Calculation of the Electrostatic Field in Non-Homogeneous Structures

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***Abstract*.** The paper examines a new approach of the finite volume method in determining the distribution of the electric field in non-homogeneous structures. The numerical calculation grid of the three-dimensional electric field is constructed using of the Delaunay triangulation concept and Voronoi cells. The purpose of the investigations is to simplify and order the algorithm for calculating the electric field distribution in non-homogeneous structures. The proposed algorithm is robust for calculating the potential and intensity of the electric field in non-homogeneous structures at the arbitrary distribution of voltage potential. The developed algorithm, during the computation process, ensures storage only the nonzero values of matrixes elements in the computer memory. The finite volume method keeps all the advantages of the finite difference method. Compared to the finite element method, the algorithm for constructing the finite difference relations is to simpler. In this case, there is no need to build a local and global rigidity matrix to form an equation solving system for the determination of capacitive coefficients of the electrical field. To solve the built system, we use the iterative method of conjugate gradients that converge very quickly for problems of the type examined. Numerical calculation of the electric field in the hollow cylinder with limited height and in the voltage divider from the glass-insulated micro conductors has been developed. The calculated the distribution of the electric field in the expanded voltage divider in the electrostatic screen. It is presented dates, which estimates the screen influence on capacitance of the voltage divider.

***Keywords*:** potential, intensity vector, capacitive coefficients, method, finite volume, conjugate gradients, voltage divider.
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Calculul câmpului electrostatic în structuri neomogene

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***Rezumat*.** În lucrare se prezintă o nouă abordare a utilizării metodei volumului finit în determinarea distribuției câmpului electric în structuri neomogene. Grila numerică de calcul a câmpului electric tridimensional este construită folosind conceptul de triangulație Delaunay și celulele Voronoi. Scopul investigații constă în simplificarea și ordinarea algoritmului de calcul a distribuției câmpului electric în structuri neomogene. Algoritmul propus este robust pentru calcularea potențialului și a intensității câmpului electric în structuri neomogene la distribuția arbitrară a potențialului de tensiune. Algoritmul dezvoltat, în timpul procesului de calcul, asigură stocarea numai a valorilor diferite ale elementelor matricei în memoria computerului. Metoda volumului finit păstrează toate avantajele metodei diferenței finite. În comparație cu metoda elementului finit, algoritmul pentru construirea relațiilor de diferență finită este mai simplu. În acest caz, nu este necesară construirea unei matrice de rigiditate locală și globală pentru a forma un sistem de rezolvare a ecuațiilor pentru determinarea coeficienților capacitivi ai câmpului electric. Pentru a rezolva sistemul construit, folosim metoda iterativă a gradienților conjugati care converg foarte rapid pentru probleme de tipul examinat. A fost executat calculul numeric al câmpului electric din cilindrul gol cu înălțime limitată și al divizorului rezistiv de tensiune construit în baza micro-conductoarelor izolate din sticlă. S-a calculat distribuția câmpului electric în divizorul de tensiune amplast constructiv într-un ecran electrostatic. Se prezentată date, care permit estimarea influenței ecranului electristatic asupra capacității divizorului de tensiune.

***Cuvinte-cheie*:** potențial, vector de intensitate, coeficienți capacitivi, metoda, volum finit, gradienți conjugati, divizor de tensiune.

Расчет электростатического поля в неоднородных структурах

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Аннотация. В статье рассматривается новый подход применения метода конечных объемов при определении распределения электрического поля в неоднородных структурах. Сетка для численных расчетов трехмерного электрического поля строится с использованием концепции триангуляции Делоне и ячеек Вороного. Цель исследований - упростить и упорядочить алгоритм расчета распределения электрического поля в неоднородных структурах. Предложенный алгоритм является надежным для расчета потенциала и напряженности электрического поля в неоднородных структурах при произвольном распределении потенциала напряжения. Разработанный алгоритм в процессе вычислений обеспечивает хранение только ненулевых значений элементов матриц в памяти компьютера. Метод конечных объемов сохраняет все преимущества метода конечных разностей. По сравнению с методом конечных элементов алгоритм построения конечно-разностных отношений является более простым. В этом случае нет необходимости строить матрицу локальной и глобальной жесткости для формирования системы решения уравнений для определения емкостных коэффициентов электрического поля. Для решения построенной системы уравнений используется итерационный метод сопряженных градиентов, которые очень быстро сходится для задач рассматриваемого типа. Выполнен численный расчет электрического поля в полом цилиндре с ограниченной высотой и в делителе напряжения изготовленый на основе микропровода в стеклянной изоляции. Расчитано распределение электрического поля в резистивного делителя напряжения из микропровода расположенного в электростатическом экране. Представлены результаты расчета, по которым оценивается влияние экрана на емкость делителя напряжения.

***Ключевые слова*:** потенциал, вектор интенсивности, емкостные коэффициенты, метод, конечный объем, сопряженные градиенты, делитель напряжения.

INTRODUCTION

Many phenomena and processes can be described through mathematical relationships with partial derivatives. For these equations, it is necessary to formulate the initial and marginal boundary conditions. Often, the only possibility to obtain the solutions of the problem is the use of numerical calculation methods and parametric analysis with the use of mathematical models. Mathematical models must match the constructive and physical particularities of the studied object (adequacy, precision, flexibility, economic resonance, etc.) [1, 2].

The emergence of computers has determined the extension of the use of the methods of obtaining solutions of multidimensional differential equations by their numerical integration [3, 4]. The solution of the differential equations depends on the values of the coefficients of these equations.

If we examine to the energy system, we can see that all elements of the power system are characterized by an obvious constructive inhomogeneity. This particularity leads to difficulties in calculating the values of the primary and secondary parameters of the functional components of the power system.

Addressing the physical heterogeneity of electrical installations is a complex scientific and technical issue, especially when there are large differences in the geometric dimensions of the zones and of the electrophysical parameters of these areas [5-6]. Maxwell equations presented in differential and integral form represent the theoretical basis for solving these problems.

The macroscopic bodies of the non-homogeneous environment are limited by surfaces, which may have an arbitrary relief. Following the structural inhomogeneity, the values of the electrophysical parameters of the macroscopic bodies can be changed by jumping to the boundaries of the selected areas. As a result, the electromagnetic fields generated by these bodies can also change very strongly when moving from one macroscopic body to another.

The paper presents an approach to the problem of elaborating numerical mathematics models for calculating the electric fields in non-homogeneous environments, based on the meshing procedure on blocks using the characteristic ideas for the finite volume method [7].

I. FORMULATION OF THE BOUNDARY VALUE PROBLEMS

To realize these methods, it is necessary to use 2D or 3D calculating grids with structured or unstructured topology. The use of unstructured calculating grids faces difficulties in examining tridimensional problems.

In order to overcome these difficulties is reasonable to dividing the volume of the non-homogeneous environment into several sub-zones with relatively simples form blocks is argued. In these blocks it is possible to construct relatively simple calculation networks, by taking into account the boundary conditions of these blocks.

This separation into structured blocks and use the finite volume method provides of avantages in making numerical calculations of the physical fields in unhomogene environments. Advantages of the finite volume method: preserving of the basic values in the highlighted volume (system energy, mass, heat fluxes etc); high speed of the calculation; the possibility of applying the method for objects with complex topology and curved borders.

We will examine the problem of calculating the distribution of the electric field in a non-homogeneous three-dimensional structure. We will determine the three-dimensional distribution of potential $u(x, y, z)$ in the multi-associate areas of the non-homogeneous environment Ω, for which the distribution function of the absolute dielectric permeability $\varepsilon_a(x, y)$ has a constant value in those areas. The function $u(x, y, z)$ inside the domain Ω satisfies the Poisson equation:

$$div(\varepsilon_a \,\text{grad}\, u) = -\sigma(x, y, z) \qquad (1)$$

in which $\sigma(x, y, z)$ - distribution density of free electrical charges.

If the function $\sigma(x, y, z) = 0$ in the domain Ω, the relation (1) will correspond to the Laplace equation $div\,(\varepsilon_a \text{grad}\, u) = 0$. We also considered, that at the boundaries $\Gamma = \partial\Omega$ of the domain Ω, the values of the potentil $u(x, y, z)$ are known:

$$u(x, y, z)\big|_{\Gamma} = \mu(x, y, z) \qquad (2)$$

The vector of the electrical field strength $\vec{E} = -\text{grad}\, u$ is determined using the known value of the potential u, and value of the electrical displacement vector will be determined from the $\vec{D} = \varepsilon_a \vec{E}$ relation. At interfaces of heterogeneous environments, the conditions of continuity for the potential and electric displacement is determined from relationships: $[u] = 0$ and $[\vec{D}, \vec{n}] = 0$. The square brackets indicate the difference between the limit values u and $\vec{D}$ to the left and to the right of the delimitation interface, and $\vec{n}$ - the normal vector to the highlighted interface.

II. Discrete Model of Calculation

The use Finite Element Method or Finite Volume Method requires calculation grids, which including many triangles and/or pyramid-shaped elements. The use of algorithms for defining geometric figures of this type based on Delaunay triangulation simplifies this problem.

The Delauney triangulation solves the problem of finding such a set of points in the plane (three-dimensional space), so that, there is no point of this ensemble to be positioned within the inside the circle (sphere) of any triangle (triangular pyramids) whose (whose) peaks are points located in this plane (three-dimensional space). Delauney triangulation ensures maximizing the minimum angle of all grid triangles (pyramids), which ensures minimization of errors conditioned by the numerical differentiation procedure.

A. *Theoretical Aspects of Applying the Finite Volume Method*

To illustrate the application of the finite volume method we will select a body with curved surfaces, fig.1. The space in which the hollow cylinder is located includes three substructures with different electrophysical parameters at the delimitation boundaries of the specified areas.

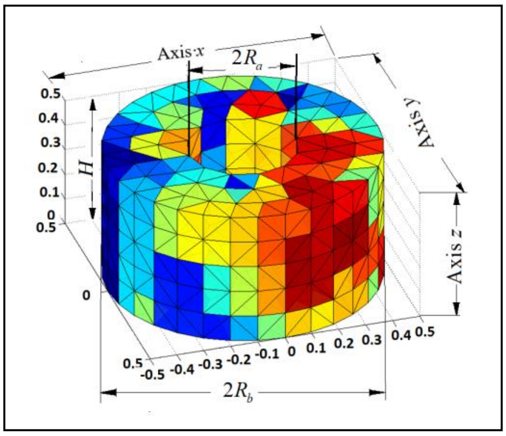

Fig.1. The grid of pyramid is for calculating the electric field distribution in the hollow cylinder with the finite volume method.

In the case of the numerical calculation of the distribution of physical fields in non-homogeneous environments, we have a Dirichlet problem, which consists in determining the function which is the solution of a differential equation in the examined space.

To solve numerically the Dirichlet problem, we divide the volumetric domain into the finite set of volumetric elements in the form of volumetric tetrahedra (pyramids). The vertices of the pyramids are called the nodes of the differential network. It is possible to construct a lot of three-dimensional domain divides into the pyramids where the nodes have fixed positions.

We will divide the hollow cylinder into volumetric elements (tetrahedra) - a total of 1980 pyramids. This provides us with the creation of a calculation grid that includes 528 nodes.

Let denote by T_h the set of grid pyramids, where *h* is the maximal value of the pyramids side lengths. Let introduce also the dual grid T_h^* that consists from so-called of the Voronoi cells. Each Voronoi cell encloses one of the inside nodes of the differential grid.

Figure 2 shows the schematic of the mutual links of a base node of the Voronoi three-dimensional cell. Voronoi cell is a polyhedron with green faces (fig.2).

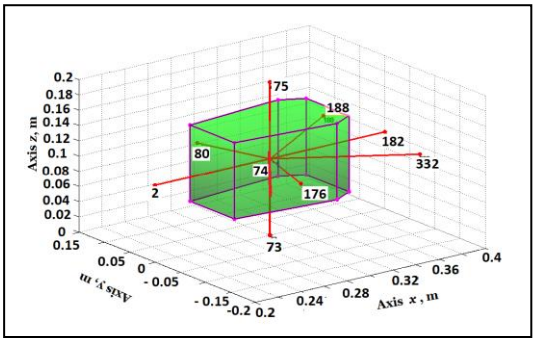

Fig.2. Voronoi cell with a polyhedron shape for the base node number 74.

To illustrate the essence of the numerical network structure is to select of the base node which number 74. This node 74 has links to base nodes 2, 73, 75, 80, 176, 182, 188 and 332, which are defined as neighboring nodes. In this case the Voronoi cell represents a polyhedron with semi-transparent faces.

Each face of the Voronoi cell is orthogonal to the segment between the base node and the neighboring node, and the point of intersection between the face and segment is located at the midpoint of that segment. Let denote by P_0 the basic node and by $K_{P_0}^*$ – the Voronoi cell. The vertices of Voronoi cell $K_{P_0}^*$ we denote by Q_i. These vertices Q_i are the centers of the spheres circumscribed around the tetrahedrons having the point P_0 as a vertex.

As an approximate solution of equations (1) and (2), we select a linear function $u_h(x, y, z)$ on portions, which must be continuous in the domain $\bar{\Omega}$ and linear on each tetrahedron $K \in T_h$. The function $u_h(x, y, z)$ on the tetrahedron T_h can be defined using the notations in fig. 3.

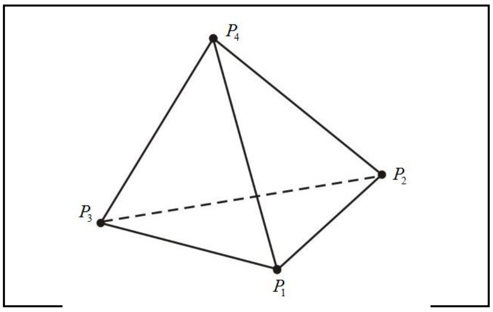

Fig.3. Nodes of tetrahedral from the calculation grid.

We will note in general the tetrahedron (fig. 3) by the relation $K = \overline{P_1P_2P_3P_4}$, which includes the nodes P_1, P_2, P_3, P_4. Let the tetrahedron $K = \overline{P_1P_2P_3P_4}$ (fig. 3) be some element of the set T_h and $P(x, y, z)$ be an arbitrary point of this element.

In this tetrahedron for each vertex we introduce the shape functions $N_i(x, y, z),\ i = \overline{1,4}$. These functions should verify the following conditions: the functions are linear and their values at the tetrahedron vertices are equal to 0 or 1, i.e. $N_i = 1$ for $i=k$ and $N_i = 0$ for $I \neq k$.

The shape functions can be represented in the explicit form through the coordinates of the vertices:

$$N_i(x, y, z) = w_{1,i}x + w_{2,i}y + w_{3,i}z + w_{4,i}\ , \qquad (3)$$

in which, $w_{1,i}$, $w_{2,i}, w_{3,i}$ and $w_{4,i}$ - are the components of the vectors $\bar{w}_i, i = \overline{1,4}$.

To determine the vectors $\bar{w}_i$, we must solve four systems of equations such as: $A\bar{w}_i = \bar{f}_i, i = \overline{1,4}$. The elements of the matrix *A* are formed from the coordinates of the vertices $P_i = P_i(x_i, y_i, z_i), i = \overline{1,4}$ of tetrahedron as follows:

$$A = \begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{pmatrix};$$

$$\bar{f}_i = (f_{1,i}, f_{2,i}, f_{3,i}, f_{4,i}),\ f_{k,i} = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases}.$$

Using the shape functions for every grid node (internal or boundary) we introduce the basic function $\varphi_i(x, y, z)$, $i = 1, 2, \dots, n, n + 1, \dots, n_1$ (n and n_1 represent here the number of internal nodes and the total number of nodes correspondingly). The function $\varphi_i(x,y,z)$ is piecewise linear, i.e. it is continuous and linear on each tetrahedron with unit value in the node P_i and with zero values in all other nodes. Following the satisfaction of these conditions, the function of the approximate solution $u_h(x,y,z)$ can be represented as a linear combination of basic functions:

$$u_h(x,y,z)=\sum_{i=1}^{n_1} u_i \varphi_i(x,y,z)\,. \tag{4}$$

The coefficients u_i from equation (4) are equal to the unknown potential values at the node

$P_i(x_i, y_i, z_i)$, i.e. $u_h(x_i,y_i,z_i)=u_i$.

In contrast to the finite element method, the generalized approach proposed by Galerkin is used in the finite volume method. This generalized approach consists in following. In the condition of orthogonality of the finite element method $\int_\Omega div(\varepsilon_a \text{grad} u_h)\varphi_k dV = -\int_\Omega \sigma\varphi_k dV,\ k=\overline{1,n}$, we use basis functions $\psi_k(x,y,z)$ of the space $W_2^0(\Omega)=L_2(\Omega)$ as follows [7,11]. Let introduce new basis functions $\psi_k(x,y,z)$ for dual grid T_h^* by the following rule: function $\psi_k(x,y,z)$ possesses the constant unit values in the Voronoi cell for internal node P_k and it possesses zero values in the rest of domain. Then the condition of orthogonality with functions $\psi_k(x,y,z)$ gets the form:

$$\int_\Omega div(\varepsilon_a \text{grad} u_h)\psi_k dV = -\int_\Omega \sigma\psi_k dV,\ k=\overline{1,n}\,. \tag{5}$$

Since, the function $\psi_k(x,y,z)$ is nonzero only in the Voronoi cell $K_{P_k}^*$, we obtain the following relation for the orthogonality condition:

$$\int_{K_{P_k}^*} div(\varepsilon_a \text{grad} u_h)dV = -\int_{K_{P_k}^*} \sigma\, dV\,, \tag{6}$$

in which $K_{P_k}^*$ represents the Voronoi cell for the node P_k.

To obtain a system of linear algebraic equations for unknown values of a function u_h in the nodes of the calculation grid to apply the finite volume method it is necessary of proceeding as follows.

To do this, we will examine in the three-dimensional space with Cartesian coordinates Oxyz of Poisson equation $div(\varepsilon_a grad\, u) = -\sigma(x,y,z)$. We will integrate the Poisson equation into the space corresponding to the $K_{P_k}^*$ cell volume with obtained of the following relationship:

$$\int_{K_{P_k}^*} div(\varepsilon_a \text{grad} u)dV = -\int_{K_{P_k}^*} \sigma(x,y,z)\, dV\,. \tag{7}$$

We will examine the left side of the relationship (7). For this we apply the divergence theorem:

$$\int_{K_{P_k}^*} div(\varepsilon_a \text{grad}\, u)dV = \int_{\partial K_{P_k}^*} \varepsilon_a \text{grad}\, u\ d\bar{S} =$$

$$= \int_{\partial K_{P_k}^*} \varepsilon_a(\text{grad}\, u, \bar{n})\, dS = \int_{\partial K_{P_k}^*} \varepsilon_a \frac{\partial u}{\partial n} dS \tag{8}$$

in which $\partial K_{P_k}^*$ - the full surface of the polyhedron; $K_{P_k}^*$; $\bar{n}$ -the external normal to the surface $\partial K_{P_k}^*$; $\partial u/\partial n$ is the derivative of function u by this normal.

In this case the equation (7) takes the form:

$$\int_{\partial K_{P_k}^*} \varepsilon_a \frac{\partial u}{\partial n} dS = -\int_{K_{P_k}^*} \sigma(x,y,z) dV\,. \tag{9}$$

Thus, it was proved that obtaining the solution of the equations (1) and (2) by applying the finite volume method is reduced to the approximation of the relation (9) in the internal nodes of the Voronoi cell calculation grid.

A similar procedure is inherent to the finite difference method for a rectangular cell grid. In this context, the finite volume method can be considered as a generalization of the finite difference method for divide case of the space in blocks with arbitrary shape cells. For this reason, the finite volume method keeps all the advantages of the finite difference method, and, compared to the finite element method and the algorithm for constructing the relationships in finite differences proves to be simpler.

Because of these consequences, in obtaining the numerical solutions of the electric field repairs in the non-homogeneous environment with the application of the finite volume method, it is not necessary to construct local and global rigidity matrices, which are necessary to the use finite element method.

B. Applying the Finite Volume Method to the Calculation of the Electric Field

Either in the Voronoi cell $K_{P_0}^*$ (fig. 2) the base node is denoted by the symbol P_0. With the symbols $P_i, i=\overline{0,8}$ we will denote the nodes of the calculation grid. Symbols $S_i, i=\overline{1,8}$ are used to describe lateral faces, which are orthogonal to the straight-line segment $\overline{P_0P_i}$. The parameter $M_i, i=\overline{1,8}$ describes the points of intersection of the orthogonal $\overline{P_0P_i}$ segments with the lateral faces S_i of the Voronoi cells. When accepting these notations for the Voronoi cell, we have the possibility to approximate of the integral on the $\partial K^*_{P_0}$ surface (9) by the following expression:

$$\int_{\partial K^*_{P_0}} \varepsilon_a \frac{\partial u}{\partial n} dS = \sum_{i=1}^{8} \int_{S_m} \varepsilon_a \frac{\partial u}{\partial n} dS \cong \sum_{i=1}^{8} \varepsilon_a(M_i) \frac{u(P_i)-u(P_0)}{\left|\overline{P_0P_i}\right|} S_i \ ,$$

in which: $\varepsilon_a(M_i)$ - the dielectric permeability value at the point of intersection of the PP segment with the side Si surface; $u(P_i)$ and $u(P_0)$ -voltage values of points P_i and P_0; $\left|\overline{P_0P_i}\right|$ - the length of the segment marked $\overline{P_0P_i}$.

The integral on the right side of the expression (9) will be approximate it with the following relationship:

$$\int_{K^*_{P_0}} \sigma(x,y) dV = \sigma(P_0)V_0 \ ,$$

where: V_0 is the volume of the Voronoi cell $K^*_{P_0}$.

As a result, the expression (9) can be transcribed in the following form:

$$\sum_{i=1}^{8} \varepsilon_a(M_i) \frac{u(P_i)-u(P_0)}{\left|\overline{P_0P_i}\right|} S_i = -\sigma(P_0)V_0 \ .$$

After some transformations, we can write for the point noted by P_0 of the calculation grid the following relationships:

$$\alpha_0 u(P_0) + \sum_{i=1}^{8} \alpha_i u(P_i) = -\sigma(P_0)V_0 \ , \qquad (10)$$

in which: $\alpha_i = \varepsilon_a(M_i) \dfrac{S_i}{\left|\overline{P_0P_i}\right|}, \ i=\overline{1,8}; \ \alpha_0 = -\sum_{i=1}^{8} \alpha_i$.

Equation (10) can be written for each internal node of the calculation grid. For the boundary nodes of the highlighted volumes it is necessary to use the limit conditions (2).

As a result, we obtain the system of linear algebraic equations with symmetrical matrix. Each type (10) equation contains only a few elements that differ from zero. The number of equations that meet this condition is not high (usually between 9-25 equations). As a result, the final matrix of the system of equations drawn up based on the finite volume method has a relatively small dimension.

Another particularity of the procedure of calculating the electric field in non-homogeneous structures is that, in the proposed algorithm, only the nonzero elements of the matrix are stored in computer memory.

To solve the obtained equation system, we use the iterative method of conjugate gradients that converge very quickly for problems of the type examined.

The solution obtained $u_h(x,y,z)$ in the domain $\bar{\Omega}$ makes it possible to construct the field of flux of the vector intensity $\vec{E} = (E_x, E_y, E_z) = -\mathrm{grad}\, u$ of the electric field.

We will note by v of the vector flux $\vec{E}$ that crosses the unitary area of the orthogonal surface with the $\vec{E}$ vector.

For these conditions, the contour lines $u(x,y,z) = const$ and the lines $v(x,y,z) = const$ form mutually orthogonal families.

The function $v(x,y,z)$ can be obtained by calculation of the following contour integral:

$$v(x,y) = \int_{(x_0,y_0,z_0)}^{(x,y,z)} \left(E_x dx + E_y dy + E_z dz\right),$$

in which: x_0, y_0, z_0 - the coordinates of an arbitrary fixed point in the domain Ω, and the path of integration is located inside it.

In the case of a multiply connected area, the integration path must not cross of domain cuts, which lead to a single connected structure.

The capacitance C between two conductive bodies is calculated by the formula:

$$C = \frac{q}{u_1 - u_2} \ , \qquad (11)$$

where: $(u_1 - u_2)$ is potential difference of these bodies.

The value of the electric charge q, located inside a volume V, is calculated according to the Gauss theorem for the body with of $S=\partial V$ surface, having as an independent variable the intensity $\vec{E}$ of the electric field:

$$q=\varepsilon\int_S \vec{E}\cdot d\vec{S}=-\varepsilon\int_S \text{grad}u\cdot d\vec{S}= =-\varepsilon\int_S (\text{grad}u\cdot\vec{n})dS=-\varepsilon\int_S (\partial u/\partial n)dS\ , \quad (12)$$

in which: S - arbitrary surface, which includes the body, with electrical charge; $\vec{n}$ – the exterior normal vector to the surface S, ε –the dielectric permissivity.

III. VERIFICATION OF THE ALGORITHM

The algorithm proposed to solve the electric field problem was made as a complex of programs in the Matlab application environment. The robustness of the algorithm and software package have been tested on an object, which having geometry of a hollow cylinder (Fig. 1). In the calculation model the initial condition was formulated, which sets a non-zero value of the potential on the internal surface of the hollow cylinder. At the same time, it was considered that the outer surface of the cylinder (lateral and its bases) having the null value of the potential.

The algorithm described was the basis of elaboration the calculation software of the three-dimensional distribution of the potential $u(r,\varphi,z)$ of the electric field in the hollow cylinder. Cylinder has the dimensions: the internal radius R_a = 0.2 m; outer radius R_b = 0.5 m; cylinder height H = 0.5 m.

The following limit conditions have been formulated:

- the potential on the outer side surface of the cylinder: $u(R_b,\varphi,z)=0$;
- the potential on the surface of the cylinder head from above:

$$u(r,\varphi,H)=U_0(0.25\cos\varphi+0.75)\left[\left(R_b-r\right)/\left(R_b-R_a\right)\right];$$

- the potential on the surface of the cylinder head from the bottom:

$$u(r,\varphi,0)=\left(U_0/2\right)(0.25\cos\varphi+0.75)\left[(R_b-r)/(R_b-R_a)\right];$$

- the potential on the inner side surface of the cylinder:

$$u(R_a,\varphi,z)=\frac{U_0}{2}(0.25\cos\varphi+0.75)\left(1-\frac{z}{H}\right).$$

We will consider that the potential U_0 has the value $U_0=10V$.

To construct a spatial discrete grid in the cylinder volume, a stairway was selected: 10 divisions along the vertical axis z, 6 divisions along the beam and 25, 32, 38, 44, 50, 56, 62 divisions along the circumference. In this case, the divisions or steps have the size were 0.05 m, and the total number nodes of grid was 3377. Taking into account these division conditions, the volume of the cylinder was divided into 15810 tetrahedra.

The solution to the test problem's (fig. 1) is shown in fig. 4. The distribution of the equipotential lines $u_i=const$ (full lines) and the lines describing the distribution of the flux intensity vector $\vec{E}$ (dotted lines) of the electric field in the transverse section placed at the half height of the cylinder (z = H / 2 = 0.25 m) is shown in fig.4a. The distribution of the electric field in the section of the vertical plane of the cylinder with the coordinate y = 0 is shown in fig. 4b.

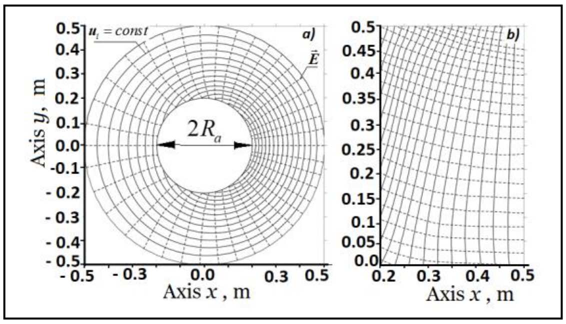

Fig. 4. Electrical field distribution in the hollow cylinder: *a*) - in the cross-section with the coordinate *z* = H / 2 = 0.25 m; *b*) - in the plane of the vertical section - *y* = 0 and *Ra* <x <*Rb*.

The proposed algorithm and numeric computing software allows the analysis of the electric field distribution in any section of the hollow cylinder. The voltage dividers are used to measure of the high voltage. The use of glass-insulated micro conductors is a reasonable solution for the manufacture of voltage dividers from resistive element [12].

The determination of the electrostatic fields and electrical capacities of the high-voltage dividers is a difficult issue because of their constructive complexity. This can be done using the finite volume method. An example of a constructive embodiment of the voltage divider is shown in fig. 5a. The resistive element has the following dimensions: H1 = 120 mm; D1 = 28 mm; D2 = 18 mm. The screen has the height H = 220 mm and the diameter D = 75 mm. Relative dielectric constant for glass $\varepsilon_1 = 6$ and $\varepsilon_2 = 1$ units for air. The potential changes linearly from zero (grounded terminal) to ten units (terminal connected to high voltage). fig. 5b and fig. 5c represents the potential and intensity curves of the field of the resistive divider with screen and without it.

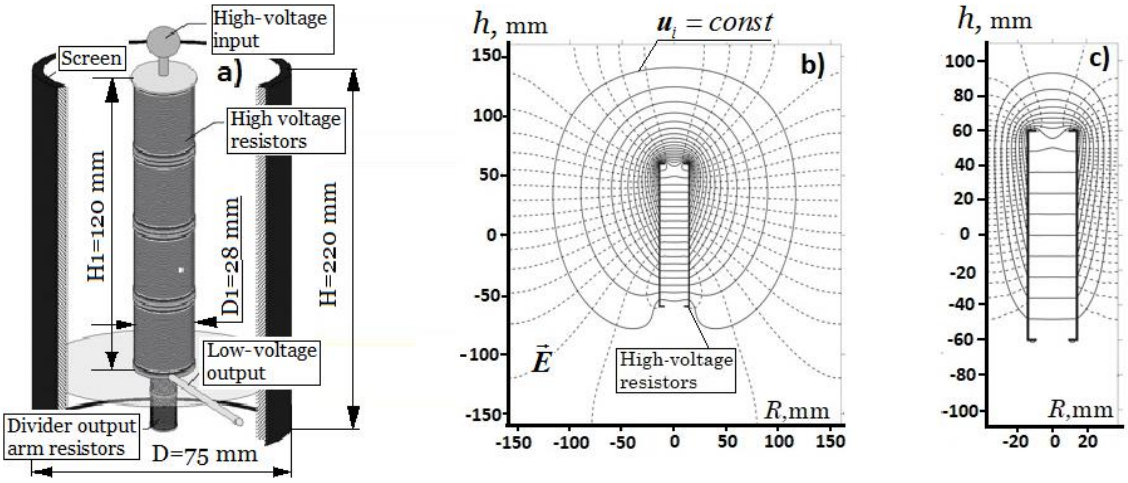

Fig. 5. General view of the voltage divider (a) made of micro conductors, the distribution of the potential and intensity of the electric field of the non-screen divider (b) with capacity of 22.0 pF (19 675 nodes the calculation grid) and the cylindrical screen divider with the capacity of 41.0 pF (calculation grid has 66164 nodes).

IV. Conclusions

An effective algorithm has been proposed to solve the problem of calculating the electric field in non-homogeneous structures based on the finite volume method. The algorithm, during the computation process, stores only the nonzero values of matrixes elements in the computer memory.

The finite volume method keeps all the advantages of the finite difference method. Compared to the finite element method, the algorithm for constructing the finite difference relations is to simpler. In this case, there is no need to build a local and global rigidity matrix to form an equation solving system for the determination of capacitive coefficients of the electrical field.

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