A Contribution to the Foundations of the Generalized Theory of Electrical Circuits: Concepts, Methodology and Axiomatics

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Abstract. The aim of this article is to create a conceptual and methodological foundation for a new, generalized theory of electrical circuits. In the generalized theory, it is assumed that the relation between voltage and current of inductors and capacitors can be described by derivatives and integrals of arbitrary order, including fractional ones. Traditional electrical engineering assumes that this relation is defined by first-order operators and represents a special case of the generalized theory. The necessity of a generalized theory is caused by a contradiction. On the one hand, there exists a broad class of elements and circuits whose behaviour is not described by the traditional theory. These include systems with memory effects, distributed and network structures, electrochemical elements, supercapacitors, and composite materials. On the other hand, their behaviour does not have a unified theoretical justification within the framework of classical circuit theory. The article analyses this contradiction and identifies the limitations inherent in the classical approach. A coherent conceptual framework of the generalized circuit theory is introduced. Methodological requirements are formulated, and a minimal axiomatics is constructed to ensure a consistent circuit description of elements with arbitrary temporal dynamics without revising the fundamental laws of electrical circuits. The scientific novelty of the obtained results lies in the formation of a conceptual basis for the further development of a generalized theory of electrical circuits. It is intended to eliminate the theoretical fragmentation and insufficient rigor characteristic of existing fractional and integral models, while preserving the basic concepts and laws of traditional circuit theory.

Keywords: generalized electrical elements, time operators, derivatives of arbitrary order, elements with memory, fractional models, element order,

DOI: https://doi.org/10.52254/1857-0070.2026.1-69.05
UDC: 621.3.01: 517.518

O contribuție la fundamentele teoriei generalizate a circuitelor electrice: concepte, metodologie și axiomatică

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Rezumat. Scopul prezentului articol este crearea unui fundament conceptual și metodologic pentru o nouă teorie generalizată a circuitelor electrice. În teoria generalizată se presupune că relația dintre tensiune și curent în bobine și condensatoare poate fi descrisă prin derivate și integrale de ordin arbitrar, inclusiv fracționar. Electrotehnica tradițională presupune că această relație este definită prin operatori de ordinul întâi și reprezintă un caz particular al teoriei generalizate. Necesitatea unei teorii generalizate este determinată de existența unei contradicții. Pe de o parte, există o clasă largă de elemente și circuite al căror comportament nu este descris de teoria tradițională. Aceasta include sisteme cu efecte de memorie, structuri distribuite și de tip rețea, elemente electrochimice, supercondensatoare și materiale compozite. Pe de altă parte, comportamentul acestora nu are o fundamentare teoretică unitară în cadrul teoriei clasice a circuitelor. În articol este realizată analiza acestei contradicții și sunt identificate limitările inerente abordării clasice. Este introdus un aparat conceptual coerent al teoriei generalizate a circuitelor. Sunt formulate cerințe metodologice și este construită o axiomatică minimă, care asigură o descriere coerentă a circuitelor pentru elemente cu dinamică temporală arbitrară, fără revizuirea legilor fundamentale ale circuitelor electrice. Noutatea științifică a rezultatelor obținute constă în formarea unui bazis conceptual pentru dezvoltarea ulterioară a unei teorii generalizate a circuitelor electrice. Aceasta este destinată eliminării fragmentării teoretice și a rigorii insuficiente caracteristice modelelor fracționare și integrale existente, păstrând în același timp conceptele și legile de bază ale teoriei tradiționale a circuitelor.

Cuvinte-cheie: elemente electrice generalizate, operatori temporali, derivate de ordin arbitrar, elemente cu memorie, modele fracționare, ordinul elementului, model operatorial, echivalența circuitelor, reducerea schemelor, metodologia electrotehnicii, axiomatica circuitelor, dinamică neîntreagă, teoria generalizată a circuitelor.

Некоторые аспекты основ обобщенной теории электрических цепей: понятия, методология и аксиоматика

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Аннотация. Целью настоящей статьи является создание концептуального и методологического фундамента для новой, обобщенной теории электрических цепей. В обобщенной теории предполагается, что связь между напряжением и током катушек и конденсаторов может описываться производными и интегралами произвольных порядков, в том числе и дробных. В традиционная электротехника предполагает, что эта связь задается операторами первого порядка и является частным случаем обобщенной теории. Необходимость обобщённой теории обусловлена противоречием. С одной стороны, существует широкий класс элементов и цепей, поведение которых не описывается традиционной теорией. Это системы с эффектами памяти, распределённые и сетевые структуры, электрохимические элементы, суперконденсаторы и композитные материалы. С другой стороны, их поведение не имеет единого теоретического обоснования в рамках классической теории цепей. В статье выполнен анализ указанного противоречия и выявлены ограничения, присущие классическому подходу. Введён согласованный понятийный аппарат обобщённой теории цепей. Сформулированы методологические требования и построена минимальная аксиоматика, обеспечивающие непротиворечивое схемное описание элементов с произвольной временной динамикой без пересмотра фундаментальных законов электрических цепей. Научная новизна полученных результатов заключается в формировании концептуального базиса для дальнейшего создания обобщённой теории электрических цепей. Она призвана устранить теоретическую разрозненность и недостаточную строгость, характерные для существующих дробных и интегральных моделей, сохраняя при этом базовые понятия и законы традиционной теории цепей.

***Ключевые слова*:** обобщенные электрические элементы, временные операторы, производные произвольного порядка, элементы с памятью, дробные модели, порядок элемента, операторная модель, эквивалентность цепей, редукция схем, методология электротехники, аксиоматика цепей, нецелочисленная динамика, обобщенная теория цепей.

INTRODUCTION. THREE CLOUDS ON THE HORIZON

In 1900, William Thomson (Lord Kelvin), summarising the development of classical physics, stated that physics as a science was essentially complete, and that on its clear horizon there remained only two small clouds: the unresolved problem associated with the absence of the ether in the Michelson–Morley experiment, and the discrepancy between classical radiation theory and experimental data. These two small clouds soon led to a radical revision of the fundamental foundations of physics and became the starting point of the theory of relativity and quantum mechanics.

On the horizon of modern electrical circuit theory, however, not two but as many as three “clouds” can be observed—three phenomena that do not fit within the framework of traditional theory. These phenomena manifest themselves in circuits with supercapacitors, batteries, and in processes occurring at the “electrode–electrolyte” interface. The first phenomenon is manifested in the fact that transient processes in such circuits do not decay exponentially, but according to a power law of the form $t^{-\alpha}$. The second phenomenon is closely related to the first and consists in the absence of a characteristic time scale (time constant) for transient processes in these devices. The third phenomenon manifests itself in the form of memory effects, when the state and response of a circuit depend on its entire past history rather than on a finite set of internal variables. The presence of such memory indicates temporal nonlocality of the processes in these devices. Modern electrical circuit theory does not describe these phenomena within the framework of its basic equations and conceptual apparatus, but merely reproduces their behaviour by means of external, empirically introduced models.

One of the most well-known examples of deviation from classical behaviour is dielectric relaxation in solid, polymeric, and composite materials. It has been experimentally established that the frequency dependence of the complex dielectric permittivity in such media does not correspond to the model of an ideal capacitor and is characterised by power laws with non-integer exponents [1]. The so-called universal dielectric response is observed over an extremely wide range of frequencies and materials, which indicates the fundamental nature of this phenomenon.

Similar effects are also widely observed in electrochemical systems. The impedance of porous electrodes, electrochemical capacitors, and batteries does not vary with frequency proportionally to $1/\omega$, but instead exhibits a more complex power-law dependence of the form $1/\omega^{\alpha}$. This dependence is not characteristic of an R–C circuit and is traditionally approximated by complex artificial models [2]. Detailed studies show that such behaviour is caused by the complex geometry of the electrodes, diffusion processes, non-uniform current distribution, and interactions at interfacial boundaries [3]. Within the framework of classical electrical engineering, such systems are often described using equivalent circuits, which, however, represent no more than a convenient approximation and do not fit into a unified structure of theoretical electrical engineering.

Another example of non-classical behaviour is provided by distributed electrical systems, in particular lossy multiconductor cable lines. Modern experimental and theoretical studies [4] show that in such systems, in the low-frequency and transient regimes, a mode of signal propagation is realised that is neither purely wave-like nor strictly quasi-stationary. Instead, a diffusion–wave dynamics is observed, characterised by power-law attenuation and phase-shift laws that cannot be reproduced by classical models with integer order.

The results of experimental studies of artificial electrical structures, such as ladder RC circuits, are particularly illustrative. It has been shown in [5] that, as the depth and complexity of such structures increase, their frequency and time characteristics approach power-law dependences with non-integer exponents. At the same time, the effective behaviour of the entire structure cannot be accurately reproduced by a finite number of traditional ideal elements, even if each individual component of the circuit is fully classical.

The examples presented demonstrate that deviations from classical models of electrical elements are not exceptions or purely mathematical abstractions. On the contrary, such effects arise systematically in real electrophysical systems. However, the existing theory of electrical circuits does not offer a generalised approach that would allow such elements to be incorporated into a unified logical and mathematical framework without violating the fundamental principles of electrical engineering.

This circumstance indicates the need to develop a generalised theory of electrical circuits capable of describing a wider class of relations between current and voltage than is permitted by classical models.

The term “generalised theory of electrical circuits” is used not in the sense of a general formulation of classical theory. On the contrary, it refers to a generalisation of electrical circuit theory with respect to the order of temporal dynamics, that is, to an extension of the basic mathematical apparatus of traditional theory (differential calculus) allowing non-integer and arbitrary orders of derivatives and integrals in the relations between electrical quantities. This theory should provide a consistent description of elements arising in complex electrophysical systems and, at the same time, preserve continuity with respect to the basic concepts and laws of classical electrical engineering.

The aim of this article is to develop the concept of a generalised theory, as well as its methodology and axiomatics.

CURRENT STATE

Classical electrical circuit theory is based on the assumption that the relationship between current and voltage in elements is local in time and is described by integer-order derivatives or integrals of the first order. This assumption works well for idealised resistors, inductors, and capacitors; however, as shown above, it proves insufficient for describing a wide class of real materials and structures. Nevertheless, in the absence of an alternative theory, numerous attempts have been made to describe their behaviour within the framework of classical theory.

For electrochemical systems and impedance spectroscopy systems, a constant phase-shift angle is observed over a wide frequency range [7], which contradicts classical electrical engineering. In practice, the circuit behaves as a Constant Phase Element (CPE), whose response cannot be described by an equivalent circuit comprising an ideal resistor and capacitor. In [8], the limiting (long-term) properties of the CPE are analysed, and it is demonstrated that an attempt to interpret the CPE as an “ordinary capacitor with a correction” encounters fundamental limitations when considering long times: the question of a physically meaningful limiting capacitance and its relation to distributions of time constants arises. However, [9] shows that the use of the CPE is often based simply on fitting impedance data to a preselected equivalent circuit, while the physical interpretation of the order of the fractional derivative characterising the temporal relationship between current and voltage remains a matter of discussion.

In [10], another fundamental aspect is discussed: the charge–voltage relationship for such an element may be nonlinear and becomes linear only in the limiting case of an ideal capacitor. This fact is important, since many circuit interpretations implicitly assume linearity of the basic elements. Taken together, works [7–10] show that even if the practical fitting of an equivalent circuit to experimental data is successful, the theoretical justification of such behaviour remains incomplete and ambiguous.

A number of works are devoted to the transition from frequency-domain models of such circuits to time-domain descriptions that make it possible to analyse transient processes in them. In [11], a time-domain model is proposed that allows the behaviour of a battery under various current regimes to be described by means of a correct transition from the frequency-domain description of CPE branches to the time domain. This makes the model convenient for practical calculations; however, the order of the fractional derivative relating current and voltage is still fitted to experimental data and is not derived from general principles of electrical circuit construction. In [12], the problem of correct initialisation (the specification of initial conditions) when transferring the CPE to the time domain is discussed: it turns out that the choice of the initialisation procedure itself can significantly affect the modelling result, that is, the “same” circuit in the frequency domain does not give rise to a unique canonical time-domain description without additional conventions. This class of works demonstrates a high level of engineering elaboration of the approach, but at the same time emphasises its descriptive nature: the parameters and the order of the temporal dependence are introduced solely to reconcile the model with experimental data and are not derived from general principles of interconnection and transformation of electrical circuits.

Fractional circuit models are also widely used to describe the behaviour of supercapacitors, lithium-ion, and lithium-polymer batteries. Compared with integer-order models, such models better describe real operating regimes; however, the order of the fractional temporal dependence is introduced as a parameter approximating distributed relaxation and is almost always limited to the interval from 0 to 1. Despite the development of physically consistent formulations for individual types of energy storage devices, correctness in this case is likewise achieved through particular assumptions rather than within the framework of traditional electrical circuit theory.

In the field of transmission lines and distributed structures, “non-classical” effects are often associated with the fact that losses and inhomogeneities are distributed along the length and involve a wide spectrum of time scales. In [16] and [17], fractional variants of the telegrapher’s equations are considered, which make it possible to reproduce the power-law attenuation and phase shifts observed in lossy lines: replacing ordinary derivatives with fractional ones indeed yields a qualitatively correct picture, but the order is again introduced as a model parameter. It is significant that in such works the order of the fractional temporal dependence is almost always considered within the interval from 0 to 1, since the aim is to describe slowed dynamics associated with memory effects and diffusion processes. Cases in which the order exceeds unity are, as a rule, either not considered or are left without an explanation of how such an order should be interpreted and preserved in the analysis and transformation of electrical circuits.

Numerical studies (for example, those focused on the stability and accuracy of solving fractional equations) reveal another feature: the very formulation of the problem requires additional conventions regarding initial conditions and the procedures for transitioning between the time and frequency domains [17]. This once again demonstrates that, in the absence of a generalised theoretical foundation, fractional models are applicable only within the framework of specific equations and a specific problem formulation.

Another group of works addresses the inverse problem: rather than modelling real circuits with fractional equations, it seeks or constructs electrical structures whose behaviour can be described by fractional models. However, in this approach as well, the fractional order is understood as a means of describing behaviour lying between resistive, capacitive, and inductive regimes, and therefore is almost always considered only within the range from 0 to 1.

In [19], it is shown that certain real capacitor structures can behave non-ideally and be described by CPE-type models, and that the degree of this non-ideality can be varied and measured. This confirms the physical nature of the parameter “order of the derivative”; however, the models considered are still limited to the particular case of capacitive behaviour and do not lead to a general theory of circuits of arbitrary order.

In a number of recent applied works, models with non-integer order of the temporal dependence between current and voltage are beginning to be used not only to describe material properties, but also in the analysis of electrical circuits, for example LCL filters in power electronics. In [20, 21], it is shown that replacing classical integer-order models of individual components with fractional-order models makes it possible to describe changes in the shape of the amplitude–phase characteristics of the filter, including a different slope of the frequency characteristics and a phase shift in the operating range. What is essential here is the very fact that such models are beginning to be used for the calculation and analysis of circuits, and not only for fitting experimental data. At the same time, this fact once again emphasises the need for a generalised theoretical foundation for the application of derivatives of arbitrary order in a generalised theory of electrical circuits. A well-developed mathematical apparatus of operators of arbitrary order already exists for such a generalisation [22, 23, 24, 25]; however, it was developed mainly with reference to abstract dynamical systems and has so far not been used for the description and analysis of electrical circuits [26].

Thus, an analysis of the current state of the art shows that existing approaches either attempt to describe non-classical behaviour of electrical circuits within the framework of classical theory by introducing additional parameters and fractional orders, or use such orders as a convenient tool for approximating specific effects. In all the cases considered, the order of the temporal dependence is introduced as an external characteristic of the model and is not derived from general principles of analysis and transformation of electrical circuits. This indicates the absence of a methodological and theoretical foundation in which derivatives of arbitrary order would be consistent with the basic concepts of electrical engineering and the general principles of analysis and transformation of electrical circuits.

In the next chapter, the basic concepts, methodological requirements, and axiomatic principles of the generalised theory of electrical circuits will be formulated.

TERMINOLOGY AND METHODOLOGY OF THE GENERALISED THEORY

The aim of the present chapter is to formulate the methodological foundations of a generalised theory of electrical circuits, which extends the domain of applicability of classical theory without undermining its basic principles. The discussion does not concern the replacement or revision of classical electrical engineering, but rather its refinement and generalisation in such a way that, within a unified theoretical framework, a wider range of elements and operating regimes can be correctly described than is permitted by the traditional approach.

Invariants of the theory

The generalised theory of electrical circuits is constructed as an extension of classical electrical engineering and therefore preserves all of its fundamental provisions that do not depend on the specific form of the temporal relationship between current and voltage of elements. These provisions are regarded as invariants and are accepted without modification.

The physical meaning and units of measurement of the basic electrical quantities – current, voltage and charge – as well as their interrelation through fundamental relations, are preserved. These quantities remain basic and do not depend on the chosen element model.

The fundamental laws of electrical circuits, including Kirchhoff’s laws and the commutation law, also remain unchanged. These laws are applicable regardless of the order of the derivatives used in element models.

The possibility of representing a circuit as a directed graph is also preserved. For linear elements, the principle of superposition is retained. Accordingly, traditional methods of circuit analysis and transformation remain admissible, including series and parallel connection of elements, as well as the concept of equivalent transformations.

In the generalised theory of electrical circuits, the concept of instantaneous power as the product of the voltage and current of an element is fully preserved. Power remains a basic physical quantity characterising the exchange of energy between circuit elements and external sources and does not depend on the specific form of the temporal relationship between current and voltage. The principle of power balance is also preserved: at any instant in time, the total power supplied by the sources is equal to the total power consumed and stored by the circuit elements.

Time-domain and frequency-domain descriptions of processes in a circuit retain the status of equivalent representations of the same physical system, provided that the correct conditions for transition between them are satisfied. The general theory does not abolish these representations, but specifies the domain of their applicability.

Thus, classical electrical circuit theory is fully preserved as a particular case of the generalised theory. All subsequent generalisations and extensions of the class of passive element models pertain not to the laws and basic physical quantities and concepts, but to the element models and the rules governing their temporal properties.

Terms and definitions

Basic electrical quantities. Current *i(t)*, voltage *u(t)* and charge *q(t)* are regarded as basic electrical quantities with classical physical meaning and dimensions and are used as the main variables of circuit description.

Electrical circuit element. An electrical circuit element is the minimal part of a circuit for which a rule defining the relationship between current and voltage at its terminals is specified. An element is defined by its external behaviour and is considered independently of its internal physical realisation.

Depending on the nature of the relationship between current and voltage, elements may be linear or nonlinear. In what follows, the main attention is devoted to linear elements.

Generalised element. A generalised element is defined as an electrical circuit element for which the relationship between current and voltage is specified by a temporal operator and is not limited to first-order integer derivatives. Classical electrical circuit elements are regarded as special cases of generalised elements.

Temporal relationship between current and voltage. The temporal relationship between current and voltage is understood as the rule that relates these quantities in time. This relationship may be local or nonlocal in time. The relationship is local in time if it is determined by instantaneous values of the quantities and their derivatives, and nonlocal if it depends on their prehistory. In classical electrical circuit theory, the resistor, inductor and capacitor belong to elements with a temporally local relationship between current and voltage, since their behaviour is described by differential equations of integer order.

Order of an element. The order of an element is a characteristic of the temporal operator (derivative or integral) that relates the current and voltage of the element. In classical electrical circuit theory, the order of an element is not introduced explicitly; however, it is implicitly fixed through the form of the temporal relationship between current and voltage. For a resistor, this relationship does not contain temporal operators, which corresponds to zero order. For an inductor, the relationship includes the first derivative of current with respect to time, which corresponds to order “+1”. For a capacitor, the relationship is expressed through the first derivative of voltage or, equivalently, the integral of current with respect to time, which corresponds to order “–1”. Thus, classical theory uses a fixed and limited set of orders (–1, 0, +1) without identifying it as an independent characteristic of an element. In the generalised theory, the order of an element is introduced explicitly and may take arbitrary values.

Equivalence and circuit transformation. Two circuits, or any of their parts, are called equivalent if, under identical external excitations, they exhibit identical external behaviour at a specified set of external terminals. A circuit transformation is the replacement of one circuit (or its part) by another circuit while preserving their equivalence. Reduction is a special case of transformation and is associated with the elimination of internal variables, nodes or branches, as a result of which their number is reduced.

The definitions introduced establish the conceptual foundation of the generalised theory of electrical circuits, but by themselves do not determine which element models and circuit operations are admissible. For this purpose, it is necessary to explicitly formulate methodological requirements that establish the rules for working with temporal relationships, element orders and circuit transformations.

Methodology of the generalised theory

In classical electrical circuit theory, element models and operations on circuits were formed historically and were largely used as self-evident. Extending the theory to a more general class of temporal relationships renders these implicit conventions insufficient and requires their explicit formulation. In this connection, methodological requirements are formulated below that define the boundaries of admissible models and transformations and serve as the conceptual basis for the axiomatic construction of a generalised theory of electrical circuits.

Preservation of the fundamental laws of electrical circuits. The general theory does not modify the basic laws of electrical circuits.

Kirchhoff’s laws are preserved in unchanged form, and elements and their models must be correctly embedded in the standard circuit description with nodes, branches and loops. This ensures continuity of the generalised theory with respect to classical electrical engineering.

Applicability of analysis methods without additional conditions. Analysis methods based on circuit topology and rules of element interconnection are applicable to generalised circuits without any modifications. These include nodal and loop methods, rules for formulating equations according to Kirchhoff’s laws, as well as the principle of superposition for linear elements. These methods do not depend on the specific form of the temporal relationship between current and voltage.

Applicability of analysis methods with account taken of the temporal properties of elements. Methods of traditional theory that use standard differential equations and the customary specification of initial conditions are applicable to generalised circuits, taking into account the specific features of the temporal relationships of elements. Such methods include classical transient analysis, standard application of the Laplace method, representation of circuits in the form of state-space models, as well as methods of stability and oscillation analysis. When generalised elements are used, these methods require careful interpretation of temporal operators and of the conditions for specifying the initial state.

Explicit operator specification of elements. The temporal relationship between current and voltage of an element must be specified explicitly in operator form, with indication of the order of the element and its sign. The order of an element is part of its model and must be unambiguously defined. The specific analytical realisation of the temporal operator is not fixed at the methodological level.

Invariance of the element order. The order of an element is a characteristic of its model and must not change as a result of formal operations of element interconnection. A change in order is permitted only as a consequence of reduction associated with the elimination of internal elements or variables and must be explicitly indicated and justified.

Explicitness of equivalent replacement of elements. When several elements are interconnected, the order of each element and its sign must be explicitly preserved in the circuit equations. The order of an element cannot be changed or “hidden” as a result of formal combination of elements.

If the combined behaviour of several elements is replaced by a single equivalent element, such a replacement must lead to a new model with an unambiguously specified order and sign. Such a replacement is regarded as a change of the element model and requires explicit indication.

This requirement is necessary in order for element orders and the results of equivalent transformations to remain controllable and unambiguously interpretable.

Correctness of equivalent transformations. Equivalent circuit transformations are admissible only if their external behaviour at specified terminals is preserved. Equivalence is established with respect to a specific class of excitations and observed quantities, rather than on the basis of coincidence of individual element parameters.

The explicitness of equivalent element replacement shows what happens to the element model during replacement, while the correctness of equivalent transformations determines when such a replacement is admissible at all.

Correctness of transition between the time and frequency domains. An electrical circuit may be described both in the time domain and in the frequency domain. Both descriptions refer to the same physical system but are applied for different tasks. When transitioning from one description to the other, it is necessary to explicitly indicate under what conditions such a transition is admissible and what exactly is being compared.

Continuity with classical theory. The general theory must reproduce classical elements and the results of traditional electrical circuit theory as special cases for appropriate choices of element order. This ensures continuity of the transition from classical models to generalised ones.

The formulated methodological requirements are not axioms of the theory and do not contain statements about the properties of elements and circuits. They define the framework of admissible models, operations and interpretations within which a consistent formal theory can be constructed. In the next section, on the basis of these requirements, the axiomatics of the generalised theory of electrical circuits is formulated, defining the minimal set of initial statements used for further analysis and derivation of consequences.

AXIOMATICS OF THE GENERALISED THEORY OF ELECTRICAL CIRCUITS

For the formal construction of a generalised theory of electrical circuits, a minimal set of initial statements accepted without proof is fixed. These statements define the structure of the objects of the theory and the admissible level of their description, without substituting methodological requirements or repeating the definitions introduced earlier. The axioms are formulated in such a way that classical electrical circuit theory is reproduced as a special case.

Axiom 1. Circuit representability

Any electrical circuit can be represented as a collection of elements interconnected at nodes and branches and described by currents and voltages at the terminals of the elements.

This axiom establishes the circuit-based nature of the description and introduces the level of abstraction at which electrical circuits are considered, independently of the physical realisation of the elements.

Axiom 2. Fundamental laws of electrical circuits

For any electrical circuit, the fundamental laws of electrical engineering hold, expressing charge conservation at nodes and the potential nature of the electric field in the quasi-stationary approximation. These laws are applicable to all elements and do not depend on the form of the temporal relationship between current and voltage.

Axiom 3. External nature of the element model

The behaviour of an electrical circuit element within a circuit is fully determined by the relationship between current and voltage at its terminals and does not depend on its internal physical structure, provided that the external element model is the same.

This axiom formalises the “black box” principle and separates circuit description from microscopic models.

Axiom 4. Existence of an operator relationship of an element

For each electrical circuit element, there exists an operator relationship between current and voltage at its terminals that determines its behaviour in time.

This axiom asserts the very fact of the existence of a temporal relationship in operator form, without fixing its analytical realisation.

Axiom 5. Closedness of circuit description

The set of operator relationships of elements and the fundamental laws forms a closed system of equations sufficient to describe the behaviour of an electrical circuit under given external excitations.

This axiom guarantees that the circuit description is self-sufficient and does not require the introduction of additional relations outside the framework of the theory.

Axiom 6. Invariance of external behaviour

If two circuits, under identical external excitations, exhibit identical external behaviour at a specified set of terminals, then for the purposes of circuit analysis they are considered equivalent, regardless of their internal structure.

This axiom defines the object of equivalence at the level of the theory and serves as a basis for the analysis and comparison of circuits.

Axiom 7. Invariance of physical quantities

Currents, voltages and powers used in the circuit description of electrical circuits retain their physical meaning and interpretation regardless of the chosen element model and the form of its temporal relationship.

This axiom establishes continuity of physical interpretation when extending the class of models and prevents reinterpretation of basic quantities when transitioning to generalised elements.

The formulated system of axioms is minimal and sufficient for constructing a generalised theory of electrical circuits. Methodological requirements define the rules for the correct use of these axioms, whereas specific forms of temporal operators, issues of reduction and particular element models are considered at subsequent stages as consequences or applied realisations of the theory.

EXAMPLES OF APPLICATION

As noted in the Introduction, a wide class of experimentally observed effects in electrochemical systems, interfacial structures, and distributed electromagnetic systems can be grouped into three characteristic phenomena that are not explained by the traditional circuit theory: the power-law character of transient processes, the absence of a characteristic time scale, and pronounced memory effects and temporal nonlocality.

In this section, each of these phenomena is analysed in terms of the concepts, methodology, and axiomatics introduced in the present paper. This demonstrates that the proposed generalised theory of electrical circuits is structurally sufficient and can be applied to describe all three classes of phenomena within a unified circuit-based approach.

Power-law character of transient processes. In classical lumped-parameter circuits, the time dependence of currents and voltages (in general terms,

a(t)) is proportional to an exponential function:

$$a(t) \sim e^{t/\tau},$$

where τ – is the time constant characteristic of the circuit. However, in electrochemical interfaces, porous electrodes, supercapacitors, and heterogeneous dielectrics, power-law decay of transient processes is observed:

$$a(t) \sim t^{\beta},\ \ 0 < \beta < 1.$$

In the frequency domain, this corresponds to the universal dielectric response (UDR), in which the complex impedance obeys a fractional power-law dependence

$$\underline{Z} = \frac{1}{K \cdot (j\omega)^{\alpha}},\quad 0 < \alpha < 1,$$

as systematised in [1] and confirmed in a large number of subsequent studies [27, 28].

In the time domain, this is equivalent to the relation

$$i(t) = K \cdot \frac{d^{0,5}}{dt^{0,5}} u(t). \tag{1}$$

Such a relationship between the current and the voltage of an element is neither described nor derived within the traditional theory of electrical circuits. At the same time, the methodology and axiomatics formulated in this work directly imply the possibility of the existence of such an element (Axiom 4), while the invariance of Kirchhoff's laws and the closedness of the circuit-based description (Axiom 5) guarantee that the introduction of fractional-order elements does not violate the fundamental laws of the generalised theory. Within the developed methodology, the power-law form of the transient process is a direct consequence of the chosen operator order of the element and does not require the introduction of exotic semi-infinite distributed circuits. Thus, the phenomenon of power-law decay is described as a structural property of the element rather than as a model approximation.

Absence of a characteristic time scale of transient processes.

For classical RC circuits, a single time constant $\tau = R \cdot C$, is characteristic, which naturally determines the time scale of a given transient process and makes it possible to estimate its duration. However, for elements governed by equation (1) and exhibiting power-law decay in time, the time constant is absent. Moreover, in circuits with such elements, the phase shift between current and voltage, when passing to the frequency domain, remains practically constant:

$$\varphi = -\frac{\pi}{2} \cdot \alpha.$$

Such scale-invariant behaviour is widely described in the literature on electrochemical impedance spectroscopy [27–29].

The absence of a characteristic time scale in the generalised circuit theory follows directly from the assumption of the admissibility of scale-invariant operators introduced in the methodological part of the paper. An element with a fractional operator order does not contain a distinguished relaxation time, which is consistent with the principle of explicit specification of the operator order of an element and with the methodological distinction between equivalence and reduction of circuits formulated in the paper. Within the axiomatics (Axiom 4–5), the absence of a time constant is interpreted as a fundamental property of the element and does not require the use of complex equivalent circuits. Thus, scale-invariant behaviour is naturally incorporated into the generalised theory.

Memory effects and temporal nonlocality.

For many systems with a fractional impedance response, a dependence of the response on the entire history of the excitation is characteristic. Mathematically, this is expressed by hereditary relations of the form

$$u(t) = \int_0^t k(t-\tau) \cdot i(\tau) d\tau$$

where the kernel $k(t)$ has a power-law character.

Such a relation gives rise to a temporally nonlocal nature of the voltage evolution. In contrast to circuits with classical elements, the voltage at a given instant of time depends on the history of the voltage evolution prior to that instant. Within the framework of the classical theory, this leads to the well-known problem of specifying initial conditions and to ambiguities in the transition between time-domain and frequency-domain descriptions, which are discussed in detail in the literature [12, 30].

The axiomatics of the generalised circuit theory naturally presupposes the presence of such phenomena. Memory effects and temporal nonlocality are directly covered by the assumption of temporal nonlocality of operators introduced in the methodological part of the paper. Within Axiom 4, memory is regarded as an intrinsic property of an element, defined by the relation between its voltage and current, rather than as some externally induced property. At the same time, the invariance of the description and of the power balance (Axiom 5) is preserved regardless of the presence or absence of memory.

In addition, the methodological requirement of correctness of the transition between time-domain and frequency-domain descriptions, formulated in the paper, ensures a consistent treatment of memory effects and initial conditions, excluding formal contradictions between spectral and temporal analyses.

Thus, all three phenomenological “clouds” indicated in the Introduction receive a direct interpretation within the framework of the generalised theory of electrical circuits:

– power-law transient processes correspond to elements with a non-integer operator order;

– the absence of a characteristic time follows from the scale invariance of operators;

– memory effects reflect the temporal nonlocality of derivatives of arbitrary order.

All these effects are described without modification of the fundamental laws of electrical circuits, which confirms the universality and internal consistency of the axiomatics and methodology of the proposed theory.

CONCLUSION

In the present work, a conceptual, methodological, and axiomatic foundation is established for the construction of a generalised theory of electrical circuits, aimed at extending the admissible class of temporal relations between the current and voltage of elements without revising the fundamental laws of electrical engineering. The proposed approach preserves the physical meaning of basic electrical quantities, the terminal-based formulation, and the fundamental laws of circuit theory, while creating strict formal prerequisites for incorporating elements with arbitrary, including non-integer, orders of temporal dynamics into circuit descriptions.

A key result of the paper is the consistent introduction of operator-based temporal relations and the order of an element as independent characteristics at the level of the general theory. Unlike existing approaches, where fractional or integral models are mainly used as approximations of experimental data, the order of temporal dynamics is here treated as a structural property of an element, subject to explicit specification and control in circuit analysis and transformations. The specific analytical realisation of the temporal operator is deliberately not fixed, allowing a clear separation between circuit-structural properties and particular mathematical representations.

The formulated methodological requirements and the minimal system of axioms ensure the closedness and internal consistency of circuit descriptions involving elements of arbitrary order. It is shown that extending the class of admissible element models does not affect the invariants of the classical theory—Kirchhoff’s laws, the terminal-based formulation, the principle of power balance, and the equivalence of time- and frequency-domain descriptions under a correct transition. Thus, the classical theory of electrical circuits is naturally recovered as a special case of the generalised theory for appropriate choices of element order.

It is further demonstrated that the developed methodology and axiomatics are not purely formal but encompass a wide class of experimentally observed phenomena not explained by traditional circuit theory. In particular, the power-law character of transient processes, the absence of a characteristic time scale, and memory effects with temporal nonlocality receive direct and internally consistent interpretations within the proposed framework.

Power-law transients are interpreted as a direct consequence of the operator order of elements, rather than as artefacts of artificial equivalent circuits or distributed models. The absence of a characteristic time constant follows naturally from the scale invariance of fractional-order operators, while memory effects and temporal nonlocality are treated as intrinsic properties defined by operator relations between current and voltage, without violating circuit description or power balance.

Overall, the generalised theory of electrical circuits does not adapt known phenomena to predefined models, but provides a unified conceptual and formal language in which these phenomena arise as natural consequences of element structure and circuit analysis principles. This eliminates the theoretical fragmentation and methodological uncertainty typical of existing fractional and integral models and creates a basis for their systematic inclusion in a unified theory.

The results presented should be regarded as an initial stage in the development of a generalised theory of electrical circuits. Future work will address specific classes of generalised elements, formal rules for their interconnection and reduction, analysis of operator-order changes under equivalent transformations, and the formulation of circuit equations for complex distributed and network structures. In the longer term, this should lead to a comprehensive theory in which phenomena appearing as “clouds on the horizon” of classical electrical engineering are explained as direct consequences of circuit structure and universal analytical principles.

REFERENCES

[1] Jonscher A.K. The universal dielectric response. Nature, 1977, vol. 267, no. 5613, pp. 673–679.

[2] Bisquert J., Garcia-Belmonte G., Fabregat-Santiago F., Bueno P.R. Theoretical models for ac impedance of finite diffusion layers exhibiting constant phase element behavior. Journal of Electroanalytical Chemistry, 1999, vol. 475, no. 2, pp. 152–163.

[3] Barsoukov E., Macdonald J.R. Impedance spectroscopy: theory, experiment, and applications. 2nd ed. Hoboken, Wiley, 2005. 595 p.

[4] Eriksen T., Rehman M.U. Diffusion-wave behavior in lossy transmission lines. Scientific Reports, 2023, vol. 13, Article number 28390. https://doi.org.10.1038/s41598-023-28390-w .

[5] Karpilow I., Kvitka O., Stetsyuk D., et al. Diffusive signal propagation in transmission lines with distributed losses. Electric Power Systems Research, 2022, vol. 208, Article number 108527. https://doi.org/10.1016/j.epsr.2022.108527 .

[6] Sierociuk D., Dzielinski A., Sarwas G., Petras I., Podlubny I., Skovranek T. Experimental evidence of variable-order behavior of ladders and nested ladders. IEEE Transactions on Control Systems Technology, 2013, vol. 21, no. 2, pp. 459–466. https://doi.org/10.1109/TCST.2012.2186774 .

[7] Gateman S.M., Gharbi O., de Melo H.G., Ngo K., Turmine M., Vivier V. On the use of a constant phase element (CPE) in electrochemistry. Current Opinion in Electrochemistry, 2022, vol. 36, art. 101133. https://doi.org/10.1016/j.coelec.2022.101133 .

[8] Balaguera E.H., Allagui A. Limit capacitance of the constant phase element. Journal of Energy Storage, 2024, vol. 90, art. 111801. https://doi.org/10.1016/j.est.2024.111801 .

[9] Gateman S. M., Gharbi O., Gomes de Melo H., Ngo K., Turmine M., Vivier V. On the use of a constant phase element (CPE) in electrochemistry, Current Opinion in Electrochemistry, 2022, vol 36, 101133, https://doi.org/10.1016/j.coelec.2022.10113 .

[10] Fouda M.E., Allagui A., Elwakil A.S., Das S. Nonlinear charge-voltage relationship in constant phase element. AEU - International Journal of Electronics and Communications, 2020, vol. 117, art. 153104. https://doi.org/10.1016/j.aeue.2020.153104 .

[11] Cheng C.-S., Chung H.S.-H., Lau R.W.-H., Hong K.Y.-W. Time-Domain Modeling of Constant Phase Elements for Simulation of Lithium Battery Behavior. IEEE Transactions on Power Electronics, 2019, vol. 34, no. 8, pp. 7573–7587. https://doi.org/10.1109/TPEL.2018.2881200 .

[12] López-Villanueva J.A., Utrilla-Manso M., Díaz-Méndez J.A. Constant Phase Element in the Time Domain: The Problem of Initialization. Energies, 2022, vol. 15, no. 3, art. 792. https://doi.org/10.3390/en15030792 .

[13] Bertrand N., Sabatier J., Briat O., Vinassa J.-M. Embedded fractional nonlinear supercapacitor model and its parametric estimation method. IEEE Transactions on Industrial Electronics, 2010, vol. 57, no. 12, pp. 3991–4000. https://doi.org/10.1109/TIE.2010.2076307 .

[14] Allagui A., Freeborn T.J., Elwakil A.S., Maundy B. Energy and power analysis of fractional-order electrical energy storage devices. Energy, 2016, vol. 111, pp. 785–792. https://doi.org/10.1016/j.energy.2016.05.104 .

[15] Cruz-Duarte J.M., Guía-Calderón M., Rosales-García J.J., Correa R. Determination of a physically correct fractional-order model for electrolytic computer-grade capacitors. Mathematical Methods in the Applied Sciences, 2021, vol. 44, pp. 4366–4380. https://doi.org/10.1002/mma.7037 .

[16] Radwan A.G., Salama K.N. Fractional-order RC and RL circuits. Circuits, Systems, and Signal Processing, 2012, vol. 31, no. 6, pp. 1901–1915. doi: 10.1007/s00034-012-9432-z.

[17] Cvetićanin, S.M., Zorica, D. & Rapaić, M.R. Generalized time-fractional telegrapher’s equation in transmission line modeling. Nonlinear Dyn 2017, vol. 88, pp. 1453–1472. https://doi.org/10.1007/s11071-016-3322-z

[18] Wang B , Li S. E., Peng H., Liu Z. Fractional-order modeling and parameter identification for lithium-ion batteries. Journal of Power Sources, 2015, Vol. 293, pp. 151-161, https://doi.org/10.1016/j.jpowsour.2015.05.059 .

[19] Cheng C.-S., Chung H.S.-H., Hong K.Y.-W. Time-domain modeling of constant phase element for simulation of lithium batteries under arbitrary charging and discharging current profiles. 2017 IEEE Applied Power Electronics Conference and Exposition (APEC), 2017. https://doi.org/10.1109/APEC.2017.7930816 .

[20] Agambayev A., Patole S.P., Farhat M., Elwakil A., Bagci H., Salama K.N. Ferroelectric Fractional-Order Capacitors. ChemElectroChem,

2017, vol. 4, no. 11, pp. 2807–2813. https://doi.org.10.1002/celc.201700663 .

[21] Xu J., Zeng E., Li X., He G., Liu W., Meng X. Fractional-Order LCL Filters: Principle, Frequency Characteristics, and Their Analysis. Fractal and Fractional, 2024, vol. 8, no. 1, art. 38. https://doi.org.10.3390/fractalfract8010038.

[22] Tarasov V.E. Fractional Dynamics: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media. Springer, 2011. https://doi.org.10.1007/978-3-642-14003-7

[23] Lorenzo C.F., Hartley T.T. Variable order and distributed order fractional operators. Nonlinear Dynamics, 2002, vol. 29, pp. 57–98. https://doi.org.10.1023/A:1016586905654 .

[24] Valério D., Sá da Costa J.Variable-order fractional derivatives and their numerical approximations. Signal Processing, 2011, vol. 91, Issue 3, pp. 470-483, https://doi.org.10.1016/j.sigpro.2010.04.006.

[25] Sun H., Zhang Y., Baleanu D., Chen W., Chen Y. A new collection of real-world applications of fractional calculus in science and engineering. Communications in Nonlinear Science and Numerical Simulation, 2018, vol. 64, pp. 213–231. https://doi.org.10.1016/j.cnsns.2018.04.019 .

[26] Sansit Patnaik, John P. Hollkamp, Fabio Semperlotti; Applications of variable-order fractional operators: a review. Proc. A 1 February 2020; 476 (2234): 20190498. https://doi.org/10.1098/rspa.2019.0498

[27] Brug, G.J.; van den Eeden, A.L.G.; Sluyters-Rehbach, M.; Sluyters, J.H. The analysis of electrode impedances complicated by the presence of a constant phase element. J. Electroanal. Chem. 1984, 176, 275–295.

[28] Barsoukov, E.; Macdonald, J.R. Impedance Spectroscopy: Theory, Experiment, and Applications, 2nd ed.; Wiley: Hoboken, NJ, USA, 2005.

[29] Bisquert, J. Theory of the impedance of electron diffusion and recombination in a thin layer. J. Phys. Chem. B 2002, 106, 325–333.

[30] Westerlund, S.; Ekstam, L. Capacitor theory. IEEE Trans. Dielectr. Electr. Insul. 1994, 1, 826–839.

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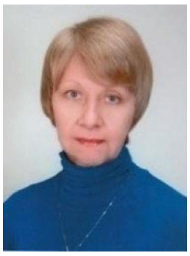

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