Development of a Method for the Analytical Synthesis of Standards for Navigation Control Systems of Unmanned Aerial Vehicles with a Deficit of Informative Features

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Abstract. The aim of this article is to synthesize reference images with high invariance and sufficient information content to improve the operational stability of unmanned aerial vehicles (UAVs) under conditions where informative features of target objects are insufficient. This goal is achieved through parametric optimization of the transformation order and regularization of the object image reconstruction process in fractional Fourier transform space. A method for analytically synthesizing reference images is proposed, enabling the extraction of the integral phase structure of an object while preserving the information invariance of the signal under critical noise and a deficiency of informative features. An analytical expression for synthesizing an optimal reference image is derived. The solution effectively suppresses noise through the use of a regularization function and a regularization coefficient, followed by the return of the reference image to the coordinate plane. An analytical model of an ensemble of optimal reference images is synthesized. It is shown that this approach ensures high noise immunity of machine vision systems under changing viewing angles and weather conditions, while maintaining the low computational complexity of image matching algorithms. The effectiveness of the developed method was assessed using noise immunity criteria and a regularized structural quality criterion for the reference standards. It was demonstrated that, under low signal-to-noise ratio conditions, the use of synthesized reference standards ensures reliable system operation, while maintaining a standard deviation of 2–4 meters. The effectiveness of the method was confirmed through numerical simulation of real scenes in the Python software environment.
Keywords: unmanned aerial vehicle, machine vision system, reference object, informative features, destructive impact.

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Dezvoltarea unei metode pentru sinteza analitică a standardelor pentru sistemele de control al navigației vehiculelor aeriene fără pilot cu deficit de caracteristici informative

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Rezumat. Scopul principal al acestui articol este sintetizarea imaginilor de referință caracterizate printr-un grad ridicat de invarianță și un conținut informațional suficient pentru a îmbunătăți semnificativ stabilitatea funcționării vehiculelor aeriene fără pilot în condiții de deficit sever al caracteristicilor informative ale obiectelor. Acest obiectiv este atins prin optimizarea parametrică a ordinului transformării fracționare Fourier și prin regularizarea riguroasă a procesului de reconstrucție a imaginii. Se propune o metodă originală de sintetizare analitică, care extrage structura integrală de fază a obiectului, păstrând în același timp invarianța informațională a semnalului în

condiții de zgomot critic și de reducere a elementelor geometrice locale. În cadrul cercetării este derivată o expresie analitică fundamentală pentru imaginea de referință optimă în domeniul transformării fracționare Fourier. Soluția propusă suprimă eficient zgomotul de fond prin utilizarea unei funcții de regularizare și a unui coeficient adaptiv de regularizare, urmată de revenirea precisă în planul inițial de coordonate. Modelul analitic al ansamblului de imagini de referință asigură o imunitate ridicată la zgomot a sistemelor de viziune artificială în condiții meteorologice variabile și la unghiuri diferite de vizualizare, menținând în același timp o complexitate computațională redusă a algoritmilor de potrivire. Eficacitatea metodei dezvoltate a fost evaluată pe baza unor criterii matematice de imunitate la zgomot și a unui criteriu regularizat de calitate structurală. În condiții de raport semnal-zgomot scăzut, utilizarea imaginilor de referință sintetizate asigură funcționarea fiabilă a sistemului de orientare, generând o deviație standard cuprinsă între doi și patru metri. Simularea numerică complexă a unor scene reale, realizată în mediul software Python, a confirmat pe deplin eficacitatea practică a metodei.
Cuvinte-cheie: vehicul aerian fără pilot, sistem de vedere artificială, obiect de referință, caracteristici informative, impact distructiv.

Разработка метода аналитического синтеза эталонов для систем управления навигацией беспилотных летательных аппаратов при дефиците информативных признаков

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Аннотация. Целью статьи является синтез эталонных изображений, обладающих высокой инвариантностью и достаточной информативностью, для повышения устойчивости функционирования беспилотных летательных аппаратов в условиях дефицита информативных признаков объектов визирования. Поставленная цель достигается на основе параметрической оптимизации порядка преобразования и регуляризации процесса восстановления образа объекта в пространстве дробного преобразования Фурье. Впервые предложен метод аналитического синтеза эталонов, основанный на поиске оптимального пространственно-частотного представления входного изображения в рассматриваемом базисе. Метод позволяет извлекать интегральную фазовую структуру объекта, сохраняющую информационную инвариантность сигнала даже при критическом зашумлении и дефиците информативных признаков исходного кадра. В работе получено аналитическое выражение для синтеза оптимального эталонного изображения в пространственно-частотной области. Данное решение обеспечивает эффективное подавление помех за счет применения регуляризирующей функции и коэффициента регуляризации с последующим возвратом эталона в координатную плоскость. Синтезирована аналитическая модель ансамбля оптимальных эталонных изображений, описывающая процедуру итерационного формирования их спектральных характеристик с учетом регуляризатора Тихонова. Показано, что такой подход обеспечивает высокую помехоустойчивость систем технического зрения при изменении ракурса и метеоусловий, сохраняя при этом низкую вычислительную сложность алгоритмов сопоставления изображений. Эффективность разработанного метода оценена с использованием критериев помехоустойчивости и регуляризованного структурного критерия качества эталонов. Доказано, что в условиях низких значений отношения сигнал/шум применение синтезированных эталонов гарантирует надежную работу системы, обеспечивая при этом среднеквадратическое отклонение локализации в пределах 2…4 метров. Путем численного моделирования реальных сцен в программной среде Python подтверждено, что метод позволяет формировать отклик решающей функции с высокой точностью локализации центрального пика даже в условиях интенсивного зашумления.
Ключевые слова: беспилотный летательный аппарат, система технического зрения, синтез эталонов, дробное преобразование Фурье, регуляризация Тихонова, информативные признаки.

INTRODUCTION

The rapid advancement of unmanned aerial vehicles (UAVs) and their continuous modernization have led to a significant expansion of their practical applications [1]. UAVs have become indispensable for monitoring critical infrastructure facilities, including energy, transportation, and industrial facilities, and are also widely used in agriculture and forestry.

The expansion of UAV capabilities has been driven by the improvement and application of high-precision information acquisition sensors and modern information technologies that enable near real-time onboard data processing. These components form the basis of machine vision systems (MVS), which serve as a key tool for extracting informative features (IFs) of target objects [2].

Unlike rigid navigation systems, such as correlation-extremum systems, modern intelligent machine vision systems (MVSs) must maintain operational capability under dynamic conditions caused by changes in UAV navigation and attitude parameters, variations in illumination, changes in the informative parameters of target objects under the influence of various external factors, and the impact of interference, including meteorological effects. However, these factors inevitably lead to a mismatch between the current frames acquired during flight and the reference images (RIs) stored in the onboard computer memory. The latter often fail to account for the dynamic variability of informative features (IFs) and the possibility of their partial disappearance under the influence of external factors. This necessitates the adaptation of the reference images used in MVSs to conditions of informative feature deficiency and the compensation of possible distortions caused by noise and interference at the stage of their synthesis [3–5].

At present, the following approaches to reference image synthesis are employed, which can be classified into three main groups:

Geometric and structural methods. These methods are based on the extraction of contours, corner points, or skeletal models. They are effective when applied to objects with deterministic geometry [4–5].

Frequency-domain and spectral methods (Fourier, Haar, wavelets) [7]. These methods provide invariance to rotation and scaling but are characterized by high algorithmic complexity and sensitivity to nonlinear effects in the generated image.

Local feature-based methods (SIFT, SURF, ORB) [8]. These methods are based on the detection of key points and are also used for the analysis of static scenes. They are ineffective under nonlinear illumination variations or significant perspective distortions caused by changes in the UAV viewing angle.

A promising approach to reference image synthesis is the use of neural networks, which enable the formation of compact vector representations of target objects. The drawback of this approach is its dependence on the size and quality of the training dataset, as well as the high computational complexity of the training process, which requires significant computational resources. This considerably limits their application to the monitoring of unique critical infrastructure facilities.

Thus, none of the existing approaches simultaneously provides low computational complexity and high invariance under conditions of informative feature deficiency.

Under these conditions, analytical methods capable of synthesizing reference images that are robust to informative feature deficiency appear to be the most suitable. This approach makes it possible to construct a mathematically grounded representation of an object using its integral phase characteristics.

Unlike empirical approaches, analytical methods ensure the invariance of reference images to a wide range of distortions and provide robust image matching under extremely low signal-to-noise ratio conditions by extracting the internal structure of the signal that remains inaccessible to conventional algorithms. The application of this approach can ensure the reliable operation of MVSs even in situations where most of the object's informative features are obscured by interference and the remaining data are insufficient for the correct operation of conventional or neural network-based algorithms [9–10].

Thus, there is a need to develop a method for the analytical synthesis of invariant reference images adapted to informative feature deficiency and stochastic noise.

The implementation of this method is possible through the use of the fractional Fourier transform (FrFT), which enables the synthesis of reference images in the domain of optimal spatial-frequency representations [11]. This ensures the formation of a reference image of the object that is invariant to informative feature deficiency in the original coordinate space.

LITERATURE REVIEW

The effectiveness of machine vision systems (MVSs), based on their design principles, is determined by two key factors: the quality of a priori prepared reference images and the reliability of current images (CIs) formed during navigation. The combination of these components defines the resulting accuracy of UAV positioning achieved through the matching of CIs and reference images (RIs). The formation of CIs, as well as methods and algorithms for their comparison, have been addressed in fundamental studies [1–6].

A number of publications [7–9] are devoted to improving the accuracy characteristics of MVSs through the use of additional informative features (IFs). These works consider various algorithms that allow for a more detailed representation of the structure of both RIs and CIs.

Study [10] proposes an object recognition algorithm based on contours, which provides effective identification in the presence of well-defined boundaries.

To compensate for visual mismatches between the reference map and onboard data, study [12] proposes an unsupervised deep feature extraction method based on the triplet loss function. Additional integration of visual odometry with a hypothesis verification mechanism for position estimation makes it possible to minimize navigation error. However, these approaches are designed for UAV flight over terrain with a high diversity of features, which limits their applicability under conditions of visual uncertainty.

A study of the impact of meteorological interference on UAV operation in automatic landing mode is presented in [13]. Simulation of combined factors (fog, wind, precipitation) demonstrated a critical reduction in the operational execution rate.

In [14], the use of topological descriptors and semantic analysis is proposed to improve robustness to scale and viewpoint variations. Despite effectively reducing onboard memory requirements, these methods remain dependent on the stability of feature extraction, which is problematic under conditions of dynamic variability and visual uncertainty.

Study [15] presents principles for the synthesis of optimal reference images based on the concepts of measurement theory. The authors propose a formalization of the brightness component, geometric transformations (translations and rotations), as well as fragment search procedures in terms of Grenander's mathematical theory of patterns. This approach provides a strict systematization of features; however, it requires high computational stability when operating with noisy data.

Issues of real-time reference image formation are addressed in [16], where reference image synthesis is implemented based on a fractal analysis field. The use of fractal dimensions makes it possible to identify stable structural characteristics of objects; however, the formation of an excessive set of images required for reliable navigation under dynamically changing conditions was not considered in this work.

In [17], the destabilizing influence of the atmosphere on the quality of generated images is investigated. The authors propose a software complex for compensating atmospheric distortions and filtering dark background, aimed at improving the visual distinguishability of objects under complex observation conditions.

Approaches to object identification and contour localization based on the Hough transform are presented in [18]. This framework enables effective extraction of geometric features; however, it remains sensitive to contour discontinuities under high noise levels.

Issues of robustness of object localization to scale distortions are also considered in [18]. Within the proposed approach, the problem is solved using a single reference image, which imposes limitations on system invariance under significant variations in viewing angle and observation distance.

In [19], a hybrid CNN–Transformer network architecture designed for feature detection and matching is presented. A ResNet50 neural network is used as a basic feature extractor, supplemented by a modified multiscale data fusion module. The application of an encoder–decoder Transformer structure enables the formation of preliminary correspondences between images, while the introduction of a geometric outlier removal method provides filtering of incorrect matches based on geometric similarity criteria.

Issues of optimization of the training process for navigation algorithms are considered in [20], where a combined approach integrating deep reinforcement learning (DQN) and self-learning mechanisms is proposed. The use of the Bellman loss function together with a contrastive loss function during the self-learning stage made it possible to accelerate the encoding of input scenes acquired by the UAV onboard camera. In addition, the integration of an obstacle detection model into the control loop improved navigation safety by reducing the probability of collisions in dynamic environments.

Study [21] presents a method for constructing a decision function for UAV positioning based on the fusion of radiometric and electro-optical data channels. The authors propose using the most high-contrast stationary objects as reference landmarks, which makes it possible to minimize the influence of perspective and scale distortions. However, a significant limitation of this approach is its low efficiency in environments with developed industrial and urban infrastructure characterized by a large number of small-scale and low-contrast targets.

Issues related to the selection of small-scale and extended monitoring objects are examined in detail in [21–23]. These studies propose adaptive thresholding algorithms followed by processing using specialized differential operators.

Thus, the conducted analysis of existing studies in the field of machine vision system (MVS) development shows that, despite the achieved results, the problem of synthesizing robust reference images under critical informative feature deficiency and a high level of stochastic noise remains only partially solved, which confirms the relevance of developing the proposed method for analytical synthesis of reference images.

The aim of this study is to synthesize reference images with high invariance and sufficient informativeness to ensure robust operation of UAV MVSs under conditions of informative feature deficiency or significant variability of target object features, based on parametric optimization of the transformation order and regularization of the object image reconstruction process in the fractional Fourier transform domain. This approach makes it possible to a priori synthesize mathematically grounded reference images. It also transforms the problem of decision function construction from the domain of visual search to the domain of statistical estimation in phase space.

To achieve this goal, the following tasks must be addressed:

1. To formulate the problem of developing a method for the analytical synthesis of invariant reference images for MVSs.
2. To develop a method for the analytical synthesis of reference images in the domain of optimal time–frequency representations using Tikhonov regularization.
3. To evaluate the effectiveness of the developed method for synthesizing a set of reference images.

METHODS, RESULTS, AND DISCUSSION

The spatial position of a UAV at each instant of flight is determined by the navigation parameter vector (altitude h, latitude θ, and longitude β) and the Euler angles (roll, pitch, and yaw). Any variation in these parameters causes a transformation of the geometric structure of the generated current image (CI). This necessitates the use of an ensemble of reference images (RIs) onboard the UAV. During the formation of this ensemble, it is also necessary to account for the possibility of a priori uncertainty caused by adverse weather conditions and potential interference of both artificial and natural origin, leading to significant image noise and informative feature (IF) deficiency.

1. Problem Formulation for the Method Development

To ensure the robust operation of an MVS under conditions of uncertainty, it is necessary to synthesize an ensemble of reference images $\mathbf{\Omega} = \{\mathbf{S}^i_{RI}\}$, the use of which minimizes the effect of informative feature (IF) deficiency and provides the highest UAV positioning accuracy:

$$\mathbf{\Omega}_{opt} = \arg\min_{\Omega} \mathbf{M}\left\{\left\|\mathbf{X}(t) - \mathbf{X}_{MVS}(t, \mathbf{S}_{CI}, \mathbf{S}_{RI})\right\|^2\right\} \quad (1)$$

subject to smoothness constraints imposed on each reference image in the ensemble $\|QS_{RI}\|^2 \le C, \forall_i = 1,2,...N$:

where:

$\mathbf{\Omega}_{opt}$ – the optimal ensemble of reference images (RIs) that minimizes the UAV positioning error;

$\mathbf{M}\{...\}$ – the mathematical expectation operator that averages the positioning error over all possible realizations of random noise and UAV flight trajectory conditions;

$\mathbf{X}(t)$ – the true UAV coordinate vector at time t;

$\mathbf{X}_{MVS}(t, \mathbf{S}_{CI}, \mathbf{S}_{RI})$ – the UAV coordinate vector estimated by the MVS from the noisy current image (CI) $\mathbf{S}_{CI}$ using the selected reference image $\mathbf{S}_{RI}$ from the ensemble $\mathbf{\Omega}$;

$\|QS_{RI}\|^2$ – the stabilizing constraint imposed on the norm of the derivatives, ensuring the robustness of each i-th reference image to informative feature deficiency;

C – the admissible smoothness constant of the reference image, ensuring the stability of the synthesis process under conditions of informative feature deficiency.

The desired result can be achieved by minimizing the functional that describes the trade-off between preserving the structural details of the image (data fidelity condition) and the need for noise suppression:

$$M^a[S_{RI}] = \| F^a[S_{RI}] - S^a_{CI} \|^2 + \gamma \| QS_{RI} \|^2 \to min\ , \quad (2)$$

where:

$\| F^a[S_{RI}] - S^a_{CI} \|^2$ – the measure of similarity between the synthesized reference image (RI) and the desired response in the optimal fractional Fourier transform domain;

$F^a[S_{RI}]$ – the reference image transformed into the fractional Fourier transform domain with the rotation angle in the spatial-frequency plane a (transformation order);

S^a_{CI} – the noisy current image (CI) transformed into the fractional Fourier transform domain;

Q – the differentiation operator;

γ – the regularization parameter selected according to the noise level.

Thus, the mathematical formulation of the problem of developing the analytical synthesis method is reduced to a two-stage optimization procedure. At the first stage, the optimal order of the fractional Fourier transform is selected automatically.

The second stage consists of the noise-robust analytical synthesis of a reference image (RI) that is invariant to informative feature (IF) deficiency through the application of the Tikhonov regularization method, which imposes stabilizing constraints on the smoothness of the RI (the norm of its derivatives).

2. Development of a Method for the Analytical Synthesis of an Ensemble of Reference Images in the Domain of Optimal Spatial-Frequency Representations Using Tikhonov Regularization

At the first stage of the study, the reference image model was substantiated, including the formalization of dynamic distortions, transformation into the fractional Fourier transform (FrFT) domain, as well as regularization and analytical synthesis.

Based on the above, the current image (CI) can be represented in operator form as:

$$S_{CI}(x,y) = G \cdot M \cdot B\{S_{RI}(x,y)\} + n(\mathrm{x},\mathrm{y})\ , \quad (3)$$

where:

G, M, and B are the operators of geometric, meteorological, and brightness distortions, respectively;

$n(\mathrm{x},\mathrm{y})$ is the additive noise.

According to expression (3), the fractional spectrum model of the reference image (RI) is given by [23–26]:

$$W_a(\mathrm{u},\mathrm{v}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_{RI}(x,y) \cdot K_a(x,y,u,v)dxdy\ , \quad (4)$$

where:

$K_a(x,y,u,v)$ is the kernel of the two-dimensional fractional Fourier transform;

u,v denotes the fractional frequency-coordinate variables.

Using expressions (3) and (4), the expression for the fractional spectrum of the current image (CI) can be written as:

$$W_a(\mathrm{u},\mathrm{v}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \{G \cdot M \cdot B\{S_{RI}(x,y)\} + n(\mathrm{x},\mathrm{y})\} \times \times K_a(x,y,u,v)dxdy\ . \quad (5)$$

The transformation order a was selected using the classical entropy minimization criterion [11, 23, 26]:

$$J(a) = \frac{\max|W_a(\mathrm{S}_{OI})|^2}{Ent\{W_a(\mathrm{S}_{OI})\}\sigma_n^2(\mathrm{S}_{OI})} \to \max\ , \quad (6)$$

where:

$\max|W_a(\mathrm{S}_{OI})|^2$ – the maximum value of the squared magnitude (intensity) of the FrFT image;

$Ent\{W_a(\mathrm{S}_{OI})\}$ – the entropy of the spatial-frequency energy distribution of the FrFT image;

$\sigma_n^2(\mathrm{S}_{OI})$ – the variance of random noise in the original image (OI).

Maximization of the criterion ensures the simultaneous fulfillment of three conditions: maximization of the peak intensity of the focused reference image (RI) (numerator), minimization of its structural entropy, and suppression of the influence of input noise variance (denominator).

The use of this criterion makes it possible to determine the spatial-frequency projection of the input image (OI) at which the energy of its fractional spectrum is maximized within a compact region, ensuring the formation of a high-contrast contour or geometric representation of the object.

Based on expression (6), and omitting intermediate transformations, the expression for the optimal fractional Fourier transform order is obtained:

$$a_{opt} = \arg\max_{a\in(0,1)} \frac{\|W_a(S_{OI})\|^4}{\int |W_a(S_{OI})|^4 \, dS}, \qquad (7)$$

The procedure for finding the optimal transformation order a_{opt} is performed iteratively until the maximization condition of criterion (6) is satisfied. After this, the direct synthesis of the reference image (RI) is reduced to solving an ill-posed inverse problem (2). To ensure noise-robust synthesis of the RI, the Tikhonov regularization method is applied, according to which the required RI S_{RI} is obtained by minimizing the smoothing functional (2).

Applying the reconstruction operator (Tikhonov filter) and the inverse FrFT to model (5), the expression for the synthesized reference image (RI) is obtained:

$$\hat{S}_{RI}(x,y) = F^{-a_{opt}}\left[\frac{F^{a_{opt}}\{S_{CI}\}}{1+\gamma Q^2(u,v)}\right], \qquad (8)$$

Let us analyze the components of expression (7).

$F^{a_{opt}}\{S_{CI}\}$ – adaptive numerator. The current image (CI) S_{CI} is transformed into the fractional domain with order α_{opt}, which is computed in real time via image moments. This provides geometric focusing of the object energy distorted by UAV maneuvers and meteorological conditions.

Operator $Q^2(u,v)$ – adaptive denominator (Tikhonov filter). This is a stabilizing kernel that "rotates" in the time–frequency space synchronously with the signal. Such a structure allows the filter to adapt to the edges and contours of buildings while ignoring chaotic noise from the underlying surface.

Parameter α controls the "rigidity" of noise suppression. Under conditions of informative feature (IF) deficiency, α increases in order to stabilize the synthesized phase portrait.

$F^{-a_{opt}}$ – inversion operator. It returns the cleaned data from the optimal basis to the coordinate space, completing the formation of the synthesized reference image (RI).

Expression (8) establishes a fundamental analytical relationship between the kinematic parameters of UAV flight and the informational characteristics of the generated current image (CI) under a priori uncertainty conditions. It also allows the synthesis of reference images (RIs) to be interpreted not as a static filtering process, but as a dynamic mapping of the CI into the space of optimal features. Optimal features are understood as components of the focused phase spectrum that are invariant to viewpoint in MVSs, and as regions of maximum energy concentration of the CI in the fractional Fourier domain.

To complete the mathematical description of the method, the regularization parameter is selected based on the discrepancy principle, while the noise variance is estimated using a robust algorithm [25].

A robust algorithm for estimating the standard deviation and noise variance using the Donoho–Johnstone method for digital images is given in [24]:

$$\hat{\sigma}_\eta^{\,2} = \left(\frac{median(|HH\{S_{CI}\}|)}{0{,}6745}\right)^2, \qquad (9)$$

where:

$HH\{S_{CI}\}$ – high-frequency coefficients of the CI decomposition;

0.6745 – the scaling coefficient that converts the median to the standard deviation for a normal distribution.

Thus, the joint adaptation system of equations takes the following form:

$$\begin{cases} \hat{\sigma}_\eta^{\,2} = \left(\dfrac{median(|HH\{S_{CI}\}|)}{0{,}6745}\right)^2, \\ a_{opt} = \arg\max\limits_{a\in(0,1)} \dfrac{\|W_a(S_{OI})\|^4}{\int |W_a(S_{OI})|^4 \, dS}, \\ \iint\limits_{u,v} \dfrac{a^2 Q^4(u,v,a_{opt}) \cdot |W_{a_{opt}}(u,v)|^2}{(1+aQ^2(u,v,a_{opt}))^2} = N\hat{\sigma}_\eta^{\,2} \end{cases}, \qquad (10)$$

where:

N – sample size (number of CI pixels);

$\hat{\sigma}_\eta^{\,2}$ – real-time estimate of the variance of additive noise η.

The quantity $|W_{a_{opt}}(u,v)|^2$, included in (10), is the squared magnitude of expression (5). In the selection equation, $\hat{\sigma}_\eta^{\,2}$, the spectrum (4) is used to separate the useful component (result of the action of operators (G, M, B)) and the noise component $n(x,y)$.

Analysis of the spectrum $W_a(u,v)$ allows the system to estimate the energy distribution density of IFs in the selected fractional basis.

Through the estimate $\hat{\sigma}_{\eta}^{2}$, the system determines the optimal value of the smoothing parameter. Parameter α is tuned to filter exactly the portion of the spectrum $W_a(\mathrm{u},\mathrm{v})$ that is generated by additive noise $n(\mathrm{x},\mathrm{y})$.

Consequently, analytical model (8) serves as an informational basis for the algorithm of adapting reference images (RIs) to informative feature (IF) deficiency and noise conditions. It confirms that the synthesis of RIs and the selection of parameter α are based on an adequate physical description of the RI formation process, taking into account the combined influence of geometric, meteorological, and brightness distortions. Such a comprehensive consideration of interfering factors ensures high reliability of RI formation under real operating conditions of MVSs. Based on the solution of the joint system of equations (10), the analytical synthesis of individual invariant elements of the RI ensemble $\mathbf{S}^{i}_{RI}$ is implemented, adapted to different operating conditions of MVSs. As a result, an a priori database of coupled fractional Fourier spectra of RIs is formed, which are invariant to noise and capture the phase structure of objects under conditions of informative feature deficiency:

$$\mathbf{\Omega} = \left\{ \mathbf{W}^{*}_{(i)a_{opt}}(u,v,S^{i}_{RI}) \right\}_{i=1}^{N} = \left\{ \frac{\mathbf{W}^{*}_{(i)a_{opt}}(u,v,S^{i}_{01})}{1 + a^{(i)} Q^{2}(\mathrm{u},\mathrm{v},\mathrm{a}^{i=1}_{opt})} \right\}_{i=1}^{N}, \qquad (11)$$

where:

N – the number of RIs in the ensemble covering a given range of viewing angles and UAV flight altitudes;

i – discrete index of spatial-geometric observation.

The decision on spatial localization of the object is implemented by the UAV onboard computer based on the search for the global maximum of the mutual correlation response function in the fractional Fourier domain. The two-dimensional correlation response function $R(\mathrm{u},\mathrm{v})$ is formed by computing the inverse fractional Fourier transform of the product of the current fractional spectrum of the current image (CI) $\mathbf{W_a}(\mathbf{S_{01}})$ and the complex-conjugate spectrum of the reference image (RI) $\mathbf{W}^{*}_{(k)a_{opt}}(\mathbf{u},\mathbf{v},\mathbf{S}^{(k)}_{RI})$, dynamically selected from the a priori synthesized ensemble $\mathbf{\Omega} = \{\mathbf{S}^{i}_{RI}\}$ according to the current viewing angle:

$$\mathbf{R}(\mathbf{u},\mathbf{v}) = F_{a}^{-1}\left\{ \mathbf{W_a}(\mathbf{S_{01}}) \cdot \mathbf{W}^{*}_{(k)a_{opt}}(\mathbf{u},\mathbf{v},\mathbf{S}^{(k)}_{RI}) \right\}, \qquad (12)$$

Thus, a method for the analytical synthesis of reference images (RIs) has been developed, based on a two-stage optimization procedure in the fractional Fourier transform domain. The application of the proposed approach to RI ensemble synthesis enables compensation for informative feature (IF) deficiency under non-stationary noise conditions.

3. Evaluation of the Effectiveness of the Method for Analytical Synthesis of a Reference Image Ensemble

The problem of evaluating the effectiveness of the proposed analytical synthesis method for the RI ensemble is addressed using criteria that take into account the mathematical properties of the FrFT and the stabilizing properties of Tikhonov regularization.

Noise Immunity Criterion

This parameter evaluates the quality of spatial alignment between the synthesized reference image (RI) and the current noisy image. The computation of the criterion is based on the two-dimensional correlation response function $R(\mathrm{u},\mathrm{v})$, formed according to relation (12). To ensure UAV navigation robustness under intense interference conditions ($q < 10\,\partial Б$), the criterion maximizes the signal-to-noise ratio of the response $R(\mathrm{u},\mathrm{v})$, normalized by the slope of the MVS correlation response at the point of true alignment $(0,0)$:

$$\mathbf{K_{ni}}(a_{opt},\gamma) = \frac{\max \mathbf{R}(\mathbf{u},\mathbf{v}) - \mu_{sb}}{\sigma_{sb}} \cdot \left| \frac{\partial^{2}\mathbf{R}(\mathbf{u},\mathbf{v})}{\partial u^{2}} \right|_{(0,0)} \rightarrow \max_{a_{opt},\gamma}, \qquad (13)$$

where μ_{sb} and σ_{sb} are the mathematical expectation and standard deviation of the fluctuation background of the side lobes of matrix $\mathbf{R}(\mathbf{u},\mathbf{v})$ outside the main lobe.

Maximization of $\mathbf{K_{ni}}(a_{opt},\gamma)$ is equivalent to minimization of the linear error in UAV coordinate estimation on the terrain.

Regularized Structural Quality Criterion of the RI

This criterion evaluates the internal quality of the reference image (RI) by finding a trade-off between the spectral discrepancy relative to noisy input data $\mathbf{S}_{CI}$ and the degree of smoothness (suppression of spurious oscillations). Considering the use of Tikhonov regularization in the fractional Fourier transform (FrFT) spectral domain, the criterion is expressed as:

$$\mathbf{K}_{\text{str}}(a_{opt},\gamma)=\sum_u\sum_v\frac{\gamma^2(u^2+v^2)\cdot\left|F^{a_{opt}}\{S_{CI}\}(u,v)\right|^2}{\left[1+\gamma(u^2+v^2)\right]^2}+,$$

$$+\sum\sum\frac{\gamma(u^2+v^2)\cdot\left|F^{a_{opt}}\{S_{CI}\}(u,v)\right|}{\left[1+\gamma(u^2+v^2)\right]^2}. \quad (14)$$

The physical meaning of criterion (14) lies in penalizing the algorithm both for failure to suppress high-frequency noise at small values of γ (first term of the numerator) and for excessive smoothing ("blurring") of stable geometric landscape contours at large values of γ (second term).

The results of numerical evaluations of the method's effectiveness using the noise immunity criterion and the regularized structural quality criterion of the RI for a given signal-to-noise ratio $q<6\ дБ$ are presented in Tables 1 and 2.

The analysis of the results shows that the noise level ($q<6\ дБ$) shifts the global optimum toward higher values of the regularization parameter ($\gamma=0{,}25$). At the same time, adaptive selection of the fractional spectrum rotation angle $a=0{,}75$ makes it possible to extract invariant spatial frequencies of the landscape that are least susceptible to distortion.

Table 1.

Navigation robustness evaluation of the VES measurement loop.

Algorithm Tuning Mode	a	γ	R(u,v)	σ_{sb}	$\mathbf{K}_{\mathbf{ni}}(a_{opt},\gamma)$	UAV Navigation Loop Status
Mode 1: Under-regularization	0,50	0,01	14,5	3.35	**2,00**	Tracking failure. False noise-induced peaks
Mode 2: Optimum	0,75	0,25	42	2.12	**15,00**	Stable tracking. Minimal RMS error
Mode 3: Over-regularization	0,75	2,00	16.00	1.08	**6,02**	Accuracy degradation. Correlation peak smearing

Table 2.

Structural quality analysis of reference image synthesis within the FrFT domain.

Algorithm Tuning Mode	Spectral Residual (Proximity to Input Data)	Smoothness Penalty (Derivative Norm)	Total Structural Criterion, $\mathbf{K}_{\text{str}}$	Physical State of the Synthesized Reference Image
Mode 1: Under-regularization	0,05	3,80	**3,85**	Reference texture is heavily degraded by intensive white noise.
Mode 2: Optimum	12,40	37,60	**50**	Landmark boundaries are preserved; high-frequency noise is suppressed.
Mode 3: Over-regularization	72,15	16,74	**88,89**	Image is over-smoothed; stable edge features are completely lost.

Thus, the results of the numerical evaluation of the method's effectiveness using the noise immunity criterion and the regularized structural quality criterion of the RI demonstrate that, under low signal-to-noise ratio conditions and informative feature (IF) deficiency, the use of synthesized reference images ensures robust operation of the MVS, providing a UAV positioning root mean square error (RMSE) on the order of 2–4 meters in terrain conditions.

Computational Efficiency of the Method.

Let us compare the computational complexity of the proposed method with the classical correlation algorithm for different image size values $M \times N$.

Table 3.

Computational costs dependency on the sensor resolution

Frame Size, (M x N)	Number of Operations: Classical Spatial Cross-Correlation, $O(N^4)$	Number of Operations: Developed Method with 2D FRFT Algorithm, $O(5N^2 log_2 N)$	Execution Speed-Up Factor (Gain)	Frame Processing Time (Developed Method), t
64 x 64	1.68 * 107	1.27 * 105	132.1 times	~ 1.2 ms
128 x 128	2.68 * 108	5.90 * 105	455.1 times	~ 5.3 ms
256 x 256	4.29 * 109	2.69 * 106	1598.4 times	~ 11.2 ms
512 x 512	6.87 * 1010	1.21 * 107	5698.8 times	~ 24.5 ms

The analysis of the results presented in Table 3 demonstrates a significant advantage of the developed method for RI synthesis compared to the classical correlation algorithm (CCA) used in correlation-extremum navigation systems.

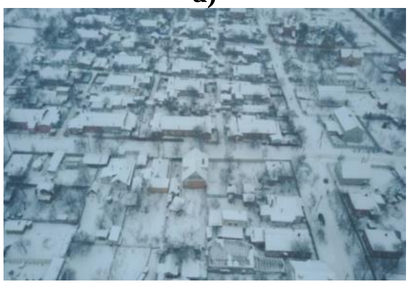
In particular, when transitioning to higher MVS resolution, which is required for high-precision UAV positioning at high altitudes, the CCA becomes inefficient due to a substantial increase in computational complexity.

In contrast, the use of the FrFT-based algorithm reduces the frame processing time to 25 ms, ensuring stable updating of navigation parameters at a rate of 30–40 frames per second in real time. For a standard CI frame of size 128 × 128, the CCA requires 268 million operations, whereas the proposed FrFT-based approach requires only 590 thousand operations.

Thus, a computational speedup of 455 times is achieved. To confirm the effectiveness of the proposed method, experiments were conducted using real images obtained with a DJI Mavic Pro quadcopter equipped with a gyro-stabilized 4K camera. The original images of snow-covered terrain and suburban areas (with an initial sensor resolution of 4000 × 3000 pixels) are shown in Fig. 1. The images were acquired at different altitudes (50 m, 100 m, 200 m).

The experimental frames shown in Fig. 1 contain natural geometric distortions (caused by changes in altitude and viewing angle) as well as opto-atmospheric interference (haze, blizzard effects, and low contrast).

a)

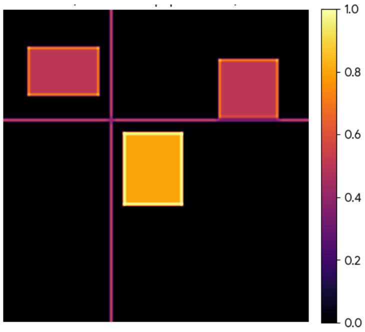


b)

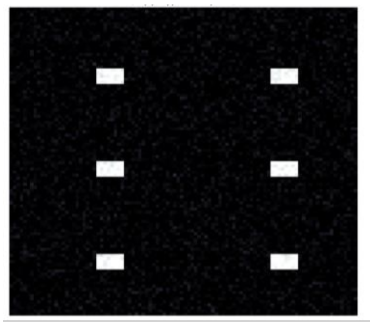
c)

Fig. 1. Original aerial photographs of the underlying surface obtained from a DJI Mavic Pro UAV:

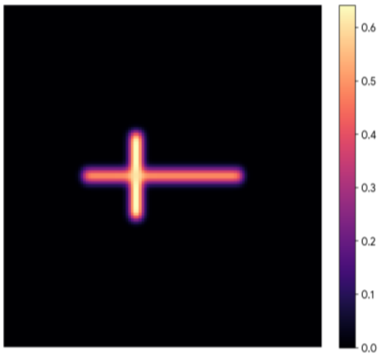
a) from a height of 50 m; b) from a height of 100 m; c) from a height of 200 m.



a)



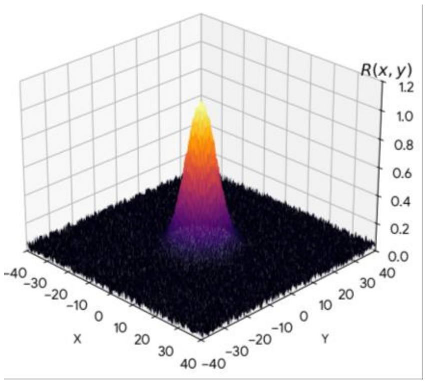
b)



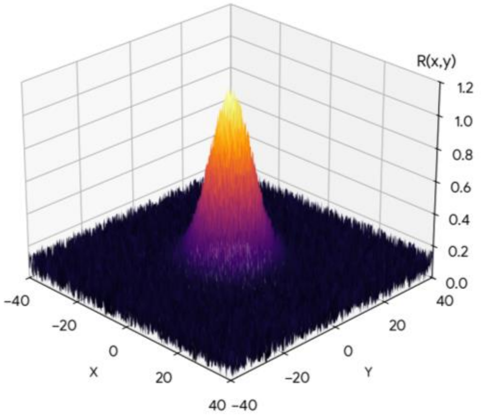
c)

Fig. 2. Synthesized reference images generated using fractional Fourier transform and Tikhonov regularization:
a) from a height of 50 m; b) from a height of 100 m; c) from a height of 200 m.

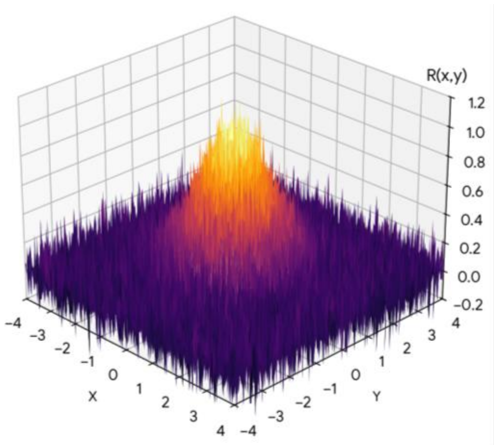
The results of constructing the decision function obtained using the synthesized reference images (RIs) are shown in Fig. 3.



a)

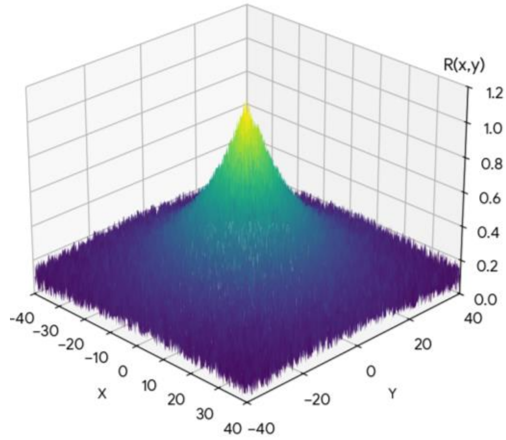


b)

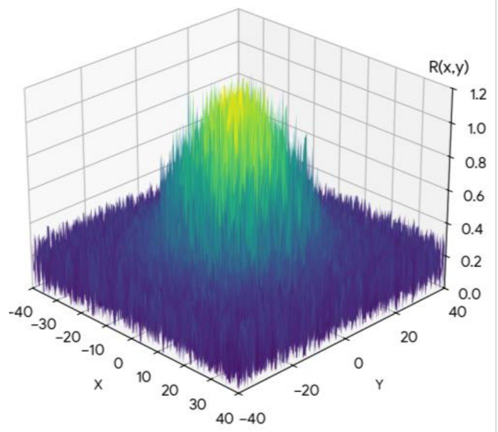


c)

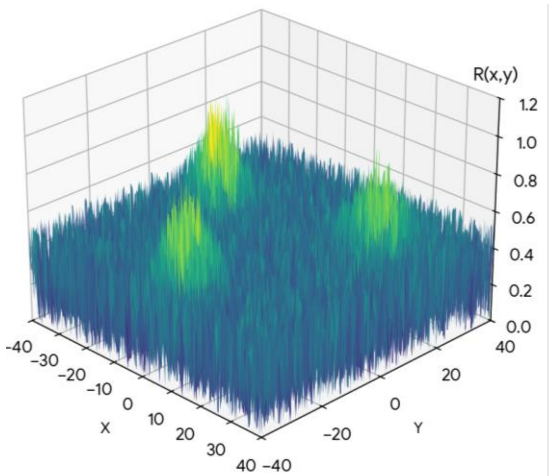
Fig. 3. Spatial distribution of decision functions obtained using synthesized reference images:
a) from a height of 50 m; b) from a height of 100 m; c) from a height of 200 m.



a)



b)



c)

Fig. 4. Spatial distribution of decision functions obtained using the classical correlation algorithm:
a) from a height of 50 m; b) from a height of 100 m; c) from a height of 200 m.

The analysis of the RF simulation results presented in Figs. 3 and 4 shows that, compared to the CCA, the proposed method of analytical RI synthesis using the FrFT and the Tikhonov regularizer provides significantly higher noise immunity in the RI formation process. The developed method enables the synthesis of RIs with high accuracy in locating the central peak even under conditions of strong initial noise, which confirms its applicability in MVSs. This effect is clearly illustrated in Fig. 3(c). Under similar noise conditions, the CCA completely loses stability and fails to determine the spatial position of the UAV with the required accuracy.

CONCLUSIONS

1. A method for analytically synthesizing RIs based on finding the optimal spatial-frequency representation of the original image in FrFT space is proposed for the first time. Unlike traditional methods of extracting IFs, the developed approach enables the extraction of the integral phase structure of an object, preserving the information invariance of the signal even under critical noise and IF deficiency.

2. An analytical expression for synthesizing the optimal RIs is obtained, implementing image processing in the spatial-frequency domain based on the combined use of an optimal-order FrFT and Tikhonov regularization.

3. An analytical model for an ensemble of optimal RIs is synthesized, describing the iterative generation of their spectral characteristics, taking into account the Tikhonov regularizer. This approach ensures high stability of the VS when changing the viewing angle and weather conditions, while maintaining low computational complexity of image matching.

4. The results of a numerical evaluation of the method's effectiveness using the developed noise immunity criteria and a regularized structural criterion for the RI quality showed that, under conditions of low signal-to-noise ratios and a shortage of information sources, the use of synthesized RI ensures stable operation of the VS system, while providing a standard deviation of UAV location of approximately 2–4 meters on the ground.

Simulations in the Python software environment demonstrated that the developed method enables the synthesis of RI with high accuracy in localizing the central peak, even in conditions of intense noise in the original image.

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