Theoretical Examination of the Volume Concentration and Nanoparticles Density Influence on the Convective Heat Transfer Enhancement of Nanofluid in 2D Cavity Including the Square Heater

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Abstract. Nanofluid is one of the solutions of heat transfer, which can apply in devices fields as nuclear power, Nano-electronics systems, and solar fluid heating. In this work, the natural convection of Cu-water nanofluid is examined using the Lattice Boltzmann Method (LBM) as a mesoscopic approach. The main objectives of this work are to study the performance and pattern of Cu-water nanofluid and to demonstrate that the nanofluids behave differently while improving their energy transfer compared to pure fluids. The set goal was achieved by solving the tasks: based on the streamlines and isotherms profiles to demonstrate how the convection process and temperature gradients improve. Also, conducting study on heat transfer of nanofluid by calculate the Nusselt number. Depending on the nanoparticle volume percentage φ, the Grashof number Gr, and the hot obstacle have a significant impact on the convection flow and rate of heat transfer. The most important result is the enhancement of heat transfer with the increasing of volume fraction for a particular Grashof number, also it improves with the rising of Grashof number for a particular nanoparticles volume fraction. Therefore, the aspect ratios of the enclosure have played a significant part in Nusselt number variation. In addition, we found that the Nusselt number Nu_{avg} is higher in the case of cavity without hot obstacle more than for cavity with hot obstacle, so the heat transfer improves in the case of cavity without hot obstacle. The significance of the obtained results consists that the nanofluid is one of the ways to improve heat transfer due to its specific characteristics and properties.
Keywords: convecție naturală, metoda Boltzmann latice, nanofluid, transfer de căldură, număr Grashof.

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Studiu teoretic al influenței concentrației volumetrice și densității nanoparticulelor asupra îmbunătățirii transferului de căldură convectiv al unui nanofluid într-o cavitate bidimensională, inclusiv un încălzitor pătrat

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Rezumat. Obiectivele principale ale acestei lucrări sunt de a studia performanța și modelul nanofluidului ”Cu-apă” și de a demonstra că nanofluidele se comportă diferit în timp ce își îmbunătățesc transferul de energie în comparație cu fluidele pure. Scopul stabilit a fost atins prin rezolvarea sarcinilor: pe baza profilurilor de fluidizare și izoterme pentru a demonstra modul în care procesul de convecție și gradienții de temperatură se îmbunătățesc. De asemenea, efectuarea unui studiu privind transferul de căldură al nanofluidului prin calcularea numărului Nusselt. În funcție de procentul de volum al nanoparticulelor φ, numărul Grashof Gr și obstacolul fierbinte au un impact semnificativ asupra fluxului de convecție și ratei transferului de căldură. Cel mai important rezultat este îmbunătățirea transferului de căldură odată cu creșterea fracției de volum pentru un anumit număr Grashof, de asemenea, se îmbunătățește odată cu creșterea numărului Grashof pentru o anumită fracție de volum de nanoparticule. Prin urmare, raporturile de aspect ale incintei au jucat un rol semnificativ în variația numărului Nusselt. În plus, am constatat că numărul Nusselt Nu_{avg} este mai mare în cazul cavității fără obstacol fierbinte mai mult decât în cazul cavității cu obstacol fierbinte, astfel încât transferul de căldură se îmbunătăţeşte în cazul cavității fără obstacol fierbinte. Semnificația rezultatelor obținute constă în faptul că nanofluidul este una dintre modalitățile de îmbunătățire a transferului de căldură datorită caracteristicilor și proprietăților sale specifice.
Cuvinte-cheie: convecție naturală, metoda ecuației Boltzmann latice, nanofluid, transfer de căldură, număr Grashof.

Теоретическое исследование влияния объемной концентрации и плотности наночастиц на усиление конвективного теплопереноса наножидкости в двумерной полости, включая квадратный нагреватель

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Аннотация. Наножидкость — это одно из решений теплопередачи, которое может применяться в таких областях, как ядерная энергетика, системы наноэлектроники и нагрев солнечной жидкости. В этой работе естественная конвекция наножидкости Cu-вода исследуется с использованием метода решеточных уравнений Больцмана при мезоскопическом подходе. Основные цели этой работы — изучить характеристики и структуру наножидкости «медь-вода» и продемонстрировать, что наножидкости ведут себя по-другому, улучшая при этом передачу энергии по сравнению с чистыми жидкостями. Исследование проведено для объемной доли твердых наночастиц и числа Грасгофа Gr в пределах от 0 до 8% и от 10^3 до 10^5, соответственно. Поставленная цель была достигнута путем решения следующих задач: на основе профилей линий тока и изотерм продемонстрировать, как улучшается процесс конвекции и температурные градиенты. Также проводятся исследования теплопередачи наножидкости путем расчета числа Нуссельта. В зависимости от объемной доли наночастиц число Грасгофа Gr и горячее препятствие оказывают существенное влияние на конвекционный поток и скорость теплопередачи. Наиболее важным результатом является рост теплопередачи с увеличением объемной доли наночастиц для определенного числа Грасгофа, а также она расчете увеличением числа Грасгофа при определенной объемной доле наночастиц. Таким образом, соотношение сторон корпуса сыграло значительную роль в изменении числа Нуссельта. Кроме того, мы обнаружили, что число Нуссельта выше в случае полости без горячего препятствия больше, чем в полости с горячим препятствием, поэтому теплообмен улучшается в случае полости без горячего препятствия. Значимость полученных результатов состоит в том, что применение наножидкости является одним из способов улучшения теплоотдачи благодаря своим специфическим характеристикам и свойствам.
Ключевые слова: естественная конвекция, метод решеточных уравнений Больцмана, наножидкость, теплопередача, критерий Грасгофа.

INTRODUCTION

In recent years, several researches on systems filled with nanofluids have been conducted due to the unsolved phenomena of nanofluids [1-5]. Heat transmission through natural convection in a nanofluid, which solid nanoparticles are suspended in a pure fluid, is a significant phenomenon due to its extensive application in science and engineering systems. Natural convection heat transfer of nanofluids is used in a wide range of industries fields, such as electronics, automotive, and nuclear reactor cooling, biomedical purposes. Dong et al. [6] simulated a natural convection around the dome in the cooling system. Li et al. [7] studied, in hydrothermal reactor, a natural convection flow and heat transfer. Shahrooz et al. [5] investigated the natural convection of nanofluid flow in a confined space in a study using the first and second laws of thermodynamics, Saha et al. [8] investigated thermal influence of heated fin on MHD natural convection flow of nanofluids inside a wavy square cavity, Saleem et al. [9], and Hu et al. [10] studied Oscillatory natural convection of nanofluid near its density maximum in a narrow horizontal annulus.

The main goal of using nanofluids is to improve energy transmission and heat exchange. Nanofluid displays several benefits compared to pure fluid (water, oil, or ethylene glycol) such as thermal conductivity and heat transfer coefficient. The addition of nanoparticles, such as aluminum oxide (Al_2O_3) [9, 10], copper (Cu) [11, 12], zinc oxide (ZnO) [15], copper oxide (CuO) [16], silicon dioxide (SiO_2) [17], silver (Ag) [18], Multi-walled carbon nanotubes (MWCNT) [19], titanium oxide (TiO_2) [20] to pure fluid enhances the heat transfer properties. Nanoparticles have a range of factors and characteristics that impact heat transfer such as the type, the size, the form, and the concentration of nanoparticles as well as the surface layer on the nanoparticles.

This property was studied by Khanafer et al. [21] in two-dimensional cavity containing nanofluids. They demonstrated that the heat transmission improves as the volume percentage of solid nanoparticle rises. Shanmugapriya et al. [22] studied the impact of nanoparticle shape in enhancing heat transfer of magnetized ternary hybrid nanofluid, Maleki et al. [23] investigated experimentally heat transfer enhancement in heated copper tube for nanofluids, Ma et al. [24] studied heat transfer enhancement of nanofluid flow at the entry region of microtubes, and Halawa et al. [25] investigated the optimum design of magnetic field arrangement to enhance heat transfer performance of magnetic nanofluid. They found an enhancement in heat transmission by rising the solid nanoparticles volume percentage. In the same direction Ali et al. [26] experimentally studied nanofluids for heat pipes used in solar photovoltaic panels, also Najim et al. [27] simulated performance improvement of shell and tube heat exchanger by using nanofluid. They reported that adding nanoparticles concentration improve Nusselt number, so heat transfer. Contrary to the general expectation, Abu-Nada et al. [28] investigated the effect of nanofluid variable properties on natural convection. The results showed a reduction in the nanoparticles volume percentage causes an enhance in heat transmission.

The reason of these unexpected findings may be understood by taking into account that the performance of nanofluids is a direct result of the two contradictory effects on heat transmission in natural convection flows. This includes effective dynamic viscosity and thermal conductivity, which influence temperature and flow fields, respectively. These parameters increase with volume fraction of suspended nanoparticles [29].

The Lattice Boltzmann method (LBM) has proved its capabilities to study many complicated fluids flow phenomena. By using this approach, Sheikholeslami et al.[30] studied a nanofluid forced convection heat transfer around the elliptic obstacle by using LBM, also Hssikou et al. [31] investigated the variation of heat transfer in a partially heated or cooled enclosure. Moreover, Li et al. [32] conducted a 3D Lattice Boltzmann method study on condensation and condensate on textured structure.

In this study, the LBM is used to simulate natural convection fluid behaviour and enhancement of heat transmission inside an enclosure.

The effect of nanoparticles enhanced thermophysical properties on fluid, presented in terms of streamlines and isotherms distribution as well as the average Nusselt number quantifying the heat transfer, is discussed in this article. The main parameters used in this study are Grashof number Gr, the percentage of the concentration of the nanoparticles φ, the aspect ratio of the closed cavity, and the hot obstacle added to the center of the cavity.

I. PROBLEM STATEMENT

A two-dimensional configuration enclosure of height H and width W filled with Cu-water nanofluid is considered in this study (Fig. 1). The size and shape of the nanoparticles are considered homogeneous and uniform. In addition, we assume that the base fluid's velocity moves at the same of solid nanoparticles and are in a condition of thermal equilibrium. The nanofluid inside the enclosure is heated from the left wall kept at hot temperature T_H, and cooled from the right-hand side wall at a cold temperature T_C, whereas the top and bottom walls are considered adiabatics.

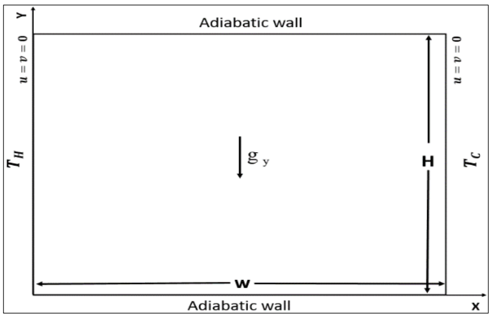


Fig. 1. Geometry enclosure

For the present study, the aspect ratio Ar is the ratio of the enclosure width to its height $\mathrm{Ar} = W/H$, which equals to 1 (square form). Note that the block dimension is fixed as $\mathrm{A_O} = L/W = 0.25$, in this study.

I. NANOFLUID AND NUMERICAL METHOD

1. NANOFLUID

Table 1 presented the thermos-physical characteristics of the pure fluid (water) and nanoparticle (Cu).

The external force term F_y at y direction and the effective thermos-physical characteristics of nanofluid are taken as.

$$F_y = (\rho\beta)_{nf}\, g_y \left(T - \frac{T_H + T_C}{2} \right), \tag{1a}$$

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_{nf}\,, \tag{1b}$$

$$\left(\rho c_p\right)_{nf} = (1-\varphi)\left(\rho c_p\right)_{nf} + \varphi\left(\rho c_p\right)_{np}, \tag{1c}$$

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_{nf} + \varphi(\rho\beta)_{np}\,, \tag{1d}$$

where φ stands for the solid volume percentage, ρ is the density, c_p represents the specific heat capacity at constant pressure, β is the thermal expansion coefficient, and subscripts f, D_2Q_9 np and nf refer, respectively, to fluid pure, solid nanoparticle, and nanofluid.

In this research, the nanofluid effective dynamic viscosity μ_{nf} and the effective thermal conductivity κ_{nf} are defined, respectively, according to the Graham [33] (2a) and Patel et al. [34] (2b) models by:

$$\mu_{nf}=\mu_f\left(1+2.5\varphi+4.5\left[\frac{1}{4\frac{h}{d_{np}}\left(1+\frac{h}{d_{np}}\right)\left(1+\frac{2h}{d_{np}}\right)^2}\right]\right), \quad (2a)$$

$$\kappa_{nf}=\kappa_f\left(\frac{\kappa_{np}}{\kappa_f}\left(1+25000\frac{u_c d_{np}}{\alpha_f}\right)\frac{d_f}{d_{np}}\frac{\varphi}{1-\varphi}+1\right), \quad (2b)$$

Where $h=9nm$ is the inter-nanoparticles spacing, d_{np} is nanoparticle diameter, d_f is the pure fluid diameter, and u_c is Brownian velocity for nanoparticles which is calculated by:

$$u_c=\frac{2k_BT}{\pi\mu_f d_{np}{}^2}, \quad (3)$$

Where $k_B=1.3807\times10^{-23}$ is the Boltzmann constant.

2. NUMERICAL METHOD

For the fluid flow and heat transfer fields, LBM based on two distribution functions to calculate the flow (f) and the temperature (g) patterns.

The governing equations for f and g are respectively:

$$f_i(\mathbf{x}+c_i\Delta t,t+\Delta t)-f_i(\mathbf{x},t)=\ldots$$
$$\ldots-\frac{1}{\tau_f}\left(f_i(\mathbf{x},t)-f_i^{eq}(\mathbf{x},t)\right)+\Delta tF_i, \quad (4a)$$

$$g_i(\mathbf{x}+c_i\Delta t,t+\Delta t)-g_i(\mathbf{x},t)=\ldots$$
$$\ldots-\frac{1}{\tau_g}\left(g_i(\mathbf{x},t)-g_i^{eq}(\mathbf{x},t)\right), \quad (4b)$$

where Δt represents lattice time step $\Delta t=1$, $\mathbf{c}_i$ is the discrete particle velocity vector in the i-direction, and τ_f (τ_g) is the relaxation times for the flow (temperature) field. f_i^{eq} (g_i^{eq}) is the flow's (temperature) local equilibrium distribution function [35, 36] at second and first orders, respectively, as:

$$f_i^{eq}(x,t)=\omega_i\rho\left(1+\frac{3(c_i.u)}{c^2}+\frac{9(c_i.u)^2}{2c^4}-\frac{3u^2}{2c^2}\right), \quad (5a)$$

$$g_i^{eq}(x,t)=\omega_i'T\left(1+\frac{3(c_i.u)}{c^2}\right), \quad (5b)$$

***Table* 1. Thermophysical characteristics of pure fluid and suspended nanoparticles.**

Physical characteristics	Fluid phase (Water)	Solid phase (Cu)
ρ (kg/m^3)	997.1	8954
C_p (J/kg K)	4179	383
k (W/mK)	0.613	400
$\beta\times10^{-5}$ (K^{-1})	21	1.67
μ (kg/ms)	8.55×10^{-4}	-
$d_{f/np}$ (nm)	0.29	10

where the quantities ρ and $\mathbf{u}$ stand for macroscopic density and velocity, respectively. ω_i (ω_i') the weight for flow (temperature). Figure 2 illustrates D_2Q_9 model used in this study.

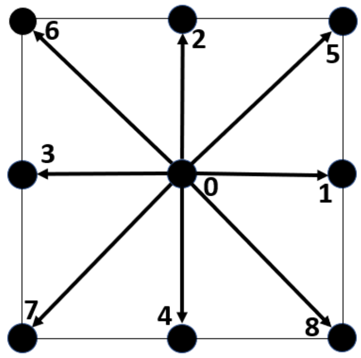


Fig. 2. The discrete velocity sets of the D_2Q_9 model.

The kinematic viscosity ν is given as:

$$\nu=\left[\tau_f-\frac{1}{2}\right]c_s{}^2\Delta t. \quad (6a)$$

And the thermal diffusivity α is defined by:

$$\alpha=\left[\tau_g-\frac{1}{2}\right]c_s{}^2\Delta t. \quad (6b)$$

By fixing Grashof (Gr), Mach (Ma), and Prandtl (Pr) numbers, the viscosity ν is calculated by:

$$\nu=c_s.Ma.N\sqrt{1/G_r}, \quad (7)$$

where N denotes the lattices number in the y-direction. Prandtl (Pr) and Grashof (Gr) numbers are given by:

$$Gr = \frac{\beta g_y N^3 (T_H - T_C)}{\nu^2}, \qquad (8a)$$

$$Pr = \frac{\nu}{\alpha}. \qquad (8b)$$

The external force term $\mathbf{F_i}$ of this problem of natural convection, which is applied in collision step, as shown in Eq. (3a) is provided by:

$$\mathbf{F_i} = \frac{\omega_i}{c_s^2} F_y \mathbf{c_i}, \qquad (9)$$

Where $F_y = \rho . g_y . \beta . \Delta T$ is the buoyancy force term, g_y is the gravitational vector, and Δt is computed as $\Delta T = T - \frac{T_H + T_C}{2}$.

Finally, the macroscopic quantities of density ρ, Momentum $\rho\mathbf{u}$, and temperature T are calculated by using equations as follows:

$$\rho = \sum_{i=0}^{8} f_i, \qquad (10a)$$

$$\rho\mathbf{u} = \sum_{i=0}^{8} f_i \mathbf{c_i}, \qquad (10b)$$

$$T = \sum_{i=0}^{8} g_i. \qquad (10c)$$

Along the heated wall, the local (Nu) and the average Nusselt number (Nu_{avg}) are determined respectively, as follow:

$$Nu = -\frac{\kappa_{nf}}{\kappa_f} \frac{\mathrm{H}}{T_H - T_C} \left(\frac{\partial T}{\partial x}\right)_{x=0}, \qquad (11a)$$

$$Nu_{avg} = \frac{1}{\mathrm{H}} \int_0^{\mathrm{H}} Nu . dy. \qquad (11b)$$

In this work, note that in part of results and discussion, all the values of stream function have been calculated by $|\psi'_{max}|$, which is defined as:

$$|\psi'_{\max}| = |\psi_{\max}| \times \nu_{nf}. \qquad (12)$$

II. RESULTS AND DISCUSSION

1. HOICE OF MESH AND VALIDATION

To ensure a grid independent solution, a thorough mesh testing process was conducted. Seven alternative mesh configurations were explored for the case of $Gr = 10^5$ and $\varphi = 5\%$. The average Nusselt number Nu_{avg} was calculated on the heated side wall.

We concluded that a grid independent solution is ensured with an $\mathrm{N} = 140$ grid size, as shown in *Table* 2. The second step is to validate our results with those of the literature.

So, our findings are compared to Khanafer et al. [21] in terms of the temperature distribution on axial midline (Fig. 3).

***Table* 2. Grid independent test on the hot wall for $\varphi = 0\%$ and for $\varphi = 5\%$.**

Grid size	Nu_{avg}	
	$\varphi = 0\%$	$\varphi = 5\%$
60×60	7.30436087	9.08368015
80×80	7.74876928	9.31092167
100×100	7.83996344	9.41771507
120×120	7.88659382	9.47491074
140×140	7.90700579	9.50557423
160×160	7.90773249	9.50557423
180×180	7.89107037	9.51982403

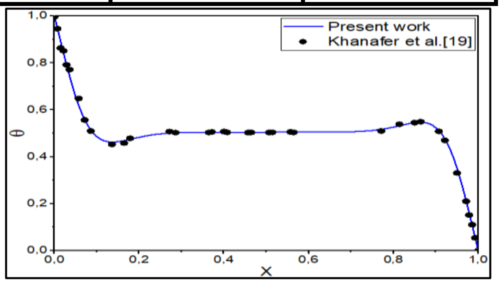
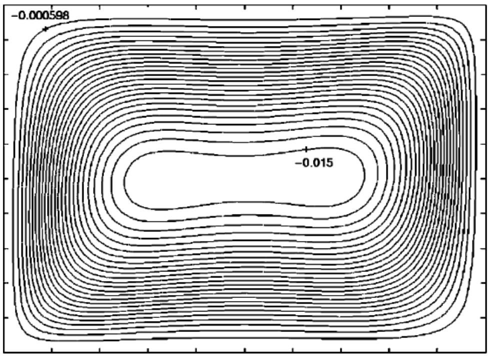


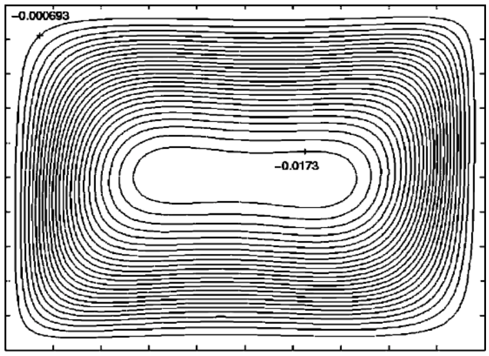
Fig. 3. The temperature profile at the vertical mid-section for $Gr = 10^5$ and $\varphi = 5\%$.

The Fig. 4 and 5, which also are compared with those of Khanafer et al. [21], illustrate the isotherms and streamlines profiles.

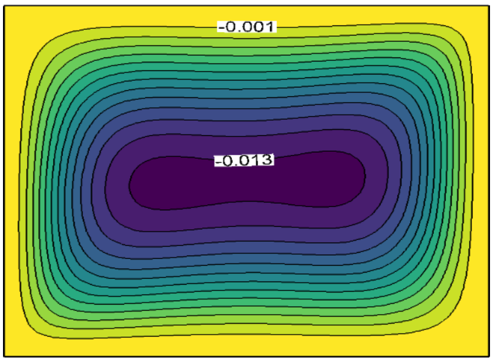
This shows the good agreement of C the present results with literature. So, the ability of lattice Boltzmann method for solving the problems of nanofluid.



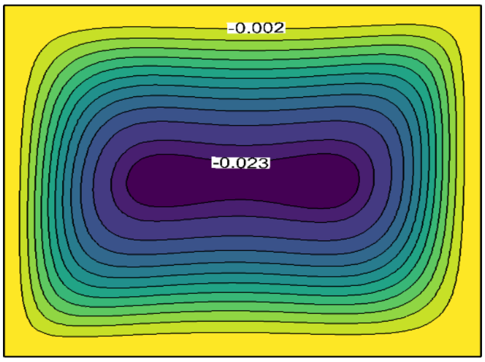
(a) φ = 0%, *Khanafer et al.'s* [21] *result.*



(b) φ = 8%, *Khanafer et al.'s* [21] *result.*

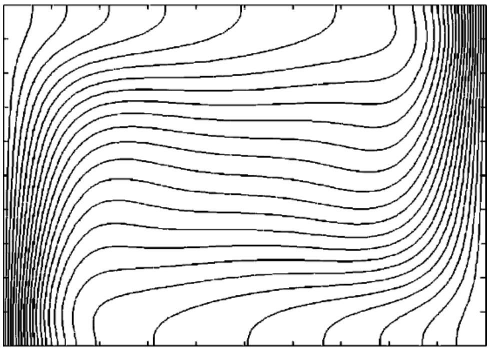


(c) φ = 0%, *present result.*

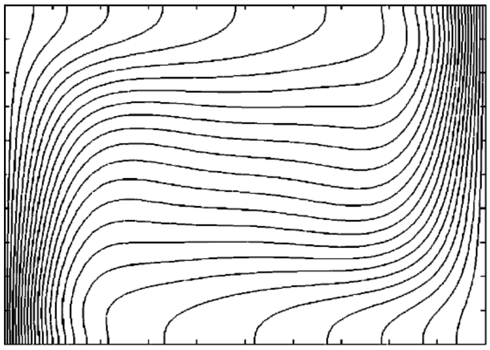


(d) φ = 8%, *present result.*

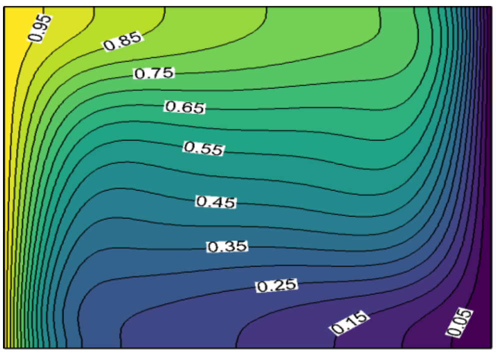
Fig. 4. Streamlines for $\varphi = 0\%$ and $\varphi = 8\%$, $Gr = 10^4$.



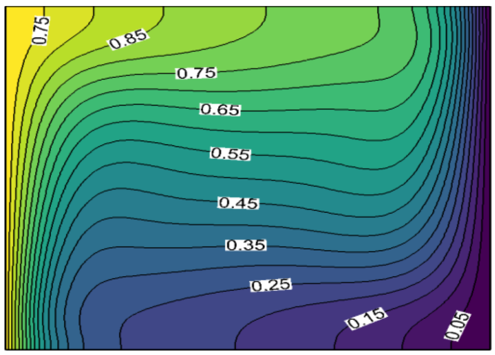
(a) $\varphi = 0\%$, *Khanafer et al.'s* [21] *result.*



(b) $\varphi = 8\%$, *Khanafer et al.'s* [21] *result.*



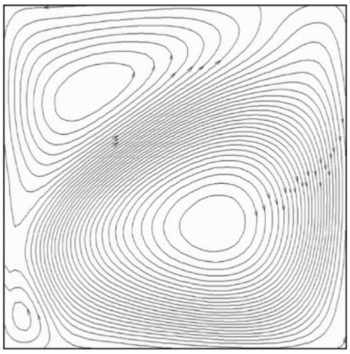
(c) $\varphi = 0\%$, *present result.*



(d) $\varphi = 8\%$, *present result.*

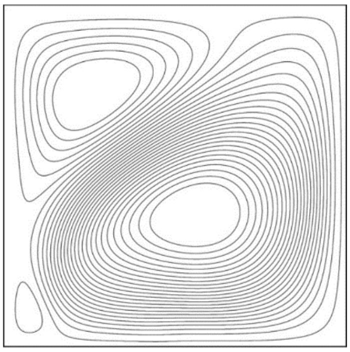
Fig. 5. Isotherms for $\varphi = 0\%$ and $\varphi = 8\%$, $Gr = 10^4$.

In order to validate the elaborated code, we also compare our findings with the experimental results of natural convection in a cavity filled with air $Pr = 0.71$ [37]. The heat source is located at $x_d = 0.25$ on the bottom side of the enclosure; its length is $\frac{2}{5}\mathrm{H}$. Both vertical walls are maintained at a cold temperature, while the top wall is adiabatic. From this examination, it shows that there is a good agreement between the experimental and numerical data of Corvaro and Paroncini [37] and our numerical findings (Fig. 6).

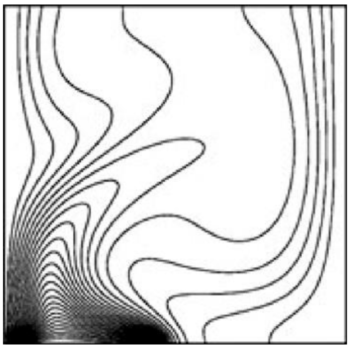


(a) *Experimental result of Corvaro et Paroncini* [37].

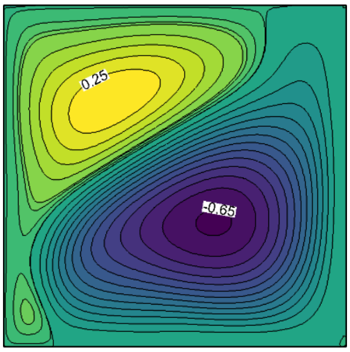
(b) *Experimental result of Corvaro et Paroncini* [37].



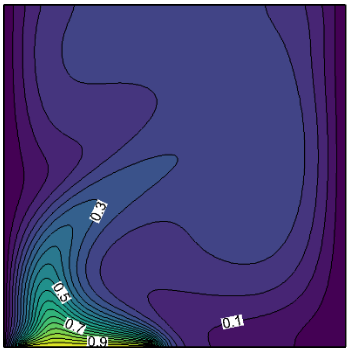
(c) *Numerical result of Corvaro and Paroncini* [37].



(d) *Numerical result of Corvaro et Paroncini* [37].



(e) *Present result.*



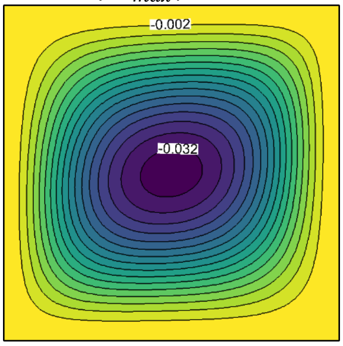
(f) *Present result.*

Fig. 6. Streamlines (a, c, and e) and isotherms (b, d, and f) for $Ra = 2.12 \times 10^5$ in the case of air.

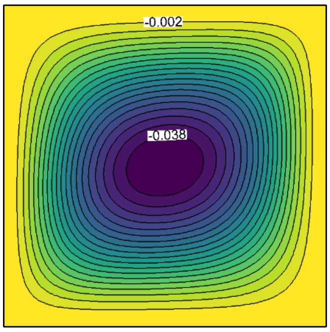
2. EFFECT OF NANOPARTICLES VOLUME FRACTION φ AND GRASHOF NUMBER Gr

The graphical findings are presented to determinate the impacts of solid volume percentage φ on the streamlines, isotherms profiles, and Nusselt number for two different Grashof number values Gr. Figures 6 and 7 show the streamlines and isotherms profiles for $Gr = 10^3$ with various nanoparticles concentration φ $(0-8\%)$. The streamlines show that the maximum stream function absolute value $|\psi'_{max}|$ has increased as volume fractions of nanoparticles rise (Fig. 7). The formation of new vortexes, so the intensity of streamlines enhances due to change of nanofluid viscosity.

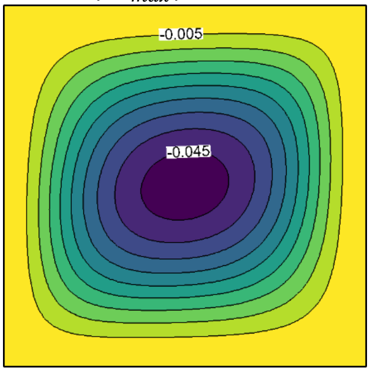
(a), $|\psi'_{max}| = 0.0331702$



(b), $|\psi'_{max}| = 0.0404312$



(c), $|\psi'_{max}| = 0.0479178$



(d), $|\psi'_{max}| = 0.0636707$

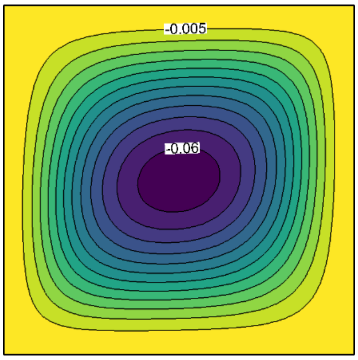
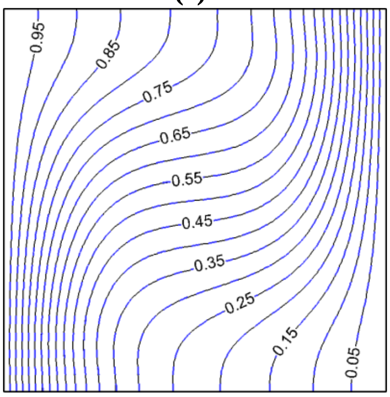
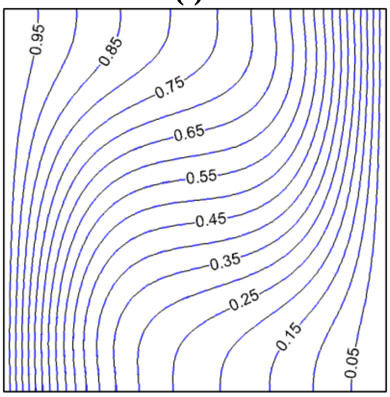


Fig. 7. Streamlines for $Gr = 10^3$, (a) $\varphi = 0\%$, (b) $\varphi = 2\%$, (c) $\varphi = 4\%$, and (d) $\varphi = 8\%$.

(e)



(f)



(g)

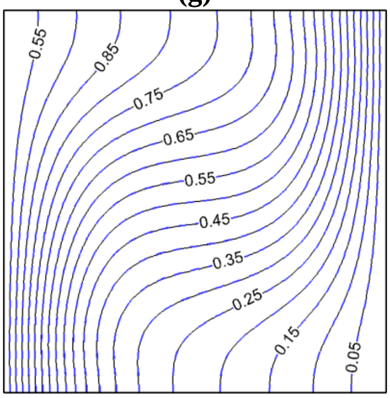
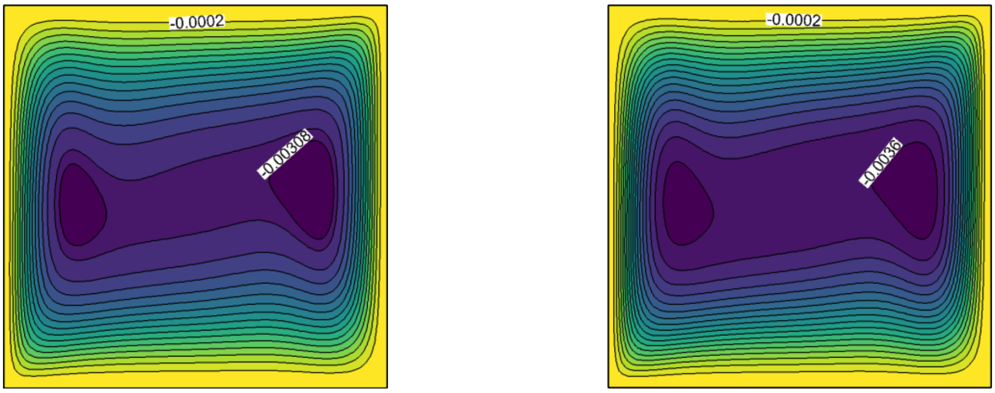


Fig. 8. Isotherms for $Gr = 10^3$, (e) $\varphi = 2\%$ (---), (f) $\varphi = 4\%$ (---), (g) $\varphi = 8\%$ (---), $\varphi = 0\%$ (−).

(a), $|\psi'_{max}| = 0.0032047$

(b), $|\psi'_{max}| = 0.0037453$



(c), $|\psi'_{max}| = 0.0043166$ **(d)**, $|\psi'_{max}| = 0.00539658$

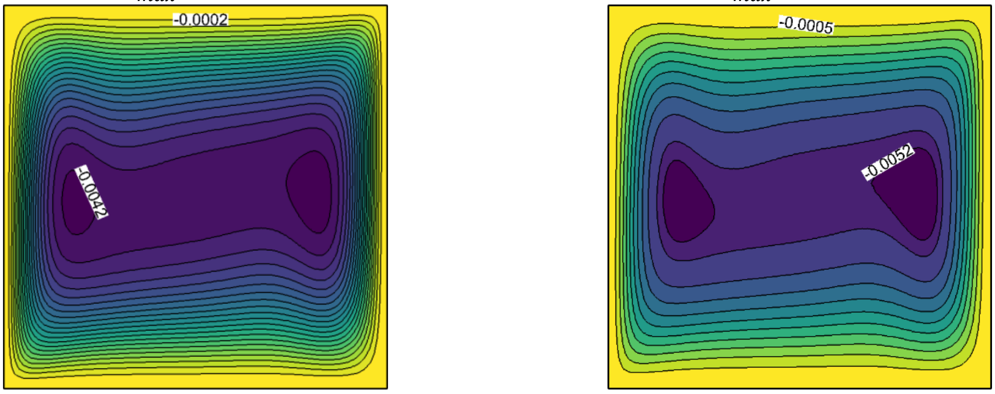
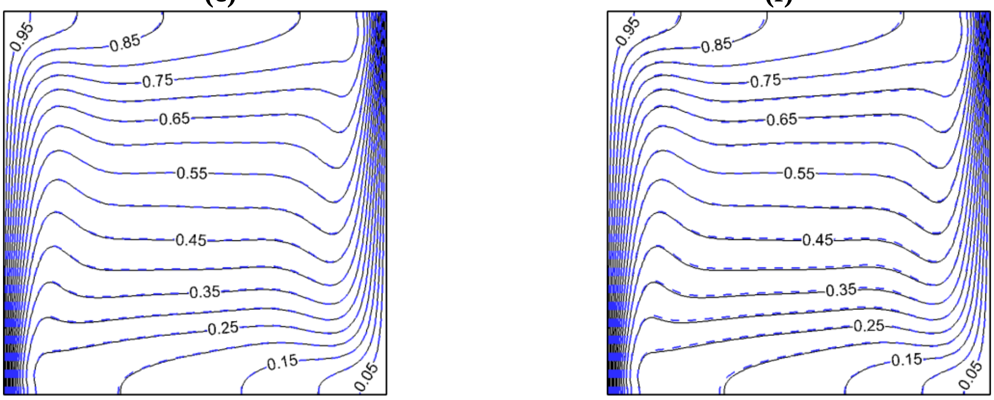


Fig. 9. Streamlines for $Gr = 10^5$, (a) $\varphi = 0\%$, (b) $\varphi = 2\%$, (c) $\varphi = 4\%$, and (d) $\varphi = 8\%$.

(e) **(f)**



(g)

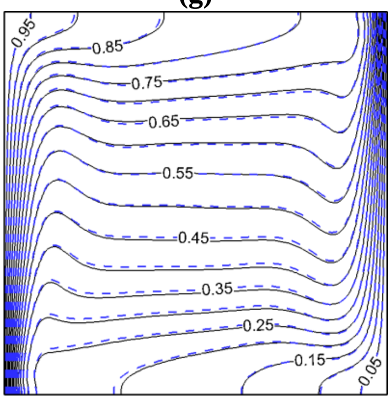


Fig. 10. Isotherms for $Gr = 10^5$, (e) $\varphi = 2\%$ (---), (f) $\varphi = 4\%$ (---), (g) $\varphi = 8\%$ (---), and $\varphi = 0\%$ $(-)$.

Figure 8 demonstrates that, at low Grashof number $Gr=10^3$, it is observed that there is no significant difference between the lines of temperature for all values of nanoparticles volume percentage due to the lowest Grashof value which make the viscous forces dominate over buoyancy forces, so, the heat transfer is dominant by conduction process.

Figure 9 illustrates that for high Grashof numbers $(Gr=10^5)$, as the solid nanoparticles volume percentage φ rises, the $|\psi'_{max}|$ of the cavity increase, indicating better convection. It shows how the Grashof number affects the isotherms, which are more sensitive for $Gr=10^5$ than $Gr=10^3$. However, when the volume percentage of nanoparticles rises, the isotherms become increasingly distorted due to an increase of buoyance forces, and the lines on the bottom wall (the top) move and come close to the right side (the left side). Due to higher conductivity of nanofluid, which might improve heat transmission, the thermal boundary layer is more sensitive (Fig. 10). Which means that the natural convection is enhanced.

Figure 11 presents the effect of Grashof number Gr, for a specific nanoparticle concentration $\varphi=4\%$, on the temperature distribution. It shows that the temperature is influenced by Grashof number; for $Gr=10^5$ the temperature profile is distorted more than for $Gr=10^4$ and $Gr=10^3$. Figure 12 elucidates that the impact of volume fraction φ, for various Grashof numbers Gr (10^3, 10^4, and 10^5), on the average Nusselt number Nu_{avg} along the hot side wall. It is clear that the Nu_{avg} rises as the volume fraction φ and Grashof number Gr augment due to reduce of the thermal boundary layer thickness which is the area near a hot surface where heat is transferred from the surface to the fluid. Accordingly, heat transfer is improved as the solid nanoparticles volume percentage φ and Grashof number augment. For $Gr=10^3$, the Nu_{avg} was not affected much by increasing nanoparticle volume percentage. By increasing Grashof number, the buoyancy forces increase which enhances energy transport. Thus, the heat transmission enhances by convection which is dominated at a high Grashof numbers. In fact, the inertia forces for $Gr=10^3$ are lower than those for $Gr=10^5$. Because of this, a greater negative impact of nanoparticles at $Gr=10^3$, which lowers the Nusselt number more compared to higher Grashof numbers.

For various aspect ratios of the enclosure Ar, the impact of the solid nanoparticle concentration φ and the Grashof number Gr on the average Nusselt number Nu_{avg} along the heated side wall, is displayed in Fig. 13. The results indicated that throughout aspect ratios and Grashof numbers, the Nu_{avg} value increased with the rise in nanoparticles volume proportion.

It is obviously observed that for all of Grashof number values the great values of the Nu_{avg} are observed for $\mathrm{Ar}=0.5$ then $\mathrm{Ar}=2$, and the small values are observed for $\mathrm{Ar}=1$.

If one of enclosure dimensions increases, either in width or height, the transfer of energy improves. Our results established that the cavity's geometry also plays a crucial role in heat transmission.

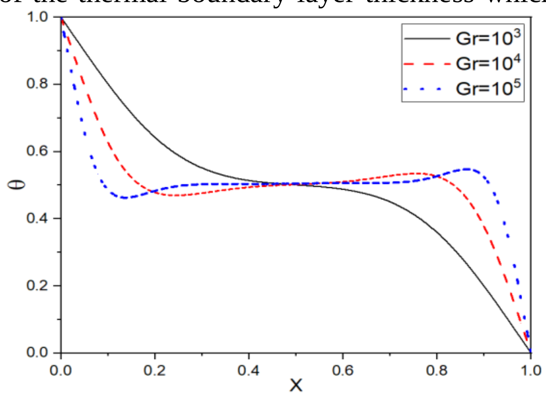


Fig. 11. Temperature profiles in the middle of vertical axis for different values for $\varphi = 4\%$ of $Gr = 10^3, 10^4$, and 10^5.

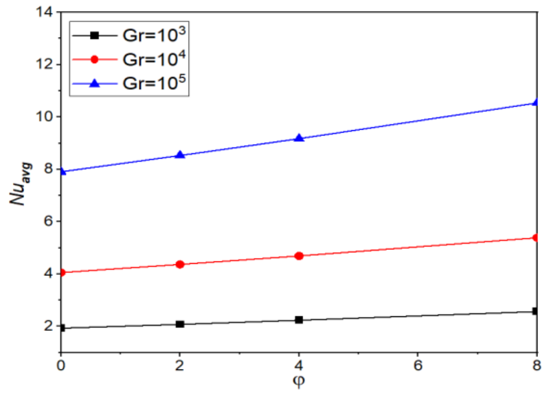


Fig. 12. Average Nusselt numbers Nu_{avg} for various values of φ along the heated surface.

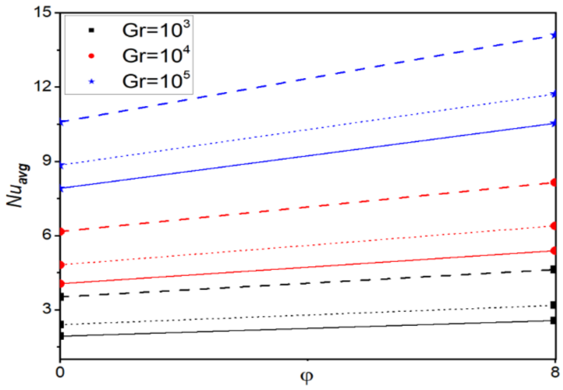


Fig. 13. Average Nusselt numbers Nu_{avg}, (−) Ar = 1, (---) Ar = 0.5, and (· · ·) Ar = 2.

3. EFFECT OF HOT OBSTACLE

In this part of this study, the effect of hot square obstacle situated in the center of a square enclosure is investigated. The block dimension is $A_o = 0.25$ (Figs. 14 and 15).

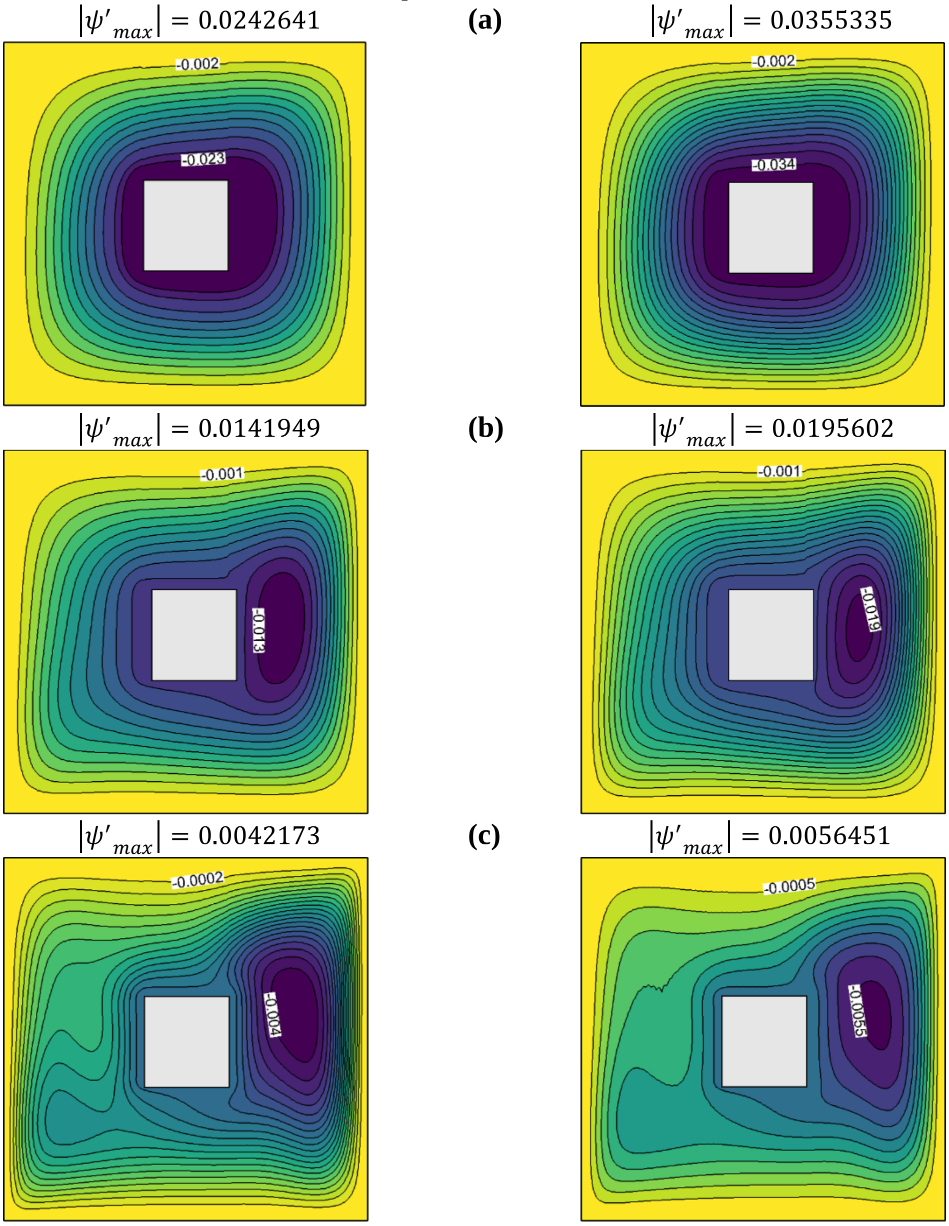


Fig. 14. Streamlines for $\varphi = 0\%$ (on the left) and $\varphi = 4\%$ (on the right), (a) $Gr = 10^3$, (b) $Gr = 10^4$, and (c) $Gr = 10^5$.

Figure 14 presents the streamlines contours for two distinct values of solid volume percentage $\varphi = 0\%$ and $\varphi = 4\%$, and for three values of Grashof numbers $Gr = 10^3$, 10^4, and 10^5. It shows that for $Gr = 10^3$, from $\varphi = 0\%$ to $\varphi = 4\%$, the streamlines have taken the same form with an increasing in the stream function absolute values $|\psi'_{max}|$.
As a consequence, the convection process is enhanced with the increasing of nanoparticle volume proportion. Moreover, we noticed that for the whole range of volume proportion of nanoparticles φ and Grashof numbers Gr, it is clear that due to the heated block, the circulation of stream tends to be elongated in rightward horizontal direction, resulting the vortex for $Gr = 10^4$ and $Gr = 10^5$. Also, the results demonstrate that with increasing Gr, the convection process dominates. For Fig. 15, the isotherms are affected by heated obstacle. We observed that for low Grashof number, the isotherms are distorted around the heated block, which causes convection to dominate. Also, we show that there is no visible difference in the lines of temperature between $\varphi = 0\%$ and $\varphi = 4\%$ due to the low Grashof number $(Gr = 10^3)$, the impact of nanoparticle volume fraction starts to appearance from $Gr \geq 10^4$.

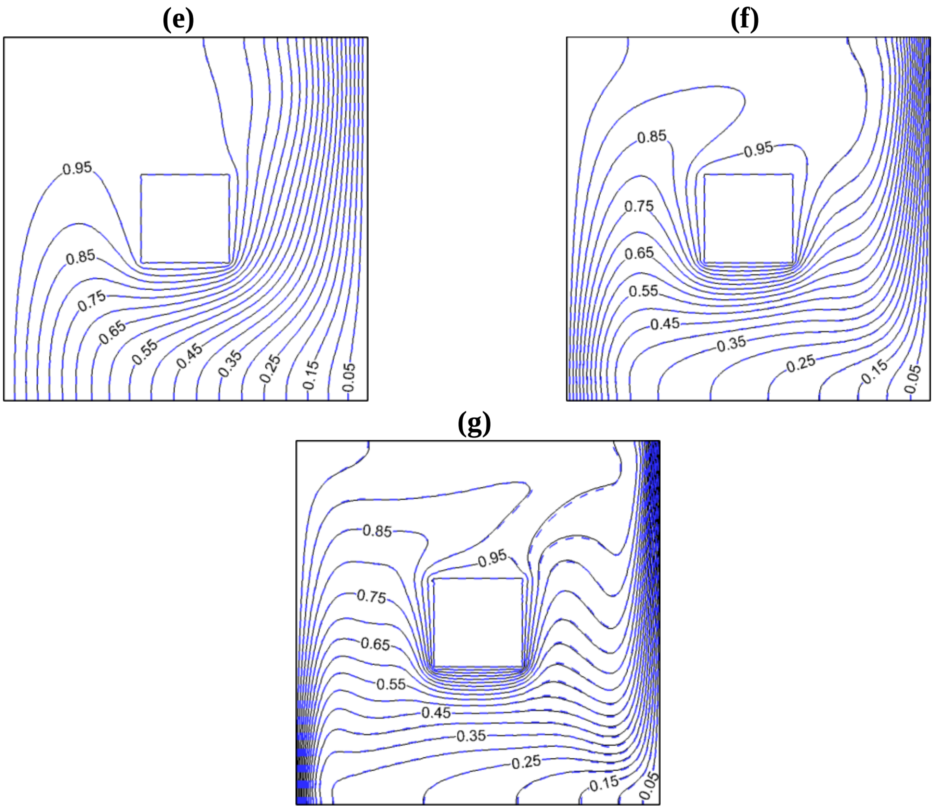


Fig. 15. Isotherms for $\varphi = 0\%$ (−) and $\varphi = 4\%$ (---), for (e) $Gr = 10^3$, (f) $Gr = 10^4$, and $Gr = 10^5$.

To investigated the effect of the existence of the hot obstacle on the Nusselt number Nu_{avg}. Figure 16 presents that the average Nusselt number calculated along the hot wall of the cavity with and without the hot obstacle for a particular Grashof number $Gr = 10^5$ and different nanoparticles concentration values $0 \leq \varphi \leq 8$. The results showed that without the existence of obstacle, the exchange between the surfaces of the cavity and the fluid is high and important and the thermal boundary layer thickness reduces, so the heat transfer enhances more than the case of cavity with obstacle.

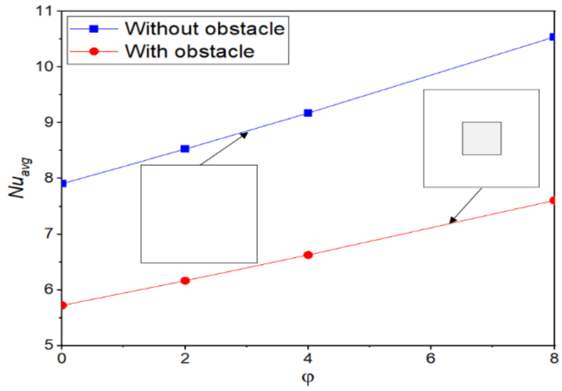


Fig. 16. Average Nusselt numbers on the hot wall of cavity with and without the obstacle for $Gr = 10^5$

CONCLUSION

A numerical study of natural convection heat transport of a nanofluid in a two-dimensional cavity (with and without obstacle), by using Lattice Boltzmann Method (LBM), is conducted. The key parameters are volume percentage of solid nanoparticles φ and Grashof numbers Gr, which are in the ranges $Gr = 10^3 - 10^5$ and $\varphi = 0 - 8\%$, respectively. The main results of this work are as follows:

- For low Grashof numbers $(Gr = 10^3)$, the heat transmission is not sufficiently sensitive to the solid nanoparticle volume percentage φ due to decrease of buoyancy forces. While with the rising of Gr to the $Gr = 10^5$, the heat transfer enhances.
- As increases the volume percentage φ for high Grashof numbers Gr leads the heat transmission enhancement efficiently. Also, by widening the cavity, either in height or width, we enhance the heat transmission rate.
- The aspect ratios of enclosure have a significant role on heat transfer of nanofluid.
- The hot square obstacle has an important effect on natural convection flow of nanofluid, which the Nusselt number Nu_{avg} is higher in the case of cavity without hot obstacle more than for cavity with hot obstacle, so the heat transfer improves in the case of cavity without hot obstacle.

Nomenclatures

Ar	Aspect ratio
Ao	Obstacle dimensions
d	Length of the heat source along x-axis
c_p	Specific heat capacity at constant pressure
d_{np}	Diameter of nanoparticles
d_f	Diameter of base fluid
h	Inter-nanoparticles spacing
$\boldsymbol{c_i}$	Discrete particle speeds
c_s	Speed of sound
f	Density distribution function
f^{eq}	Equilibrium state of f
g	Temperature distribution function
g^{eq}	Equilibrium state of g
g_y	Gravity acceleration
Ma	Mach number
κ	Thermal conductivity
Nu	Local Nusselt number
Pr	Prandtl number
Gr	Grashof number
p	Pressure
W	Enclosure length
H	Enclosure height
L	Block length
T	Temperature
$\mathbf{u}$ (u, v)	Vector of velocity
x, y	Cartesian coordinates

Greek symbols

ρ	Density
α	Fluid thermal diffusivity
ν	Kinematic viscosity
β	Thermal expansion coefficient
θ	Normalized temperature
μ	Dynamic viscosity
ω_i	Weighted factor for flow (D_2Q_9)
φ	Nanoparticle volume fraction
ψ	Stream function
τ_v	Relaxation time for flow
τ_c	Relaxation time for temperature
Δt	Time increment

Subscripts

f	Fluid
np	Nanoparticle
nf	Nanofluid
avg	Average
H	Hot
C	Cold

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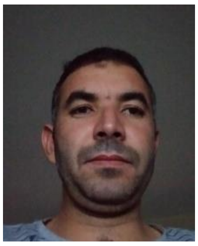

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