Theoretical and Experimental Foundations for the Development of an Electric Power Generation Unit with a Solid-State Heat Engine Based on a Ni-Ti-Cu Alloy for the Utilization of Low-Grade Thermal Energy

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Abstract. The main objectives of the study are to develop a generalized thermo-electromechanical model of a solid-state heat engine based on a Ni-Ti-Cu shape memory alloy for converting low-grade thermal energy (30-100 °C) into electrical energy, establish physically justified relationships governing reactive force generation and determine conditions for energy-efficient operation of the system as a part of an electric power generation unit. To achieve the stated objectives, the following tasks were performed: experimental studies of the thermomechanical characteristics of thermosensitive springs; derivation of a generalized relationship between maximum force, pre-deformation and temperature; development of a torque generation model, accounting for the phase position of active elements, construction of an electromechanical model of the "heat engine-generator-load" system; introduction of a system of dimensionless criteria for thermal, mechanical, electromechanical and energy consistency. Key results include analytical laws for scaling torque and mechanical power, a thermal consistency criterion defining the limiting rotational speed under complete phase transformation, the condition for a steady operating point and a material energy criterion setting the fundamental limit on the engine's thermal efficiency. The significance of these results lies in enabling a transition from the analysis of an individual prototype to a generalized description of a class of SMA engines, providing a basis for performance prediction, parameter scaling, design optimization and improvement of the efficiency of low-grade thermal energy utilization systems. The findings can be applied in designing low-power autonomous energy modules, industrial waste heat recovery systems and the development of functional alloys with narrow thermomechanical hysteresis for further applications in energy systems, including distributed and renewable energy sources of the future.
Keywords**:** shape memory alloys, Ni-Ti-Cu, solid-state heat engine, low-potential thermal energy, electric power generation, mathematical modeling.

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Bazele teoretice și experimentale pentru dezvoltarea unei unități electrogeneratoare cu motor termic în stare solidă bazat pe un aliaj Ni-Ti-Cu pentru utilizarea energiei termice cu potențial termic scăzut

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Rezumat. Principalele obiective ale studiului sunt elaborarea unui model termo-electromecanic generalizat al unui motor termic în stare solidă bazat pe un aliaj cu memorie de formă Ni-Ti-Cu pentru conversiea energiei termice de grad scăzut (30-100 °C) în energie electrică, stabilirea unor relații justificate fizic care guvernează generarea forței reactive și determinarea condițiilor pentru funcționarea eficientă din punct de vedere energetic a sistemului ca parte a unei unități de generare a energiei electrice. Pentru a atinge obiectivele stabilite, au fost efectuate următoarele sarcini: studii experimentale ale caracteristicilor termomecanice ale arcurilor termosensibile; derivarea unei relații generalizate între forța maximă, pre-deformare și temperatură; elaborarea unui model de generare a cuplului, ținând cont de poziția de fază a elementelor active, construirea unui model electromecanic al sistemului „motor termic-generator-sarcină"; introducerea unui sistem de criterii adimensionale pentru consistența termică, mecanică, electromecanică și energetică. Rezultatele cheie includ legi analitice pentru scalarea cuplului și a puterii mecanice, un criteriu de consistență termică, care definește viteza de rotație limită în condiții de transformare completă de fază, condiția pentru un punct de funcționare staționar și un

criteriu de energie materială care stabilește limita fundamentală a eficienței termice a motorului. Semnificația acestor rezultate constă în facilitarea tranziției de la analiza unui prototip individual la o descriere generalizată a unei clase de motoare SMA, oferind o bază pentru predicția performanței, scalarea parametrilor, optimizarea proiectării și îmbunătățirea eficienței sistemelor de utilizare a energiei termice de joasă putere. Constatările pot fi aplicate în proiectarea modulelor energetice autonome de mică putere, a sistemelor industriale de recuperare a căldurii reziduale și în dezvoltarea de aliaje funcționale cu histerezis termomecanic îngust pentru aplicații ulterioare în sistemele energetice, inclusiv în sursele de energie distribuită și regenerabilă ale viitorului.
***Cuvinte-cheie*:** aliaje cu memorie de formă, Ni-Ti-Cu, motor termic în stare solidă, energie termică cu potențial scăzut, generare de energie electrică, modelare matematică.

Теоретико-экспериментальные основы создания электрогенерирующей установки с твердотельным тепловым двигателем на основе сплава Ni-Ti-Cu при утилизации низкопотенциальной тепловой энергии

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Аннотация. Основными целями исследования являются разработка обобщенной термоэлектромеханической модели твердотельного теплового двигателя на основе сплава с памятью формы Ni-Ti-Cu для преобразования низкопотенциальной тепловой энергии (30-100 °C) в электрическую, установление физически обоснованных закономерностей формирования реактивного усилия и определение условий энергетически эффективной работы системы в составе электрогенераторной установки. Для достижения поставленных целей были решены следующие задачи: выполнены экспериментальные исследования термомеханических характеристик термочувствительных пружин; получена обобщенная зависимость максимального усилия от предварительной деформации и температурного состояния; разработана модель формирования крутящего момента с учетом фазового расположения активных элементов; построена электромеханическая модель системы «тепловой двигатель – генератор – нагрузка»; введена система безразмерных критериев термической, механической, электромеханической и энергетической согласованности. Наиболее важными результатами являются получение аналитических законов масштабирования крутящего момента и механической мощности, установление критерия термической согласованности, определяющего предельную частоту вращения при условии полной реализации фазового перехода, формулировка условия существования стационарной рабочей точки и определение материального энергетического критерия, который устанавливает фундаментальное ограничение термического КПД двигателя. Значимость полученных результатов заключается в том, что предложенный подход обеспечивает переход от анализа отдельного прототипа к обобщенному описанию класса SMA-двигателей, создает научно обоснованную основу для прогнозирования, параметрического масштабирования, оптимизации конструктивных параметров и повышения эффективности систем утилизации низкопотенциальной тепловой энергии различного назначения. Полученные положения могут быть использованы при проектировании автономных энергетических модулей малой мощности, систем рекуперации тепловых выбросов промышленных процессов, а также при обосновании перспектив дальнейшего совершенствования функциональных сплавов с узким термомеханическим гистерезисом для применений в энергетических системах, в том числе распределенной и возобновляемой энергетики будущего.
Ключевые слова: сплавы с памятью формы, Ni-Ti-Cu, твердотельный тепловой двигатель, низкопотенциальная тепловая энергия, электрогенерация, математическое моделирование.

INTRODUCTION

Among the various forms of environmental energy available in our surroundings, thermal energy is the most abundant and ubiquitous. Consequently, considerable efforts have been made in order to develop methods for converting thermal energy into electrical energy. According to temperature level, thermal energy is commonly classified as high-grade, medium-grade and low-grade [1]. Low-grade waste heat offers significant work potential, since it is abundantly available around us. Most industrial processes generate large amounts of waste heat with the majority of this thermal energy available in the form of low-grade waste heat [2]. The recovery of this low-grade energy is highly desirable for improving industrial process efficiency and reducing environmental impact [3]. The development of systems for the utilization and conversion of thermal energy from low-temperature sources is becoming increasingly relevant over time, driving the development of new methods and technical solutions in this field.

Power plants that utilize low-grade energy sources solve several key problems by:

- recovering low-temperature heat from industrial thermal emissions, geothermal sources, ocean thermal energy or the heat from cooling electronic devices;
- reducing greenhouse gas emissions;
- improving energy security by reducing dependence on conventional energy sources, thereby enhancing national energy security;
- improving environmental sustainability;
- expanding energy supply options;
- minimizing energy losses.

Statement of the problem. The use of unconventional advanced materials in the field of electric power engineering is a promising component of its development. One such group of materials is functional alloys (FAs), namely shape memory alloys [4, 5].

In an effort to solve the long-standing problem of utilizing secondary thermal energy, we have designed and fabricated a compact heat engine based on a shape memory alloy (SMA), specifically Ni-Ti-Cu. The Ni-Ti-Cu alloy-based engine is designed to operate at temperatures starting from +55^{o}C, with the surrounding environment acting as a heat sink. The objectives of developing an efficient solid-state engine are to investigate the characteristics and operational features of thermally activated driving elements made of Ni-Ti-Cu alloy with a narrow thermomechanical hysteresis and to determine the performance characteristics of the engine with subsequent optimization of its design parameters.

Relevance. One of the promising directions of exploiting the unique properties of functional alloys is the development of heat engines based on shape memory alloys. Heat engines based on FAs are capable of operating efficiently at small temperature differences, which increases the overall efficiency of the energy system [6, 7, 8]. This is particularly important in the context of utilizing low-grade energy sources, which are difficult to convert into electrical energy using conventional methods [9].

The implementation of new materials, such as shape memory alloys, in electric power engineering facilitates the development of more compact, efficient and reliable power generation units capable of operating under low-temperature conditions [10, 11, 12].

ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

Literature Review. Interaction of thermal and mechanical phenomena in shape memory alloys enables the design of thermal machines, i.e., heat engines, refrigerators or heat pumps. The initial concepts of such engines as well as the thermodynamic analysis of their efficiency, were evidently presented in the works of Ahlers (1975), Delaey and Lepeleire (1976). Later, in the works of Wollants et al. (1980), Cunningham and Ashbee (1977), Golestaneh (1980) a broader interpretation of the problem of their efficiency was presented. The issues discussed in recent publications concern the need to develop highly efficient solid-state engine designs, create and implement new alloys with narrow thermomechanical hysteresis and significantly reduce the cost of functional alloys [10, 13, 14].

Purpose. The aim of this work is to perform theoretical and experimental substantiation of the prerequisites for developing a system for generating electrical energy based on low-grade thermal energy sources and the physical properties of shape memory alloys.

Research Objectives:

− investigation of the properties of the functional Ni-Ti-Cu alloy under different spring deformations;

− concept development for a system to generate electrical energy based on low-temperature thermal energy sources and the physical properties of shape memory alloys;

− modeling the mechanical characteristics of the heat engine.

Scientific novelty of the work.

For the first time, a generalized thermo-electromechanical model of a solid-state heat engine based on shape memory alloys has been developed, which:

− describes the generation of reactive force as a function of pre-deformation and temperature state;

− establishes an analytical relationship for the scaling of torque and power;

− introduces a system of dimensionless criteria for thermal, mechanical and electromechanical consistency;

− formulates the fundamental limitation of thermal efficiency based on the material parameters of the alloy.

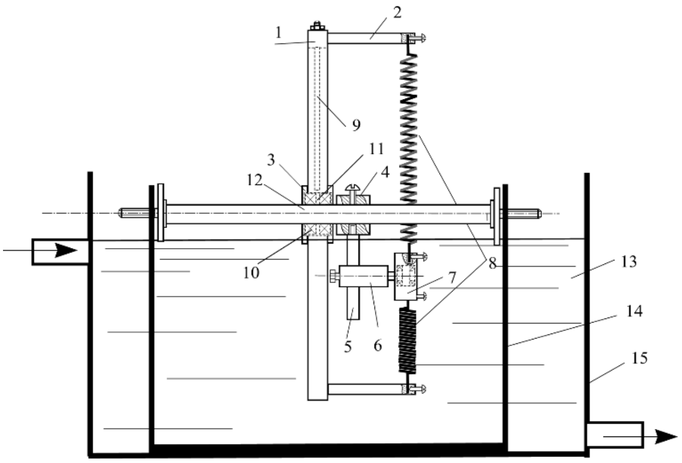
The proposed approach enables the transition from the analysis of a single experimental prototype to the investigation of an entire class of SMA engines.

MATERIALS AND METHODS OF RESEARCH

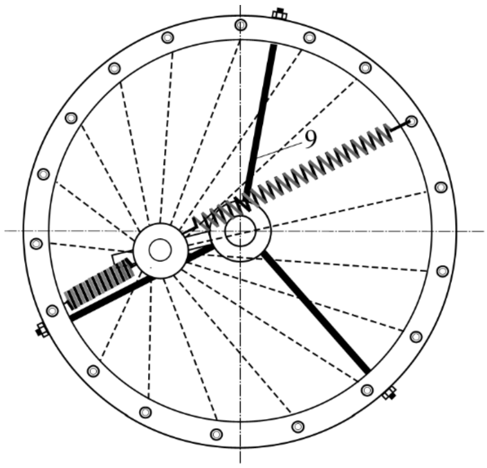
Ni-Ti-Cu shape memory alloys (Nitinol) and the group of copper-based alloys Cu-Al-Mn (Camital) and Cu-Zn-Al are being increasingly applied in electric power engineering [4, 5]. These alloys possess similar physical properties. However, the two groups differ in terms of cost, as Ni-Ti-based alloys are approximately 14…17 times more expensive than copper-based alloys. Ni-Ti alloys are characterized by good biocompatibility and a higher level of allowable recoverable strain (8…10% compared to up to 5% for copper-based alloys), which has resulted in their widespread application in medicine, aerospace and aviation engineering [15]. The physical characteristics of copper-based alloys meet the requirements of the processes in electric power installations and, therefore, their application in protective devices for electrical equipment against emergency operation conditions, thermal relays, pressure stabilization elements in separable electrical contacts, thermal engines and related systems is increasing [4].

In this study, a specific design configuration of a heat engine was considered. A theoretical model was developed, a functional prototype was fabricated and experimental investigations of the engine and its thermosensitive driving elements made of Ni-Ti-Cu functional alloy were conducted. In addition, the performance characteristics of the engine were modeled.

The kinematic diagram of the heat engine is presented in Fig. 1a and 1b.



a)



b)

1 - working wheel; 2 - rod for mounting FA springs; 3 - working wheel bushing; 4 - locking bushing; 5 - rod; 6 - slider; 7 - bushing; 8 - FA springs; 9 - spokes of the working wheel; 10 - bearings; 11 – retaining bushing; 12 – axis of the working wheel; 13 - water; 14 – engine mounting frame; 15 – water tank

Fig. 1. Kinematic diagram of the heat engine.

The maximum reactive force of the SMA element is expressed as a generalized relationship:

$$F_{\max}(L,T) = k_s \cdot L \cdot \theta(T), \quad (1)$$

where L is prior deformation; k_s – effective force utilization coefficient; $\theta(T)$ – temperature function of the phase transition (0…1).

The vertical component of the force can be expressed as:

$$F_{vert}(\varphi) = F_{\max} \cdot \sin\varphi \cdot k(\varphi), \quad (2)$$

where $k(\varphi)$ – takes kinematic corrections into account.

The generalized expression of the torque is given as follows:

$$M_{\Sigma} = N_a \cdot m \cdot \overline{F}_{vert}, \quad (3)$$

where N_a – number of active elements; m – eccentricity; $\overline{F}_{vert}$ – average vertical component.

The generalized expression for the power is given as follows:

$$P = N_a \cdot m \cdot \omega \cdot \overline{F}_{vert}(L,T,\tau). \quad (4)$$

Thus, the power is scaled according to the parameters that have the greatest influence on its value:

$$P \sim N_a \cdot m \cdot L\,. \qquad (5)$$

To move from the analysis of a single experimental sample to a generalized theory of solid-state heat engines based on SMA, a system of dimensionless criteria that define the operating modes and scaling capabilities was introduced.

Thermal compatibility criterion.

The characteristic heating time of the spring is given by:

$$t_{heat} \sim \tau\,. \qquad (6)$$

Residence time in the hot zone:

$$t_{contact} = \frac{\Delta\varphi}{\omega}\,. \qquad (7)$$

The thermal consistency criterion is introduced as follows:

$$C_{th} = \frac{t_{heat}}{t_{contact}}\,. \qquad (8)$$

Interpretation:

- $C_{th} < 1$ – the complete phase transition occurs;
- $C_{th} \approx 1$ – the boundary mode;
- $C_{th} > 1$ – thermal inertia limits the power.

This criterion defines the maximum allowable rotational speed for a specified heat exchange.

Mechanical scaling criterion of torque.

The total torque is given by:

$$M_\Sigma = N_a \cdot m \cdot \overline{F}_{vert}\,. \qquad (9)$$

Let us introduce the dimensionless mechanical force utilization coefficient:

$$C_m = \frac{\overline{F}_{vert}}{F_{\max}}\,, \qquad (10)$$

where $0 < C_m < 1$.

Then the torque scaling can be expressed as:

$$M_\Sigma \sim N_a \cdot m \cdot F_{\max} \cdot C_m\,. \qquad (11)$$

Accordingly, the power is:

$$P \sim N_a \cdot m \cdot \omega \cdot F_{\max} \cdot C_m\,. \qquad (12)$$

Thus, the power scales linearly with:

- the number of active elements;
- the eccentricity;
- the amplitude of the reactive force.

Electromechanical compatibility criterion.

Generator braking torque:

$$M_{gen} \sim \frac{k_e^2}{R_\Sigma}\,\omega\,. \qquad (13)$$

Let us introduce the operating point existence criterion:

$$C_{em} = \frac{M_{engine}}{M_{gen}}\,. \qquad (14)$$

The steady-state mode is possible only under the condition that:

$$C_{em} = 1.$$

Thus, the electrical load directly affects the mechanical rotational speed through the balance of torques.

Thermodynamic efficiency criterion.

Thermal energy per cycle is given by:

$$Q \sim m_s c \Delta T\,. \qquad (15)$$

Mechanical work is given by:

$$A \sim \sigma_{eff}\,\varepsilon V\,. \qquad (16)$$

The dimensionless criterion is given as follows:

$$C_{td} = \frac{\sigma_{eff}\,\varepsilon}{\rho c \Delta T}\,. \qquad (17)$$

The maximum thermal efficiency is given by

$$\eta_{th} \sim C_{td}\,. \qquad (18)$$

Thus, the efficiency of the SMA engine is fundamentally limited by the material properties of the alloy.

The analysis showed that:

1. The engine power scales linearly with the number of active elements.
2. The eccentricity is a linear multiplicative factor in torque increase.
3. The rotational speed is limited by the thermal consistency criterion C_{td} .

The introduction of the dimensionless criteria allowed the results to be generalized and enabled a transition from the description of a specific sample to the description of a class of solid-state SMA-based engines.

It is shown that when the condition $C_{td} < 1$ is satisfied, the engine operates in the mode of complete phase transition and its power is determined mainly by kinematic and geometric parameters.

The torque balance of the "engine - generator" system is described by criterion C_{em}, which allows the operating point to be determined for arbitrary electrical load parameters.

Thermal efficiency is limited by the material criterion C_{td} , which is determined by the heat capacity and the effective mechanical stress of the alloy.

The obtained results are of a generalized nature and are applicable to a wide class of SMA engines rather than only to the prototype under study.

Justification of the thermal model of spring heating. In order to estimate the heating rate of the thermosensitive spring in the hot zone, the equation of unsteady heat transfer in the lumped heat capacity approximation was used [16]:

$$\frac{dT}{dt} = \frac{1}{\tau}(T_w - T), \qquad (19)$$

where T – spring temperature, °C; T_w – water temperature, °C; τ – thermal time constant, s.

The solution of this equation has an exponential form:

$$T(t) = T_w - (T_w - T_0)e^{-t/\tau}, \qquad (20)$$

which describes the asymptotic approach of the spring temperature to the temperature of the heat transfer medium.

Angular corrections (kinematic and cooling). In order to reproduce the experimental asymmetry and the reduction in efficiency at large angles, angular factors were introduced:

$$k(\varphi) = 1 - 0{,}35\left(\frac{\varphi}{\pi}\right)^2, \theta(\varphi) = 1 - 0{,}25\frac{\varphi}{\pi}, \qquad (21)$$

where φ – spring position angle in radians ranging from 0 to π.

Force-temperature relation. Based on the experimental results, a linear approximation was generalized:

$$F(T) = \begin{cases} 0, & T < T_s, \\ F_{\max}\dfrac{T - T_s}{T_f - T_s}, & T_s \le T \le T_f, \\ F_{\max}, & T > T_f. \end{cases} \qquad (22)$$

The final model of the vertical component of force is expressed as:

$$F_{AD}(\varphi) = F_{\max} \cdot \sin\varphi \cdot k(\varphi) \cdot \theta(\varphi). \qquad (23)$$

Torque and mechanical power.

Torque of a single spring [17, 18]:

$$M_1 = \int_0^{\pi} mF_{AD}(\varphi)d\varphi, \qquad (24)$$

where m – eccentricity (taken as m = 0,04m).

Load characteristic. The electrical unit includes a direct-current generator with permanent magnets. Its electromechanical relationships are expressed as [19, 20]:

$$E = k_e\omega, \qquad (25)$$

$$I = \frac{E}{R_{in} + R_n}, \qquad (26)$$

$$M_{gen} = k_e I, \qquad (27)$$

where E – electromotive force (EMF) of the generator, V; k_e – generator electromechanical constant (coefficient of conversion), V·s/rad; ω – rotor angular velocity, rad/s; I – current in the external circuit, A; R_{in} – internal resistance of the generator windings, Ohm; R_n – resistance of the external load, Ohm; M_{gen} – electromagnetic (braking) torque of the generator, N·m.

By substituting the first equation into the second and the third one, we obtain:

$$M_{gen} = \frac{k_e^2}{R_{in} + R_n}\omega. \qquad (28)$$

Thus, the braking torque of the generator is directly proportional to the angular velocity and inversely proportional to the total resistance of the electrical circuit.

Mechanical characteristic of the thermal engine. The obtained experimental results show that, for the investigated prototype, the dependence of the thermal engine torque on the speed in the operating range can be approximated by the linear function:

$$M_d(\omega) = M_0\left(1 - \frac{\omega}{\omega_{xx}}\right), \qquad (29)$$

where M_0 – starting torque, ω_{xx} – idle angular velocity.

Equation of the operating point. In the steady-state mode, the torque equilibrium condition is satisfied:

$$M_d(\omega) = M_{gen}(\omega). \qquad (30)$$

Geometrically this corresponds to the point of intersection of the mechanical characteristic of the thermal engine and the load characteristic of the generator in the $M - \omega$ coordinates.

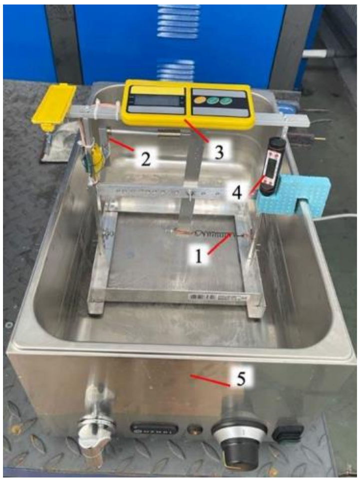
Energy balance and efficiency. Thermal energy that is supplied to a single spring during a heating cycle is given by [21]:

$$Q_s = m_s c \Delta T, \qquad (31)$$

where m_s – mass of a single spring, kg; c – specific heat capacity of the Ni-Ti-Cu alloy, J/(kg·K); ΔT – effective temperature change, K.

Springs made of the Ni-Ti-Cu functional alloy were selected as driving elements. The springs were made of wire 280 mm in length and 1 mm in diameter. The spring diameter was 10 mm and the number of turns was 9. For the purposes of the study, the pre-deformation (extension) of the springs was set to be 100, 120, 140 and 160 mm [22].

Figure 2 shows the general view of the experimental stand used to investigate the thermomechanical characteristics of the springs made of the Ni-Ti-Cu alloy. Figure 3 presents the thermomechanical characteristics of the spring at different levels of its deformation.



1 – spring made of the Ni-Ti-Cu alloy; 2 – strain gauge sensor; 3 – control and information display unit; 4 – digital thermometer; 5 – heat water tank with automatic temperature control

Fig. 2. General view of the experimental stand for investigating thermomechanical characteristics.

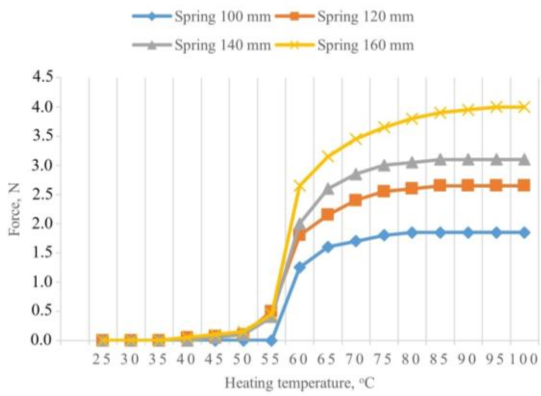


Fig. 3. Thermomechanical characteristics of Nitinol springs (wire diameter 1.0 mm) at different levels of their deformation.

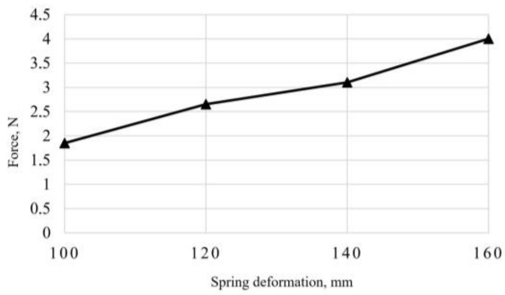


Fig. 4. Generalized graph of the dependence of the maximum generated force on spring deformation at the temperature of +100°C.

The results obtained from the conducted experimental studies allowed the following conclusions to be drawn regarding the patterns of reactive force generation by the spring made of the Ni-Ti-Cu alloy:

− the Ni-Ti-Cu alloy has a shape recovery start temperature within the range of +50…+55ºC, a shape recovery completion temperature depends on the spring deformation and ranges from +85 to +95ºC;

− the graphs of thermomechanical characteristics show that the magnitude of spring pre-deformation of 100, 120, 140 and 160 mm has a significant effect on the generated force. At the spring pre-deformation of 160 mm, the force is 4.0 N, whereas at the deformation of 100 mm, it is equal to 1.75 N, i.e., the difference is 229%.

The obtained results and conclusions were also confirmed in the works [23, 24, 25].

From Fig.4 it can be seen that the functional dependence of the maximum generated force on the spring deformation is approximately linear. Using the interpolation method, the coefficients were calculated and the functional dependence was constructed as follows:

$$F(L) = a \cdot L - b, \qquad (32)$$

де L – the length of spring deformation (extension), mm; coefficients $a = 0.0332$ N/mm; $b = 1.312$ N. The obtained dependence makes it possible to extend the range of determining the maximum generated force as a function of spring deformation.

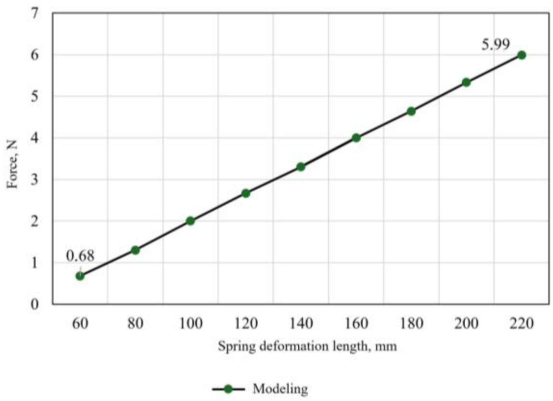


Fig. 5. Generalized graph of the dependence of the maximum generated force on spring deformation at a temperature of +100ºC, experimental and calculated data.

When heated, the spring restores it previous compressed form and generates a force ranging from $F_1 = 5.99$ N to $F_2 = 0.68$ N. During contraction (the displacement of the free end), the spring develops power, which can be determined as follows. The length of contraction (reverse deformation) of the spring during heating is 160 mm (from 220 mm to 60 mm, Fig.5). The conducted measurements showed that the contraction time of the spring at a temperature of 100ºC was equal to 0.2 s. The contraction speed of the spring was equal to 800 mm/s, or 0.8 m/s. Accordingly, the power of this spring sample varies during its contraction from $P_1 = F_1 \cdot \upsilon = 5.99 \cdot 0.8 = 4.79W$ to $P_2 = F_2 \cdot \upsilon = 0.68 \cdot 0.8 = 0.54W$.

Let us determine the specific power referred to 1 g of the Ni-Ti-Cu alloy. The density of the alloy is 6.45 g/cm^3. The volume of the spring wire in the given samples is 0.22 cm^3, and its mass is 1.42 g. Thus, the specific mechanical power developed by the spring wire varies from 3.37 W/g to 0.38 W/g. The obtained values of the specific power make it possible to approximately estimate the amount of alloy needed to generate the maximum mechanical power, for example: 1 kW – 297 g, 5 kW – 1483 g, 10 kW – 2967 g.

The modeling of force characteristics of the heat engine was carried out using the kinematic diagrams shown in Fig.6. In this version of the heat engine with the working wheel radius of $r = 130$ mm, 18 springs made of the Ni-Ti-Cu alloy were used. The point of application of the generated forces of each spring is located at point *A*, which is at a distance $m = 30$ mm from the center of the working wheel (segment *AO*). The force generated by the spring, for example F_{AC1}, has two components: a vertical component F_{AD1} and a horizontal component F_{AB1}. The vertical component F_{AD1} acts on the arm *AO* and generates a torque on the working wheel.

Point *C* lies on the circle with its center at the origin and radius *r*, its coordinates satisfy the following equation:

$$x^2 + y^2 = r^2. \qquad (33)$$

Point *C* also lies on the line *AC*. The equation of the line *AC* is: $y = kx + b$.

Since $k = tg\alpha$, and the line passes through point $A(-m;0)$, then $0 = tg\alpha \cdot (-m) + b \Rightarrow b = mtg\alpha$.

Accordingly, $y = xtg\alpha + mtg\alpha$.

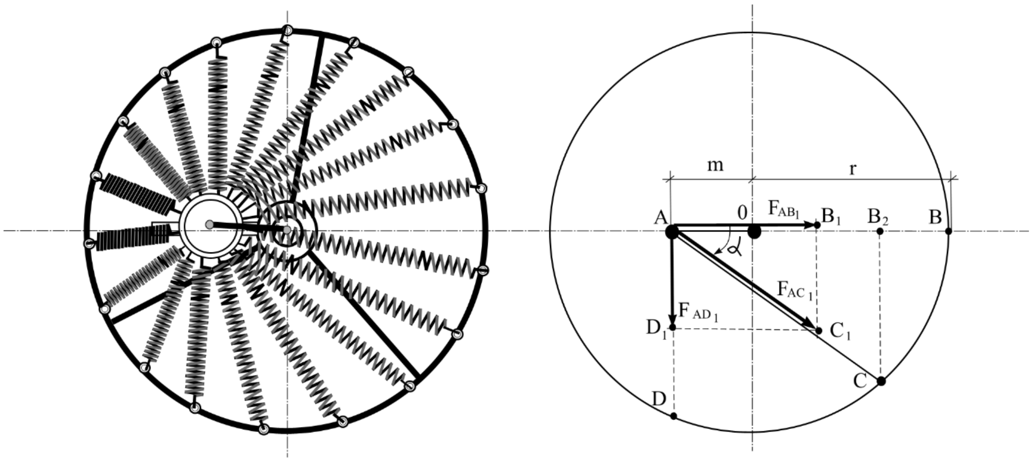


Fig. 6. Kinematic diagram of the working wheel of the heat engine and the force vectors of the spring made of functional alloy during heating.

Let us consider the system of equations that describes the interrelation and the kinematics of the unit:

$$\begin{cases} x^2+y^2=r^2 \\ y=(x+m)tg\alpha \end{cases} \Rightarrow \begin{cases} y^2=r^2-x^2 \\ y^2=(x+m)^2 tg^2\alpha \end{cases} \Rightarrow \quad (34)$$

$$\Rightarrow (x+m)^2 tg^2\alpha = r^2-x^2.$$

$$(x^2+2mx+m^2)tg^2\alpha = r^2-x^2, \quad (35)$$

$$x=-m\sin^2\alpha+\sqrt{r^2-m^2\sin^2\alpha}\cdot\cos\alpha \Rightarrow$$

$$OB_2=-m\sin^2\alpha+\cos\alpha\sqrt{r^2-m^2\sin^2\alpha}. \quad (36)$$

Taking into account equation (32) and the obtained values of the coefficients, the dependence of the force generated by the spring in the heat engine on the temperature of the heat transfer medium is given by:

$$F(AC_1)=0.0332\cdot AC-1.312, \quad (37)$$

Thus, as a result of solving the system of equations (34)-(36) and taking into account (37), we obtain:

$$F_{AC_1}(\alpha)=0.0332\cdot(m\cos\alpha+\sqrt{r^2-m^2\sin^2\alpha})-1.312.; \quad (38)$$

$$F_{AD_1}(\alpha)=[0.0332\cdot(m\cos\alpha+\sqrt{r^2-m^2\sin^2\alpha})-1.312]\sin\alpha. \quad (39)$$

Table 1

Results of modeling the forces of a single spring acting on the working wheel of the heat engine depending on the angle α (spring position relative to the horizontal axis of the wheel)

Angle α	Calculated spring deformation length, mm	Force F_{AC1}, N	Force F_{AD1}, N
0	160.00	4.00	0
20	157.79	3.93	1.34
40	151.54	3.72	2.39
60	142.38	3.41	2.96
80	131.81	3.06	3.02
100	121.39	2.72	2.68
120	112.38	2.42	2.09
140	105.58	2.19	1.41
160	101.40	2.05	0.70
180	100.00	2.01	0

Figure 7 shows a graph of the dependence of the reactive force (1) and its vertical component (2) on the angle *α* of the spring position on the rotating wheel of the engine.

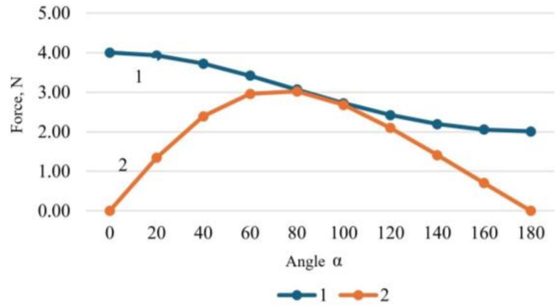


Fig. 7. Dependence of the reactive force (1) and its vertical component (2) on the angle *α* of the spring position on the rotating wheel of the engine.

Since the heating of the thermosensitive elements (springs) by the heat transfer medium and their motion on the rotating wheel can be approximately considered to occur on comparable time scales, the total action of the forces during wheel rotation can be analyzed under a quasi-steady assumption. At discrete values of the angle α (from 0 to 180°, in this example with an increment of 20°), 9 out of 18 springs of the working wheel of the engine are in contact with the heat transfer medium. Under this assumption, the total force of the components $F_{AD1}(\alpha)$ of the nine springs acting on the point A and generating a torque to drive the working wheel of the engine can be determined as $F_{AD1}(\alpha_i) = 16.59$ N.

One possible configuration of an electric power generation unit can be achieved by supplementing the technical system of the heat engine (Fig.1) by an electric generator and a mechanical transmission for transferring torque from the working wheel of the heat engine to the rotor of the generator.

In order to deepen the analysis of the characteristics of the electric power generation unit with a solid-state heat engine based on shape memory alloys, a verified and consistent thermomechanical and electromechanical model of the system is presented. This model makes it possible to quantitatively describe the spring force profile and the total torque; to construct load characteristics of the "engine-generator-load" system; to calculate the mechanical, electrical and overall energy efficiency; to formulate design measures for increasing the overall efficiency to 3-4%.

The modeling is based on the parameters of the previously selected prototype and the results of the experimental studies of thermosensitive elements (springs) made of shape memory alloy.

Table 2

Initial parameters of the prototype

Parameter	Symbol	Value
Radius of the wheel	r	130 mm
Number of springs	N	18
Active springs	N_a	9
Eccentricity, mm	m	40
Rotational speed (experimental data), rpm	n	60
Angular velocity, rad/s	ω	6.28
Wire diameter, mm	–	1
Wire length, mm	–	280
Spring mass, g	m_s	≈1.4
Start temperature, °C	T_s	50
Finish temperature, °C	T_f	90
Maximum force (experimental data), N	F_{max}	≈ 4

The calculation showed that at a characteristic thermal time constant $\tau \approx 0.1$ s, the time required to reach the phase transformation finish temperature ($T_f \approx 90$ °C) is approximately 0.2-0.25 s.

Since at the rotational speed of 60 rpm the residence time of the spring in the hot zone is equal to approximately 0.5 s, the conditions of heat transfer ensure almost complete phase transformation.

Thus, the thermal inertia of the springs is not a limiting factor at the operational rotational speed of 60 rpm. This allows a quasi-steady assumption to be applied in modeling the force characteristics and confirms that the limiting rotational speed is determined primarily by kinematic and mechanical characteristics rather than by the heat rate of the material.

A comparison of the experimental data with the model $F_{AD}(\varphi)$ (Fig.8) confirms the validity of the introduced angular correction factors $k(\varphi)$, $\theta(\varphi)$, since the mean relative error of the model with respect to the experiment in the operating range is less than 10%.

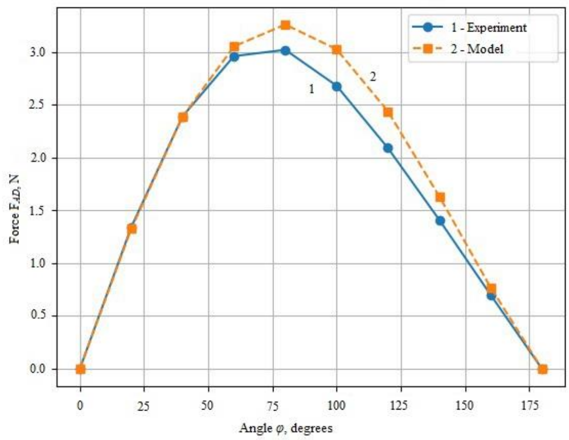


Fig. 8. Spring force characteristic (experiment / model).

Let us calculate the total torque M_Σ.

Numerically: $M_1 \approx 0.23$ N·m.

The total torque of 9 active springs:

$$M_\Sigma \approx 9 \cdot 0{,}04 \cdot 5.99 \approx 2.1 \text{ N·m}.$$

The angular velocity at 60 rpm: $\omega = 2\pi rad/s \approx 6.28 rad/s$. Accordingly, the mechanical power is: $P_{mech} = M_\Sigma \cdot \omega \approx 2.1 \cdot 6.28 \approx 5.8\,\text{W}$.

The electrical power, taking into account the electromechanical efficiency is $\eta_{em} \approx 0.55$; $P_{el} \approx 3.2$ W. Fig. 9 shows the dependence of mechanical and electrical power on temperature, illustrating the development of power at a fixed rotational speed, the activation threshold at $T_s \approx 50^o C$ and the power plateau at $T > T_f$.

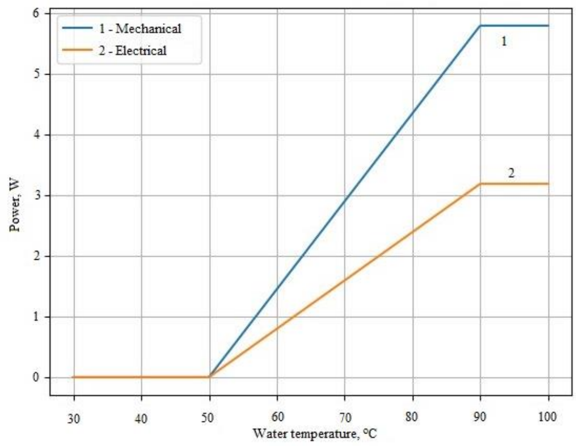


Fig. 9. Mechanical and electrical power versus temperature (60 rpm).

The operating point of the electric power system is determined by the torque balance. An increase in the electrical resistance leads to a decrease in the mechanical load on the shaft and, consequently, to an increase in the rotational speed of the working wheel.

In the numerical modeling, the following parameters were adopted: $k_e = 1.1$ V·s/rad; $R_{in} = 1.2$ Ohm; optimal load resistance $R_{n.opt} \approx 7.5$ Ohm; here $n \approx 60$ rpm, $P_{el} \approx 4.85$ W, where $M_0 \approx 2.1$ N·m, $\omega_{xx} \approx 11$ rad/s.

The equilibrium $M_d(\omega) = M_{gen}(\omega, R_n)$ determines the operating point. Fig. 10 shows the load characteristic $(M - \omega)$: the torque of the heat engine and the braking characteristics of the DC generator at different R_n, with the operating points indicated by the intersections of the lines.

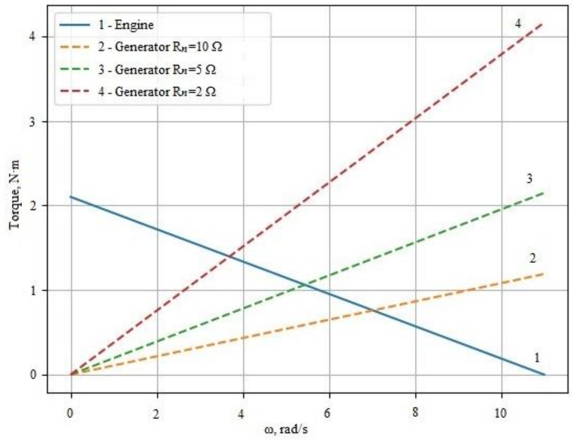


Fig. 10. Loading characteristic *M – ω*.

The graph in Fig. 11 shows the dependence of the rotational speed on the electrical load resistance (consumer).

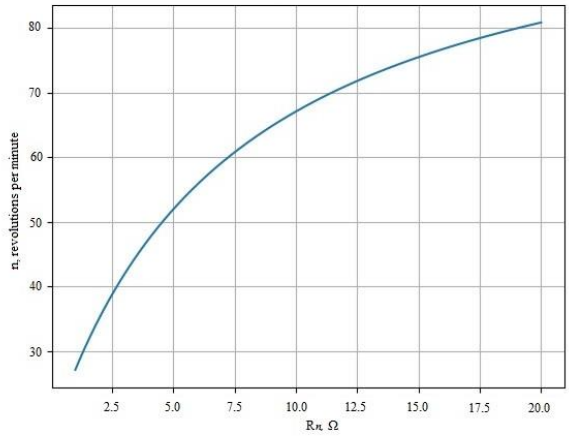


Fig. 11. Rotational speed *n(Rn)*

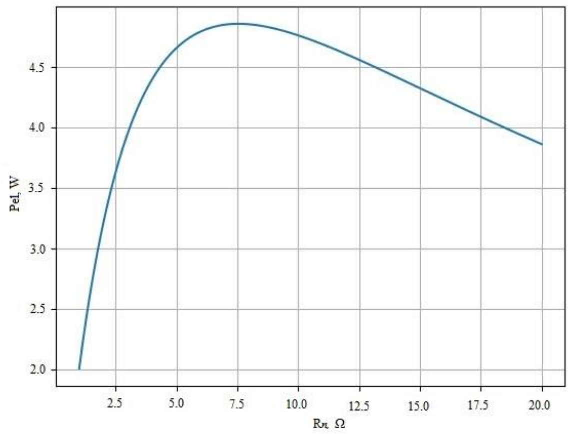


Fig. 12. Electrical power *Pel(Rn)*

From the graphs in Fig.11 and 12, it can be seen that the maximum power corresponds to the optimal load resistance of 7.5 Ohm (for the motor speed of 60 rpm, Fig.11).

Let us determine the thermal energy, which is supplied to a single spring during a heating cycle.

For the considered prototype: $m_s = 0.0014$ kg; $c = 460$ J/(kg·K); $\Delta T \approx 45$ K.

Then:

$$Q_s = 0.0014 \cdot 460 \cdot 45 \approx 29 \text{ J}.$$

Since 9 springs are simultaneously in the hot zone and the rotational speed is 1 rev/s (60 rpm), the total heat flux is $Q_{in} \approx 9 \cdot 29 \approx 260$ W.

Thus: $P_{mech} \approx 5.8$ W → thermal efficiency of the engine $\eta_{th} \approx 5.8/260 \approx 2.2$ %; $P_{el} \approx 4.9$ W → overall efficiency $\eta_{total} \approx 4.9/260 \approx 1.9$ %.

Fig. 13 shows the dependence of the electromechanical efficiency of the unit on the electrical load resistance.

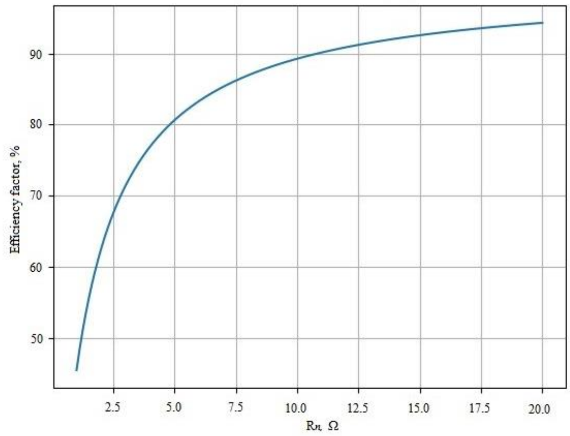


Fig. 13. Electromechanical efficiency of the unit

Table 3

Modeling results of the electric power generation unit

Parameter	Symbol	Parameter value
Torque of a single spring, N·m	M_1	0.23
Total torque of nine springs, N·m	M_Σ	2.1
Mechanical power, W	P_{mech}	5.8
Electrical power, W	P_{el}	4.8-4.9
Thermal efficiency, %	η_{th}	≈2.2
Overall efficiency, %	η_{total}	1.8-2.0
Optimal load resistance, Ohm	$R_{n.opt}$	≈7.5
Optimal rotational speed, rpm	n_{opt}	≈ 60

Results and discussions. It has been shown that the main factors contributing to an increase in power are an increase in the number of active elements and the eccentricity, provided that the thermal criterion is met. The thermal efficiency is limited by the material energy criterion and cannot be arbitrarily increased without changing the physical properties of the alloy.

The results of the experimental studies on the thermomechanical characteristics of the springs made of the Ni-Ti-Cu alloy confirmed a significant influence of the pre-deformation level on the magnitude of the generated reactive force and the temperature range of the phase transformation. It was found that the increase in the spring deformation from 100 to 160 mm leads to an increase in the maximum force from 1.75 to 4.0 N at the temperature of +100 °C, which indicated a quasi-linear relationship between these parameters.

The obtained analytical relationship between the force and the spring deformation made it possible to evaluate the specific energy characteristics of the thermosensitive elements and to estimate the required mass of the functional alloy needed to achieve the specified level of mechanical power. Calculations showed that the specific mechanical power of the springs made of the Ni-Ti-Cu alloy varies within the range of 0.38-3.37 W/g, which confirms the promising potential of such materials for the application in solid-state heat engines of low and medium power.

Modeling of the force interaction between the springs and the working wheel of the heat engine has shown that the vertical component of the reactive force is the determining factor in the formation of torque.

The obtained experimental data confirm the quasi-linear dependence of the force on the level of pre-deformation within the operating range.

The introduced dimensionless criteria made it possible to:

− establish the conditions for complete thermal realization of the phase transition;

− determine the limiting rotational speed;

− formulate the condition of the existence of the steady operating point;

− obtain the scaling law for mechanical power.

The use of the quasi-stationary assumption made it possible to determine the total force of nine springs that are simultaneously in contact with the heat carrier at the level of 16.59 N, which is sufficient to ensure stable rotational motion of the working wheel.

The obtained results are consistent with the current concepts of the operation of heat engines based on shape memory alloys and expand the scope of their practical application in the utilization of low-potential thermal energy [26, 27].

CONCLUSIONS

1. A generalized thermo-electromechanical model of a solid-state heat engine based on a shape memory alloy, which describes the interrelation of thermal, force and electrical processes, has been developed.
2. A quasi-linear pattern of the formation of the reactive force of a SMA element as a function of pre-deformation and temperature state has been established.
3. An analytical model for scaling torque and mechanical power through the number of active elements, eccentricity and rotational speed has been obtained.
4. A thermal consistency criterion, which determines the limiting rotational speed under the conditions of complete phase transformation, has been introduced.
5. An elecrtomechanical criterion of the excistence of a steady operating point in the "heat engine – generator – load" system has been formulated.
6. It has been shown that the thermal efficiency is limited by the material parameters of the alloy and is determined by a dimensionless energy criterion.
7. The proposed approach provides a scientifically grounded basis for the scaling and the optimization of SMA engines in the systems for the utilization of low-grade thermal energy.

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