2		3		4		5		6		7		8	
	$\eta/\bar{\eta}$	0/0		7/4		3/2		4/2		1/1		6/3		2/1		5/3	
l=16	Row number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	$\eta/\bar{\eta}$	0/0	15/8	7/4	8/4	3/2	12/6	4/2	11/6	1/1	14/7	6/3	9/5	2/1	13/7	5/3	10/5

For practical confirmation of the proposed hypothesis with help of the MatLAB environment, a computational experiment was performed, in which 1000 DI were used from traditional image databases in both lossy (JPEG) format (NRCS base [18]) and lossless (TIFF) (img_Nikon_D70s [19], 4cam_auth [20]). The considered image size was 400×400 pixels. During the experiment, each DI was divided in a standard way into $l \times l$-blocks, where $l \in \{4,8,16\}$

(the sizes of the blocks were chosen as the most frequently used in block steganographic methods). For each block B, the Walsh-Hadamard transform (5) (the result is a block B_{WH}) and the discrete cosine transform (the result is a block B_{DCT}) were calculated.

Then a certain element (i, j) in the B_{DCT} (the same element (i, j) for all the DI blocks) was perturbed (the result is a block $\overline{B}_{DCT}$), after which the perturbed block $\overline{B}$ was restored by inverse cosine transform. For the block $\overline{B}$ the Walsh-Hadamard transform was performed (the result is a block $\overline{B}_{WH}$), after which, by comparison of $\overline{B}_{WH}$ and B_{WH}, the most disturbed element of the block B_{WH} was found. For all the DI blocks, such an element of the block B_{WH} was determined that more often than others had the maximum perturbation as a result of perturbation of the (i, j) element of the block B_{DCT}, as well as a quantitative characteristic of this frequency, for which a $l \times l$-matrix R of maximum disturbance was built for each DI, the element $R_{ij}, i, j = 0,1,...,l-1$ of which was equal to the number of DI blocks, for which the maximum in absolute value $\overline{B}_{WH} - B_{WH}$ matrix element is located at position (i, j).

The experimental results, which fully correspond to the theoretical positions substantiated above, are presented in Table 2 and Fig. 3 (the localization of high-frequency components is highlighted by filling the corresponding elements), while the values of the R matrix elements did not depend on the magnitude of the disturbing effect on the B_{DCT} matrix elements (during the experiment, the disturbances were $\pm 1\%; \pm 10\%; \pm 100\%$, but depended only on their localization in B_{DCT}, which confirms the accuracy of the established correspondence between the elements of the frequency domain and the Walsh-Hadamard transform domain.

Note that since the low-frequency components correspond not only to the elements of the first column of the matrix of the Walsh-Hadamard transformants (Fig. 3), it is obvious that the method proposed in [8], as already noted above, cannot guarantee the reliability of perception of the generated steganographic message.

Note that for some blocks of the DI, the same maximum disturbance in blocks of the form (5) was achieved simultaneously in several elements, which was considered during the experiment. The behavior of DI in different storage formats (lossy and lossless) was slightly different. So, for the majority of DI in the lossy format, the number of blocks where the maximum disturbance in the matrix (5) occurred in one single element that meets the theoretical assumptions, often coincided with the total number of blocks or differed slightly (less than 1.5%), while in other elements the disturbance maximum in blocks was not achieved at all. For DI in a lossless format, the desired element in the Walsh-Hadamard transform domain, corresponding to a specific frequency coefficient, was determined by the absolute maximum of the blocks where it underwent maximum disturbance; the same maximum was reached in other elements of the transformed block. This situation is obviously a consequence of the fact that for DI in a lossy format, the high-frequency coefficients as a result of quantization and rounding in the process of saving and subsequent image restoration become comparable to zero in value. Because of this, even with a small absolute perturbation, their relative perturbation will be significant. The consequence of this fact is that the sufficient condition obtained below will work better for DI in a lossy format. An illustration is shown in Fig. 4. The DI matrix of the size 400×400, originally saved in the TIFF format, and then re-saved in the JPEG format, was divided into 8×8 blocks, the element (8,8) in each block B_{DCT} was perturbed.

Thus, for the matrix, which is the result of the Walsh-Hadamard transform of the matrix X of an arbitrary size, it is possible to accurately set the elements corresponding to the high-frequency components of the matrix X. The steganographic transformation, which results in a perturbation of these elements in the Walsh-Hadamard transform domain, will ensure the reliability of perception of the received steganographic message.

Table 2

Correspondence between the high-frequency components of the DI block and the elements of the result of the Walsh-Hadamard transform of the block

Block size l	Element (i, j) subjected to disturbance in B_{DCT} / element (m,n), subjected to

	maximum disturbance in the B_{WH} in maximum number of DI blocks under perturbation of the (i, j) element in B_{DCT} (the number of DI blocks (%), in which the B_{WH} element (m, n) has subjected to the maximum disturbance)		
4	(4,4)/(2,2) (97.3%)	(4,3)/(2,4) (95.6%)	(3,4)/(4,2) (96.1%)
8	(8,8)/(2,2) (99.4%)	(8,7)/(2,6) (99.4%)	(7,8)/(6,2) (99.4%)
16	(16,16)/(2,2) (99.8%)	(16,15)/(2,10) (99.9%)	(15,16)/(10,2) (99.9%)

(1,1)	(1,4)	(1,2)	(1,3)
(4,1)	**(4,4)**	(4,2)	**(4,3)**
(2,1)	(2,4)	(2,2)	(2,3)
(3,1)	**(3,4)**	(3,2)	(3,3)

a)

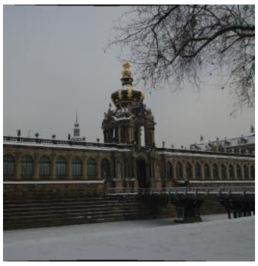
(1,1)	(1,8)	(1,4)	(1,5)	(1,2)	(1,7)	(1,3)	(1,6)
(8,1)	**(8,8)**	(8,4)	(8,5)	(8,2)	**(8,7)**	(8,3)	**(8,6)**
(4,1)	(4,8)	(4,4)	(4,5)	(4,2)	(4,7)	(4,3)	(4,6)
(5,1)	(5,8)	(5,4)	(5,5)	(5,2)	(5,7)	(5,3)	(5,6)
(2,1)	(2,8)	(2,4)	(2,5)	(2,2)	(2,7)	(2,3)	(2,6)
(7,1)	**(7,8)**	(7,4)	(7,5)	(7,2)	**(7,7)**	(7,3)	(7,6)
(3,1)	(3,8)	(3,4)	(3,5)	(3,2)	(3,7)	(3,3)	(3,6)
(6,1)	**(6,8)**	(6,4)	(6,5)	(6,2)	(6,7)	(6,3)	(6,6)

b)

(1,1)	(1,16)	(1,8)	(1,9)	(1,4)	(1,13)	(1,5)	(1,12)	(1,2)	(1,15)	(1,7)	(1,10)	(1,3)	(1,14)	(1,6)	(1,11)
(16,1)	**(16,16)**	(16,8)	(16,9)	(16,4)	**(16,13)**	(16,5)	**(16,12)**	(16,2)	**(16,15)**	(16,7)	(16,10)	(16,3)	**(16,14)**	(16,6)	(16,11)
(8,1)	(8,16)	(8,8)	(8,9)	(8,4)	(8,13)	(8,5)	(8,12)	(8,2)	(8,15)	(8,7)	(8,10)	(8,3)	(8,14)	(8,6)	(8,11)
(9,1)	(9,16)	(9,8)	(9,9)	(9,4)	(9,13)	(9,5)	(9,12)	(9,2)	(9,15)	(9,7)	(9,10)	(9,3)	(9,14)	(9,6)	(9,11)
(4,1)	(4,16)	(4,8)	(4,9)	(4,4)	(4,13)	(4,5)	(4,12)	(4,2)	(4,15)	(4,7)	(4,10)	(4,3)	(4,14)	(4,6)	(4,11)
(13,1)	**(13,16)**	(13,8)	(13,9)	(13,4)	(13,13)	(13,5)	(13,12)	(13,2)	**(13,15)**	(13,7)	(13,10)	(13,3)	(13,14)	(13,6)	(13,11)
(5,1)	(5,16)	(5,8)	(5,9)	(5,4)	(5,13)	(5,5)	(5,12)	(5,2)	(5,15)	(5,7)	(5,10)	(5,3)	(5,14)	(5,6)	(5,11)
(12,1)	**(12,16)**	(12,8)	(12,9)	(12,4)	(12,13)	(12,5)	(12,12)	(12,2)	(12,15)	(12,7)	(12,10)	(12,3)	(12,14)	(12,6)	(12,11)
(2,1)	(2,16)	(2,8)	(2,9)	(2,4)	(2,13)	(2,5)	(2,12)	(2,2)	(2,15)	(2,7)	(2,10)	(2,3)	(2,14)	(2,6)	(2,11)
(15,1)	**(15,16)**	(15,8)	(15,9)	(15,4)	**(15,13)**	(15,5)	(15,12)	(15,2)	**(15,15)**	(15,7)	(15,10)	(15,3)	**(15,14)**	(15,6)	(15,11)
(7,1)	(7,16)	(7,8)	(7,9)	(7,4)	(7,13)	(7,5)	(7,12)	(7,2)	(7,15)	(7,7)	(7,10)	(7,3)	(7,14)	(7,6)	(7,11)
(10,1)	(10,16)	(10,8)	(10,9)	(10,4)	(10,13)	(10,5)	(10,12)	(10,2)	(10,15)	(10,7)	(10,10)	(10,3)	(10,14)	(10,6)	(10,11)
(3,1)	(3,16)	(3,8)	(3,9)	(3,4)	(3,13)	(3,5)	(3,12)	(3,2)	(3,15)	(3,7)	(3,10)	(3,3)	(3,14)	(3,6)	(3,11)
(14,1)	**(14,16)**	(14,8)	(14,9)	(14,4)	(14,13)	(14,5)	(14,12)	(14,2)	**(14,15)**	(14,7)	(14,10)	(14,3)	**(14,14)**	(14,6)	(14,11)
(6,1)	(6,16)	(6,8)	(6,9)	(6,4)	(6,13)	(6,5)	(6,12)	(6,2)	(6,15)	(6,7)	(6,10)	(6,3)	(6,14)	(6,6)	(6,11)
(11,1)	(11,16)	(11,8)	(11,9)	(11,4)	(11,13)	(11,5)	(11,12)	(11,2)	(11,15)	(11,7)	(11,10)	(11,3)	(11,14)	(11,6)	(11,11)

c)

Fig. 3. Correspondence of B_{WH} elements through B_{DCT} elements for blocks of different sizes l: a) $l=4$; b) $l=8$; c) $l=16$



a)

8	8	8	8	8	8	8	8
8	2500	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	5	8
8	8	8	8	8	8	8	8
8	8	8	8	8	8	8	8
8	8	8	5	8	8	8	8
8	8	8	8	8	8	8	5

b)

10	0	0	0	0	0	0	0
0	2490	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

c)

Fig. 4. a) original DI in lossless format; b) R matrix for DI in TIFF format; c) R matrix for the original DI, resaved to the JPEG format

A slightly different view on determining a relationship between the domains of the Walsh-Hadamard transform and DCT, leading to the same results, is proposed below.

To research the physical essence of the transformants of the two-dimensional Walsh-Hadamard transform (5) and their relationship with the DCT transformants (1), it is convenient to consider their representation using row vectors.

Consider row vectors V and Y of length N^2 that are the result of sequential concatenation of rows of matrices W and X (5) of size $N \times N$, respectively. In this case, the following statement takes place.

Statement 1. Transformants of two-dimensional Walsh-Hadamard transform (5) and DCT (1) when represented as a row vector can be found using the following relation

$$V = YA_1, \quad (9)$$

where A_1 is the matrix of order N^2, the elements of which are the coefficients near $x_{i,j}$ elements of the matrix X after the expansion of the product (1) or (5).

The proof of **Statement 1** is obvious.

Moreover, in the case of using the two-dimensional Walsh-Hadamard transform (5), the following is true.

Statement 2. In the case of the two-dimensional Walsh-Hadamard transform, the matrix A_1 coincides with the Walsh-Hadamard matrix H_{N^2} of order N^2, constructed in accordance with Sylvester's construction (4) up to a coefficient $\frac{1}{\sqrt{N}}$.

To prove the **Statement 2**, we use the definitions of the one-dimensional and two-dimensional Walsh-Hadamard transform in terms of the complete binary code $b_i(k)$ of length n, where $i = 0,1,...,n-1$, $n = \log_2 N$, $k = 0,1,...,N-1$.

In this case, the one-dimensional Walsh-Hadamard transform of the vector Y is specified using the following relation

$$V_\omega = \sum_{x=0}^{N-1} Y_x (-1)^{\sum_{i=0}^{n-1} b_i(x) b_i(\omega)}, \quad (10)$$

where the sum $\sum_{i=0}^{n-1} b_i(x) b_i(\omega)$ is the dot product of the codewords of the complete code with numbers x and ω.

The relation defining the two-dimensional Walsh-Hadamard transform of the matrix X has the form

$$W_{u,v} = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} X_{x,y} \left[(-1)^{\sum_{i=0}^{n-1} b_i(x) b_i(u) + b_i(y) b_i(v)} \right]. (11)$$

Let us rewrite expression (11) with respect to row vectors V and Y, obtained as a result of row-by-row concatenation of matrices W and X

$$V_w = \\ = \frac{1}{N} \sum_{z=0}^{N^2-1} Y_z \cdot (-1)^{\sum_{i=0}^{n-1} b_i(z//N) b_i(w//N) + b_i(z \bmod N) b_i(w \bmod N)}, \quad (12)$$

where $z = 0,1,..., N^2 - 1$, and the symbol $//$ denotes an integer division operation.

In expression (12), the part of the expression under the sum $b_i(z//N) b_i(w//N)$ defines the duplication of the dot product of the codewords of the complete code with numbers z and w N times (equivalent to the low-frequency part of the complete code of length $n' = \log_2 N^2 = 2\log_2 N$), while the part of the expression under the sum $b_i(z \bmod N) b_i(w \bmod N)$ leads to the formation of the dot product of the codewords of the complete code with numbers z and w each time the value of z changes (equivalent to the high-frequency part of the full code of length $n' = \log_2 N^2 = 2\log_2 N$).

Thus, in expression (12), the sum $\sum_{i=0}^{n-1} b_i(z//N) b_i(w//N) + b_i(z \bmod N) b_i(w \bmod N)$ is equivalent to the sum $\sum_{i=0}^{n'-1} b_i(x) b_i(\omega)$ in expression (10) of the one-dimensional Walsh-Hadamard transform using the complete code $b_i(k)$, $k = 0,1,..., N^2 - 1$ with the codeword length $n' = 2\log_2 N$, while the two-dimensional Walsh-Hadamard transform of the matrix X corresponds, up to a coefficient $\frac{1}{N}$, to the one-dimensional Walsh-Hadamard transform of the vector Y, which is formed by a sequential concatenation of matrix X rows, which proves the **Statement 2**.

Let us also consider a specific example of the operation of **Statement 2** using the direct expansion of the product (5), which, for example, we perform for matrices W, X, H_4 of order $N = 4$

$$W = H_N X H_N^T = \frac{1}{N}\begin{bmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{bmatrix}\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix}\begin{bmatrix} + & + & + & + \\ + & - & + & - \\ + & + & - & - \\ + & - & - & + \end{bmatrix} =$$

$$\frac{1}{N}\begin{bmatrix}
\begin{array}{l} x_{11}+x_{12}+x_{13}+x_{14}+x_{21}+x_{22}+x_{23}+x_{24}+ \\ +x_{31}+x_{32}+x_{33}+x_{34}+x_{41}+x_{42}+x_{43}+x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}+x_{13}-x_{14}+x_{21}-x_{22}+x_{23}-x_{24}+ \\ +x_{31}-x_{32}+x_{33}-x_{34}+x_{41}-x_{42}+x_{43}-x_{44} \end{array} &
\begin{array}{l} x_{11}+x_{12}-x_{13}-x_{14}+x_{21}+x_{22}-x_{23}-x_{24}+ \\ +x_{31}+x_{32}-x_{33}-x_{34}+x_{41}+x_{42}-x_{43}-x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}-x_{13}+x_{14}+x_{21}-x_{22}-x_{23}+x_{24}+ \\ +x_{31}-x_{32}-x_{33}+x_{34}+x_{41}-x_{42}-x_{43}+x_{44} \end{array} \\
\begin{array}{l} x_{11}+x_{12}+x_{13}+x_{14}-x_{21}-x_{22}-x_{23}-x_{24}+ \\ +x_{31}+x_{32}+x_{33}+x_{34}-x_{41}-x_{42}-x_{43}-x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}+x_{13}-x_{14}-x_{21}+x_{22}-x_{23}+x_{24}+ \\ +x_{31}-x_{32}+x_{33}-x_{34}-x_{41}+x_{42}-x_{43}+x_{44} \end{array} &
\begin{array}{l} x_{11}+x_{12}-x_{13}-x_{14}-x_{21}-x_{22}+x_{23}+x_{24}+ \\ +x_{31}+x_{32}-x_{33}-x_{34}-x_{41}-x_{42}+x_{43}+x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}-x_{13}+x_{14}-x_{21}+x_{22}+x_{23}-x_{24}+ \\ +x_{31}-x_{32}-x_{33}+x_{34}-x_{41}+x_{42}+x_{43}-x_{44} \end{array} \\
\begin{array}{l} x_{11}+x_{12}+x_{13}+x_{14}+x_{21}+x_{22}+x_{23}+x_{24}- \\ -x_{31}-x_{32}-x_{33}-x_{34}-x_{41}-x_{42}-x_{43}-x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}+x_{13}-x_{14}+x_{21}-x_{22}+x_{23}-x_{24}- \\ -x_{31}+x_{32}-x_{33}+x_{34}-x_{41}+x_{42}-x_{43}+x_{44} \end{array} &
\begin{array}{l} x_{11}+x_{12}-x_{13}-x_{14}+x_{21}+x_{22}-x_{23}-x_{24}- \\ -x_{31}-x_{32}+x_{33}+x_{34}-x_{41}-x_{42}+x_{43}+x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}-x_{13}+x_{14}+x_{21}-x_{22}-x_{23}+x_{24}- \\ -x_{31}+x_{32}+x_{33}-x_{34}-x_{41}+x_{42}+x_{43}-x_{44} \end{array} \\
\begin{array}{l} x_{11}+x_{12}+x_{13}+x_{14}-x_{21}-x_{22}-x_{23}-x_{24} \\ -x_{31}-x_{32}-x_{33}-x_{34}+x_{41}+x_{42}+x_{43}+x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}+x_{13}-x_{14}-x_{21}+x_{22}-x_{23}+x_{24}- \\ -x_{31}+x_{32}-x_{33}+x_{34}+x_{41}-x_{42}+x_{43}-x_{44} \end{array} &
\begin{array}{l} x_{11}+x_{12}-x_{13}-x_{14}-x_{21}-x_{22}+x_{23}+x_{24}- \\ -x_{31}-x_{32}+x_{33}+x_{34}+x_{41}+x_{42}-x_{43}-x_{44} \end{array} &
\begin{array}{l} x_{11}-x_{12}-x_{13}+x_{14}-x_{21}+x_{22}+x_{23}-x_{24}- \\ -x_{31}+x_{32}+x_{33}-x_{34}+x_{41}-x_{42}-x_{43}+x_{44} \end{array}
\end{bmatrix}. \quad (13)$$

Performing sequential concatenation of the rows of matrices W and X, as well as writing down the coefficients near the elements x_{ij} in the resulting matrix of expression (13), it is not difficult to rewrite expression (5) with respect to vectors V and Y of length N^2

$$V = \begin{bmatrix} w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{21} \\ w_{22} \\ w_{23} \\ w_{24} \\ w_{31} \\ w_{32} \\ w_{33} \\ w_{34} \\ w_{41} \\ w_{42} \\ w_{43} \\ w_{44} \end{bmatrix} = YA_1 = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{24} \\ x_{31} \\ x_{32} \\ x_{33} \\ x_{34} \\ x_{41} \\ x_{42} \\ x_{43} \\ x_{44} \end{bmatrix}^T \begin{bmatrix}
+&+&+&+&+&+&+&+&+&+&+&+&+&+&+&+ \\
+&-&+&-&+&-&+&-&+&-&+&-&+&-&+&- \\
+&+&-&-&+&+&-&-&+&+&-&-&+&+&-&- \\
+&-&-&+&+&-&-&+&+&-&-&+&+&-&-&+ \\
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+&-&+&-&-&+&-&+&-&+&-&+&+&-&+&- \\
+&+&-&-&-&-&+&+&-&-&+&+&+&+&-&- \\
+&-&-&+&-&+&+&-&-&+&+&-&+&-&-&+
\end{bmatrix}. \quad (14)$$

Analyzing expressions (13) and (14), we can conclude that each coefficient w_{ij} of the matrix of Walsh-Hadamard transformants shows the degree of "presence" in the X matrix of one or another frequency component, which represent all possible superpositions of rows of the original Walsh-Hadamard matrix H_4. In this case, in view of the structural features of the Walsh-Hadamard matrices, these superpositions coincide with the rows of the Walsh-Hadamard matrix of order N^2, i.e. in our case H_{16}.

This is not true for DCT matrices, i.e. for a given matrix C_N, the matrix A_1 does not correspond to the DCT matrix of order N^2, in other words $A_1 \neq C_{N^2}$. However, performing calculations similar to those performed in (13), it is not difficult to find a matrix A_1 for DCT as well.

Thus, as in the case of the Walsh-Hadamard transform, expression (1) can be rewritten with respect to row vectors P and Y of length N^2 obtained by sequential concatenation of the rows of matrices S and X, respectively.

Next, consider a specific example based on the DCT matrix of order $N = 4$ constructed in accordance with (2)

$$C_4 = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.65 & 0.27 & -0.27 & -0.65 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix}, \quad (15)$$

for which we find a matrix A_1, after which we rewrite expression (1) with respect to row vectors P and Y

$$P=\begin{bmatrix} s_{11}\\ s_{12}\\ s_{13}\\ s_{14}\\ s_{21}\\ s_{22}\\ s_{23}\\ s_{24}\\ s_{31}\\ s_{32}\\ s_{33}\\ s_{34}\\ s_{41}\\ s_{42}\\ s_{43}\\ s_{44} \end{bmatrix}=YA_1=\begin{bmatrix} x_{11}\\ x_{12}\\ x_{13}\\ x_{14}\\ x_{21}\\ x_{22}\\ x_{23}\\ x_{24}\\ x_{31}\\ x_{32}\\ x_{33}\\ x_{34}\\ x_{41}\\ x_{42}\\ x_{43}\\ x_{44} \end{bmatrix}^T \begin{bmatrix}
0.25 & 0.33 & 0.25 & 0.14 & 0.33 & 0.43 & 0.33 & 0.18 & 0.25 & 0.33 & 0.25 & 0.14 & 0.14 & 0.18 & 0.14 & 0.07\\
0.25 & 0.14 & -0.25 & -0.33 & 0.33 & 0.18 & -0.33 & -0.43 & 0.25 & 0.14 & -0.25 & -0.33 & 0.14 & 0.07 & -0.14 & -0.18\\
0.25 & -0.14 & -0.25 & 0.33 & 0.33 & -0.18 & -0.33 & 0.43 & 0.25 & -0.14 & -0.25 & 0.33 & 0.14 & -0.07 & -0.14 & 0.18\\
0.25 & -0.33 & 0.25 & -0.14 & 0.33 & -0.43 & 0.33 & -0.18 & 0.25 & -0.33 & 0.25 & -0.14 & 0.14 & -0.18 & 0.14 & -0.07\\
0.25 & 0.33 & 0.25 & 0.14 & 0.14 & 0.18 & 0.14 & 0.07 & -0.25 & -0.33 & -0.25 & -0.14 & -0.33 & -0.43 & -0.33 & -0.18\\
0.25 & 0.14 & -0.25 & -0.33 & 0.14 & 0.07 & -0.14 & -0.18 & -0.25 & -0.14 & 0.25 & 0.33 & -0.33 & -0.18 & 0.33 & 0.43\\
0.25 & -0.14 & -0.25 & 0.33 & 0.14 & -0.07 & -0.14 & 0.18 & -0.25 & 0.14 & 0.25 & -0.33 & -0.33 & 0.18 & 0.33 & -0.43\\
0.25 & -0.33 & 0.25 & -0.14 & 0.14 & -0.18 & 0.14 & -0.07 & -0.25 & 0.33 & -0.25 & 0.14 & -0.33 & 0.43 & -0.33 & 0.18\\
0.25 & 0.33 & 0.25 & 0.14 & -0.14 & -0.18 & -0.14 & -0.07 & -0.25 & -0.33 & -0.25 & -0.14 & 0.33 & 0.43 & 0.33 & 0.18\\
0.25 & 0.14 & -0.25 & -0.33 & -0.14 & -0.07 & 0.14 & 0.18 & -0.25 & -0.14 & 0.25 & 0.33 & 0.33 & 0.18 & -0.33 & -0.43\\
0.25 & -0.14 & -0.25 & 0.33 & -0.14 & 0.07 & 0.14 & -0.18 & -0.25 & 0.14 & 0.25 & -0.33 & 0.33 & -0.18 & -0.33 & 0.43\\
0.25 & -0.33 & 0.25 & -0.14 & -0.14 & 0.18 & -0.14 & 0.07 & -0.25 & 0.33 & -0.25 & 0.14 & 0.33 & -0.43 & 0.33 & -0.18\\
0.25 & 0.33 & 0.25 & 0.14 & -0.33 & -0.43 & -0.33 & -0.18 & 0.25 & 0.33 & 0.25 & 0.14 & -0.14 & -0.18 & -0.14 & -0.07\\
0.25 & 0.14 & -0.25 & -0.33 & -0.33 & -0.18 & 0.33 & 0.43 & 0.25 & 0.14 & -0.25 & -0.33 & -0.14 & -0.07 & 0.14 & 0.18\\
0.25 & -0.14 & -0.25 & 0.33 & -0.33 & 0.18 & 0.33 & -0.43 & 0.25 & -0.14 & -0.25 & 0.33 & -0.14 & 0.07 & 0.14 & -0.18\\
0.25 & -0.33 & 0.25 & -0.14 & -0.33 & 0.43 & -0.33 & 0.18 & 0.25 & -0.33 & 0.25 & -0.14 & -0.14 & 0.18 & -0.14 & 0.07
\end{bmatrix}. \quad (16)$$

Using the obtained one-dimensional representation of the DCT, as well as the two-dimensional Walsh-Hadamard transform, we can establish a relationship between the transformants of both transforms. Note, however, that in view of the difference in the nature of the basis functions of the DCT and the Walsh-Hadamard transform, the effect of a change in one or another coefficient in the Walsh-Hadamard transform domain will affect a number of coefficients in the DCT domain, and vice versa.

Nevertheless, research shows that it is possible to establish the coefficients of DCT and the two-dimensional Walsh-Hadamard transform, which have the greatest influence on each other.

In accordance with the definition of the sequency [17], we find the values of the number of sign changes η for each row of the matrix A_1 (16), which we calculated for the transformants of the DCT of the matrix of order $N = 4$.

Data on the number of sign changes η for rows of matrix A_1 (16) are presented in Table 3.

Table 3

Values of η for the DCT matrix

No. of row	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A_1 (16)	0	7	8	15	1	6	9	14	2	5	10	13	3	4	11	12

Analyzing the data presented in the Table 3, and comparing it with the data presented in Table 1, we can establish a correspondence between the sequencies (frequencies) of the basis functions of the DCT and the Walsh-Hadamard transform, thus making a conclusion about which of the transformants of the Walsh-Hadamard transform leads to the greatest change in certain transformants of the DCT. For convenience, this correspondence will be written as the following expression

$$\begin{matrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{21} & s_{22} & s_{23} & s_{24} & s_{31} & s_{32} & s_{33} & s_{34} & s_{41} & s_{42} & s_{43} & s_{44}\\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow\\ w_{11} & w_{13} & w_{14} & w_{12} & w_{31} & w_{33} & w_{34} & w_{32} & w_{41} & w_{43} & w_{44} & w_{42} & w_{21} & w_{23} & w_{24} & w_{22} \end{matrix}. \quad (17)$$

The obtained correspondence (17) completely coincides with the results presented in Fig. 2.

Expression (17) allows us to establish a correspondence between the Walsh-Hadamard and DCT transformants, and also shows disturbance of which of the Walsh-Hadamard transformants affect the quality of the original image most of all.

III. CONNECTION BETWEEN THE WALSH-HADAMARD TRANSFORM AND THE SINGULAR VALUE DECOMPOSITION OF A MATRIX

As already noted, formal sufficient conditions for ensuring the reliability of perception of a steganographic message have already been proposed earlier in the domain of singular value (spectral) decomposition (of blocks) of the container matrix [13], according to which the reliability of perception of a steganographic message will be ensured if singular vectors of matrix (blocks of matrix) of the container (eigenvectors of a symmetric matrix), perturbed as a result of the steganographic transformation, correspond to small singular values (small in absolute value eigenvalues of a symmetric matrix (symmetric matrix blocks)) or singular values that have small gaps (eigenvalues of a symmetric matrix (symmetric matrix blocks) that have small absolute gaps). In this case, the smaller the perturbations of the singular values (eigenvalues of the symmetric matrix), gaps (absolute gaps), and the singular values (absolute values of the eigenvalues of the symmetric matrix) that correspond to the perturbed singular vectors (eigenvectors of the symmetric matrix), the greater the probability of observance of the reliability of perception of steganographic messages. Moreover, in the case of applying a sufficient condition to the blocks of the matrix, the blocks are obtained by its standard partitioning [22].

In order of definiteness let us consider a complete set of parameters, which completely determines the DI and consists of a singular values and singular vectors set of non-intersecting DI $l \times l$-blocks [13], an arbitrary one of which is B. The sufficient condition mentioned above, considering the fact that in the DI (digital video frame) matrix, the singular values are not simply related by the ratio: $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_l \geq 0$, but this ratio can be clarified

$$\sigma_1 >> \sigma_2 \geq ... \geq \sigma_l \geq 0, \tag{18}$$

in this case, the gap is calculated by the formula [21]: $svdgap(i, B) = \min_{j \neq i} |\sigma_i - \sigma_j|$, in accordance with which $svdgap(1, B) = \sigma_1 - \sigma_2 >> svdgap(i, B),\ i > 1$, in this case, the gap of the smallest singular value can be significantly less than one (up to comparability with 0), which leads to the fact that the singular values mentioned in the sufficient condition is the lowest singular value. Consider the singular value decomposition of the matrix B in the form of outer products [21]

$$B = \sum_{i=1}^{l} \sigma_i u_i v_i^T, \tag{19}$$

giving a representation of B in the form of a sum of matrices of rank equal to one, each of which corresponds to its own singular triple (σ_i, u_i, v_i). Taking this into account, as well as (18), the above sufficient condition can be formulated in a slightly different form: the reliability of perception of the steganographic message will be ensured in the case when, in the formal representation of the steganographic transformation in the domain of singular value decomposition of matrix blocks, this is expressed in a perturbation in matrices of rank equal to one corresponding to the smallest singular value in (19).

It should be noted that under a sufficient condition formulated in the domain of the singular value decomposition of a matrix, we do not have such a clear separation by frequency components as in the domain of DCT or Fourier transform, since each singular triple (and the corresponding matrix of rank one) carries information about all frequencies, but in varying degrees. So, the singular triples, corresponding to the minimum / maximum / average singular values, correspond mainly to the high-frequency / low-frequency / mid-frequency components. The separation of frequencies between singular triples is "softer" than directly in the frequency domain, which gives advantages in steganography [13].

It can be assumed that the absence of a clear division into frequencies will lead to a "softer" correspondence between the singular triples of the matrix and the elements of the Walsh-Hadamard transformants, and will give an opportunity for “greater maneuver” in the process of steganographic transformation, without impairing the reliability of perception of the steganographic message, expanding the possible transformation area.

For confirmation, a computational experiment was performed, in which many of the DI indicated above were involved. During the experiment, perturbations were introduced into the matrix $\sigma_4 u_4 v_4^T$ (for blocks of size 4×4), into matrices $\sigma_7 u_7 v_7^T$ and $\sigma_8 u_8 v_8^T$ (for blocks of size 8×8), and into matrices $\sigma_i u_i v_i^T$, $i\in\{13,14,15,16\}$ (for blocks of size 16×16). As a result, the area of possible disturbance of the DI blocks in the Walsh-Hadamard transform domain was expanded, which makes it possible to preserve the reliability of perception of the steganographic message (Fig. 5).

The resulting situation is absolutely natural. An additional expansion of the area of possible disturbance without violating the reliability of perception occurs, in fact, due to the use of elements that can already be attributed to those that correspond to the mid-frequency component, which, as it is known, with a significant probability does not violate the reliability of perception.

(1,1)	(1,4)	(1,2)	(1,3)
(4,1)	(4,4)	(4,2)	(4,3)
(2,1)	(2,4)	(2,2)	(2,3)
(3,1)	(3,4)	(3,2)	(3,3)

a)

(1,1)	(1,8)	(1,4)	(1,5)	(1,2)	(1,7)	(1,3)	(1,6)
(8,1)	(8,8)	(8,4)	(8,5)	(8,2)	(8,7)	(8,3)	(8,6)
(4,1)	(4,8)	(4,4)	(4,5)	(4,2)	(4,7)	(4,3)	(4,6)
(5,1)	(5,8)	(5,4)	(5,5)	(5,2)	(5,7)	(5,3)	(5,6)
(2,1)	(2,8)	(2,4)	(2,5)	(2,2)	(2,7)	(2,3)	(2,6)
(7,1)	(7,8)	(7,4)	(7,5)	(7,2)	(7,7)	(7,3)	(7,6)
(3,1)	(3,8)	(3,4)	(3,5)	(3,2)	(3,7)	(3,3)	(3,6)
(6,1)	(6,8)	(6,4)	(6,5)	(6,2)	(6,7)	(6,3)	(6,6)

b)

(1,1)	(1,16)	(1,8)	(1,9)	(1,4)	(1,13)	(1,5)	(1,12)	(1,2)	(1,15)	(1,7)	(1,10)	(1,3)	(1,14)	(1,6)	(1,11)
(16,1)	(16,16)	(16,8)	(16,9)	(16,4)	(16,13)	(16,5)	(16,12)	(16,2)	(16,15)	(16,7)	(16,10)	(16,3)	(16,14)	(16,6)	(16,11)
(8,1)	(8,16)	(8,8)	(8,9)	(8,4)	(8,13)	(8,5)	(8,12)	(8,2)	(8,15)	(8,7)	(8,10)	(8,3)	(8,14)	(8,6)	(8,11)
(9,1)	(9,16)	(9,8)	(9,9)	(9,4)	(9,13)	(9,5)	(9,12)	(9,2)	(9,15)	(9,7)	(9,10)	(9,3)	(9,14)	(9,6)	(9,11)
(4,1)	(4,16)	(4,8)	(4,9)	(4,4)	(4,13)	(4,5)	(4,12)	(4,2)	(4,15)	(4,7)	(4,10)	(4,3)	(4,14)	(4,6)	(4,11)
(13,1)	(13,16)	(13,8)	(13,9)	(13,4)	(13,13)	(13,5)	(13,12)	(13,2)	(13,15)	(13,7)	(13,10)	(13,3)	(13,14)	(13,6)	(13,11)
(5,1)	(5,16)	(5,8)	(5,9)	(5,4)	(5,13)	(5,5)	(5,12)	(5,2)	(5,15)	(5,7)	(5,10)	(5,3)	(5,14)	(5,6)	(5,11)
(12,1)	(12,16)	(12,8)	(12,9)	(12,4)	(12,13)	(12,5)	(12,12)	(12,2)	(12,15)	(12,7)	(12,10)	(12,3)	(12,14)	(12,6)	(12,11)
(2,1)	(2,16)	(2,8)	(2,9)	(2,4)	(2,13)	(2,5)	(2,12)	(2,2)	(2,15)	(2,7)	(2,10)	(2,3)	(2,14)	(2,6)	(2,11)
(15,1)	(15,16)	(15,8)	(15,9)	(15,4)	(15,13)	(15,5)	(15,12)	(15,2)	(15,15)	(15,7)	(15,10)	(15,3)	(15,14)	(15,6)	(15,11)
(7,1)	(7,16)	(7,8)	(7,9)	(7,4)	(7,13)	(7,5)	(7,12)	(7,2)	(7,15)	(7,7)	(7,10)	(7,3)	(7,14)	(7,6)	(7,11)
(10,1)	(10,16)	(10,8)	(10,9)	(10,4)	(10,13)	(10,5)	(10,12)	(10,2)	(10,15)	(10,7)	(10,10)	(10,3)	(10,14)	(10,6)	(10,11)
(3,1)	(3,16)	(3,8)	(3,9)	(3,4)	(3,13)	(3,5)	(3,12)	(3,2)	(3,15)	(3,7)	(3,10)	(3,3)	(3,14)	(3,6)	(3,11)
(14,1)	(14,16)	(14,8)	(14,9)	(14,4)	(14,13)	(14,5)	(14,12)	(14,2)	(14,15)	(14,7)	(14,10)	(14,3)	(14,14)	(14,6)	(14,11)
(6,1)	(6,16)	(6,8)	(6,9)	(6,4)	(6,13)	(6,5)	(6,12)	(6,2)	(6,15)	(6,7)	(6,10)	(6,3)	(6,14)	(6,6)	(6,11)
(11,1)	(11,16)	(11,8)	(11,9)	(11,4)	(11,13)	(11,5)	(11,12)	(11,2)	(11,15)	(11,7)	(11,10)	(11,3)	(11,14)	(11,6)	(11,11)

c)

Fig. 5. Localization of the area of possible disturbance as a result of the steganographic transformation in the Walsh-Hadamard transform domain for $l\times l$ -blocks of the DI: a) $l=4$; b) $l=8$; c) $l=16$

IV. A SUFFICIENT CONDITION FOR ENSURING THE RELIABILITY OF PERCEPTION OF A STEGANOGRAPHIC MESSAGE

Based on the results obtained in this paper, we can formulate ***a sufficient condition for ensuring the reliability of perception of the steganographic message***. To ensure the reliability of perception of the steganographic message, it is sufficient to embed additional information in such a way that in the Walsh-Hadamard transform domain, its result would be a disturbance of the elements, the localization of which is shown in Fig. 5 for $l\times l$-blocks of size $l\in\{4,8,16\}$, while the embedding process itself can be implemented not only directly in the Walsh-Hadamard transform domain, but also in any other domain of the container (spatial, transformation). If it is necessary to use blocks of a different size, it is recommended to perform embedding to the DI in such a way that the result would be a perturbation of the elements in the Walsh-Hadamard transform domain within the boundaries of the second column and the second row of the transformant matrix. For a more accurate localization of possible disturbances, it is necessary to perform additional research considering **Condition A**.

For practical verification of the obtained sufficient condition, a computational experiment was performed, in which DI from the previously listed bases were involved. Perturbations were introduced into the block matrix in the Walsh-Hadamard transform domain. The perturbations of each element were taken from the set $\{+1,-1\}$, considering the fact that during organizing a covert communication channel, additional information, as a rule, is a binary sequence. The experiment was performed for blocks of size 8×8. The results are presented in Table 4.

Table 4

The results of the embedding of the information in the Walsh-Hadamard transform domain

Perturbed block elements in the Walsh-Hadamard transform domain	(2,2)	(2,6)&(6,2)	(2,8)/(8,2)	(2,4)&(4,2)	(2,2)&(2,8)&(8,2)&(2,6)& &(6,2)&(2,4)&(4,2)	(6,6)	(8,8)	(6,6)& (6,8)&(8,6)&(8,8)	(2,2)&(2,8)&(8,2)&(2,6)& &(6,2)&(2,4)&(4,2)& &(6,6)& (6,8)&(8,6)& &(8,8)
PSNR (*dB*)	48.1	45.2	45.1	45.1	39.7	48.0	48.0	42.1	37.7

The results obtained are in full agreement with the theoretical conclusions, practically confirm the effectiveness of the obtained sufficient condition for ensuring the reliability of the perception of the steganographic message.

A visual illustration is shown in Fig. 6, where the magnitude of the perturbation of the elements in the matrices, which are the result of the Walsh-Hadamard transform of the DI 8×8-blocks, was ±1. Artifacts or differences of the original from the perturbed DI (Fig. 6, b)) are not detected by subjective ranking.

V. A SUFFICIENT CONDITION FOR ENSURING THE RELIABILITY OF PERCEPTION OF A STEGANOGRAPHIC MESSAGE

Note that the obtained sufficient condition, because of the mathematical approach used, makes it possible to prevent local violations of the reliability of perception associated not only with the steganographic transformation, which was already mentioned in the introduction (Fig. 1), and where the difference indicators are often insufficient. An illustration of this is shown in Fig. 7, where the brightness value of only one pixel is changed on the original DI. With significant DI sizes, such a change is not visually detected in the image at all, but it can be detected with a more thorough visual analysis, including the use of existing software tools.

Analyzing the 8×8-block B containing the changed pixel, it was found that all the elements in the Walsh-Hadamard transform domain underwent changes, the disturbances of which were ≈ 3.1 (matrix (5) for block B, the absolute values of the elements (except for (1,1)) are less than 1, as it is shown in (20)), which does not satisfy the obtained sufficient condition and, as expected, leads to the possibility of establishing a violation of the reliability of perception. Note that such a local violation of the reliability of perception could not be detected even if the difference indicator was applied separately for each block, which is proposed, for example, in [23]. Indeed, the authors of the method for quantitatively assessing of the reliability of the perception of a digital image, taking into account the specificity of the difference indicators, recommend to first divide the DI into non-overlapping blocks, the sizes of which are comparable with size 128×128 pixels (otherwise the technique may be ineffective), and take the minimum PSNR value for all used blocks as a quantitative estimate.

а б

Fig. 6. Effectiveness illustration of the obtained sufficient condition for ensuring the reliability of perception of the steganographic message: a) the original DI; b) DI obtained as a result of perturbation of all elements of the second column and second row in each block, which is the Walsh-Hadamard transformants of DI 8×8-blocks

a) b)

Fig. 7. Local changes of the DI: a) the original image with the selected original 8×8-area; b) image, the integrity of which is violated within the selected 8×8-area

$$W=\begin{bmatrix} 251.0781 & 0.1094 & 0.1719 & 0.3281 & -0.2656 & 0.0156 & -0.0469 & -0.0156 \\ -0.0156 & 0.0156 & -0.0469 & -0.0156 & -0.1094 & 0.0469 & -0.0156 & 0.0156 \\ 0.0781 & -0.0156 & -0.0781 & -0.0469 & -0.3281 & 0.0781 & 0.1406 & 0.0469 \\ -0.1406 & 0.0156 & 0.0781 & -0.0156 & -0.0469 & -0.0156 & 0.0469 & -0.0469 \\ -0.1719 & 0.1094 & 0.2969 & -0.0469 & -0.3906 & 0.1406 & 0.3281 & 0.1094 \\ 0.0469 & 0.0781 & 0.0156 & 0.0469 & 0.0781 & -0.0156 & 0.0469 & 0.0781 \\ 0.0156 & 0.0469 & 0.1094 & 0.0156 & -0.0156 & 0.0156 & 0.0781 & 0.1094 \\ 0.1094 & -0.1094 & -0.0469 & -0.0156 & 0.0781 & -0.0156 & -0.0781 & -0.0469 \end{bmatrix}. \qquad (20)$$

But in the case of very small local changes in the DI, as, for example, shown in Fig. 7, the block size makes it impossible to quantitatively indicate a local violation of the reliability of perception: here the PSNR for a block containing a disturbed pixel was 46.5 dB.

Thus, the obtained sufficient condition can be used not only to ensure the reliability of perception of the steganographic message, but also as a tool that will make it possible, even in the absence of a visual picture, to draw conclusions about the possible appearance of artifacts in the image or digital video frame with a high probability, including, in a small local area (up to one pixel change).

VI. CONCLUSIONS

Let's note the main results of the performed research:

1. The relationship between the Walsh-Hadamard transform and the Discrete Cosine Transform is established, which is theoretically substantiated in two ways: by considering the correspondence between the Walsh-Hadamard functions and harmonics, and also by establish-

ing a correspondence between the two-dimensional and one-dimensional Walsh-Hadamard transforms. Both methods gave the same results, which are confirmed by the conducted empirical research. At the same time, the specific localization of transformants, which correspond to high-frequency components for Walsh-Hadamard matrices is derived for practically valuable sizes $N = 4,8,16$.

2. A connection between the transformants of the Walsh-Hadamard transform and singular triples (blocks) of the DI matrix was established, which made it possible, using the existing sufficient condition to ensure the reliability of the perception of a steganographic message in the domain of singular value decomposition of the matrix, to expand the area of possible disturbances as a result of the steganographic transformation in the Walsh-Hadamard transform domain, compared to that which was determined only by high-frequency components.

3. On the basis of the performed research, a sufficient condition was obtained to ensure the reliability of perception of the steganographic message in the Walsh-Hadamard transform domain, which can be used regardless of the domain of the container (spatial, transformation) where additional information is embedded.

4. The obtained sufficient condition allows us to draw conclusions about the possible (with a high probability) appearance of artifacts in the image, frame of digital video, including in a small local area (up to a change in one pixel), even in the case when visually these changes are not easily visualized on the DI.

The results obtained in this paper, together with the advantages of the Walsh-Hadamard transform, such as the fact that the elements of its basis vectors belong to the binary alphabet, as well as the simplicity of constructing transform matrices, determine the prospects for the development and practical application of the Walsh-Hadamard transform for estimating the effectiveness of existing steganographic methods, as well as development of new promising methods for steganography and steganalysis.

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