Mathematical Model of Electrical Line with Transposition of Phase Circuits

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Abstract. The purpose of this paper is to elaborate the mathematical model and the method of calculation of the permanent regime in the line with many conductors with transposed phases. The mathematical model is based on the telegraph equations and takes into account the fact that the electric lines are lines with distributed parameters. As a subject of the study it is selected the 110 kV overhead power line with two compact circuits with the conductors placed horizontally and circularly transposed. The initial and boundary conditions are formulated for the case of two-circuit electric line and the adjustment of the phase angle of the voltages at the line input. In the transposition the values of the conductor parameters change by leap, which complicates the process of calculating the operating mode. The developed model and elaborated software include all these features. Based on the developed model, calculations of the operating mode of the two-circuit electric circuit and of the self-compensated line were performed. Numerical solutions have been obtained regarding the evolution of active and reactive power in the phases of the line in its various sections under regulation and non-regulation of the phase shift angle for the cases without and with the transposition of the phase conductors. The applicability of the model for studying power transfer processes in multi-conductor power lines has been demonstrated. There were obtained the numerical solutions useful for estimating the degree of mutual influence of phases on the ability to transfer power to load under the transposition of conductors.
Keywords: electrical line, double circuit, adjustment, phase shift angle, active power, reactive power, magnitude, deviation.
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Modelul Matematic al Liniei Electrice cu Transpunerea Conductoarelor Fazelor

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Rezumat. Scopul acestei lucrări constă în elaborarea modelului matematic și a metodei de calcul a regimului permanent în linia cu multe conductoare cu faze transpuse. Modelul matematic s-a elaborat în baza ecuațiilor telegrafiștilor, care țin cont de faptul, că liniile electrice sunt linii cu parametri distribuiți. În calitate de obiect al studiului s-a selectat linia aeriană cu tensiunea 110 kV cu două circuite de tip compact cu conductoarele amplasate în plan orizontal și transpuse circular. Sunt formulate condițiile inițiale și la limită pentru cazul liniei electrice cu dublu circuit și cu reglarea unghiului diferenței de fază al tensiunilor la intrarea liniei. În secțiunea de transpunere, valorile parametrilor conductoarelor se modifică prin salt, ceea ce complică procedeul de calcul al regimului în linie. Modelul și softul elaborat includ aceste particularități. În baza modelului elaborat s-au executat calcule ale regimului de funcționare a liniei electrice cu dublu circuit de tip compact și a liniei dirijate cu autocompensare. S-au obținut soluțiile numerice privind evoluția puterii active și reactive în fazele liniei în diferite secțiuni a ei pentru cazurile de reglare și nereglare a unghiului diferenței de fază pentru cazul fără și cu transpunere a conductoarelor fazelor liniei. S-a demonstrat aplicabilitatea modelului pentru studierea proceselor de vehiculare a puterii în liniile electrice cu multe conductoare. S-au obținut soluții numerice utile pentru estimarea gradului de influență reciprocă ale fazelor privind capacitatea de vehiculare a puterii spre sarcină la transpunerea conductoarelor.
Cuvinte cheie: linie electrică, dublu circuit, reglare, unghi a decalajului de fază, putere activă, putere reactivă, magnitudine, deviere.

Математическая модель электрической линии с транспозицией фазовых проводников

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Аннотация. Целью данной работы является разработка математической модели и метода расчета установившегося режима в линии со многими проводниками с транспонированными фазами.

Математическая модель основана на телеграфных уравнениях и учитывает, что электрическими линиями являются линии с распределенными параметрами. В качестве объекта исследования выбрана линия электропередачи 110 кВ с двумя компактными цепями с проводниками, расположенными горизонтально и циклически транспонированными. Исходные и граничные условия формулируются для случая двухцепной электрической линии при регулировки фазового угла напряжений на входе линии. При транспозиции, значения параметров проводника изменяются скачком, что усложняет процесс вычисления режима работы линии. Рассмотренная модель и разработанное программное обеспечение включают в себя все эти функции. На основе разработанной модели были выполнены расчеты режима работы двухцепной электрической цепи и управляемой линии с самокомпенсацией. Полученные численные решения позволяют проследить эволюцию активной и реактивной мощности в фазах линии в ее различных сечениях при регулировании и нерегулировании угла фазового сдвига для случаев без и с транспозицией фазных проводников. Показана применимость модели для изучения процессов передачи энергии в многопроводных линиях электропередач. Получены численные результаты которые использованы для оценки степени взаимного влияния фаз на способность передавать мощность на нагрузку при транспозиции проводников.
Ключевые слова: электрическая линия, двойная цепь, регулировка, угол фазового сдвига, активная мощность, реактивная мощность, амплитуда, отклонение.

I. Introduction

The high velocity of electromagnetic waves propagation is an advantage of electricity as a useful energy form because it provides the ability to transmit electricity at great distances and a flexible distribution. The transmission and distribution of electric power is provided by electric lines of different constructive realization. In this context, power lines are an important functional component of contemporary power systems. The constructive simplicity of the power lines does not mean the simplicity of the physical processes in these infrastructure elements of the power systems.

The first transfer of electricity at a distance of 1 km was demonstrated by Fontaine in 1873 [1], who considered that such transfers are possible only for small capacities and for short distances. The theoretical aspects regarding the transmission of electricity at long distances were developed by D.A. Lachinov and M. Deprez [1, 2], and the last one in 1882 provided the transfer of electricity by cable at a distance of 57 km between Munich and Miesbach with the yield of 22% and in 1883 the efficience reached 62% [1]. Let's mention, that the value of the yield is not the only index that can influence the economic competitiveness of the power transmission systems.

The proposal of the three phase alternating current (1888) and the increase in voltages have opened the way for the expansion of the electrical networks, which have been characterized by processes typical for the long lines. In 1891, Dolivo-Dobrovolsky with engineer Brown organized the transfer of electricity at a 170 km distance from Laufen on Neckar to the Frankfurt Electrical Engineering Exposition with the yield of approx. 75% [3]. The increase in the length of the electrical lines, the unsymmetrical location of the phase conductors led to the occurrence of unbalance in the three-phase AC system [4].

Balancing the parameters of the phases of the high voltage lines is done by several methods: changing the mutual location and the distances of the phase conductors, of the distance from the surface of the soil etc. In order to optimize the phase positioning it is recommended to use the phase equalization criterion [4], the stabilization of the distance between phase and conductor conductors [5-8], the use of new conductors with good mechanical characteristics [9, 10], transposition of phase conductors [11, 12]. As a solution to ensure the symmetry in the long lines, it was proposed to transpose the phase conductors. This technology leads to increased complexity of calculations not only of the operating regime, but also of the parameters of multi-conductor electric lines and of the distributed parameters [13-15]. The method of calculation of the power line regime in the phasor system of coordinates is presented as a robust and efficient solution, which shows the tendency of extension in its application in the engineering calculations [16 -18].

The purpose of this paper is to elaborate the mathematical model and the method of calculating the permanent regime in of the line with many phases with conductors transposed into the phasor system of coordinates.

II. Mathematical model of multi-conductor transmission line

Consider the propagation of electromagnetic energy through multi-conductor three-phase high-voltage transmission line with arbitrary number of conductors. The mathematical formulation of the problem represents the system of partial differential equations known as transmission line equations. The equations are derived from Maxwell equations and for unknown voltage vector $\boldsymbol{u}(x,t)$ and current vector $\boldsymbol{i}(x,t)$ have the following form

$$L\frac{\partial i}{\partial t}+\frac{\partial u}{\partial x}+R\boldsymbol{i}=0,$$

$$C\frac{\partial u}{\partial t}+\frac{\partial i}{\partial x}+G\boldsymbol{u}=0. \qquad (1)$$

The domain of the solution is the rectangle $D=\{(x,t): x\in(0,l), t\in(0,T_{max})\}$ where l is the length of the transmission line and $T_{\max}$ is the maximal time of calculating for vectors $\boldsymbol{u}(x,t)$ and $\boldsymbol{i}(x,t)$. In n-conductor line the vector functions $\boldsymbol{u}(x,t)$ and $\boldsymbol{i}(x,t)$ have n components each, but L, C, R and G in (1) are symmetrical matrices of linear inductances, capacitances, wire resistances and conductivities of insulation:

$$\boldsymbol{u}(x,t)=\begin{pmatrix}u_1(x,t)\\u_2(x,t)\\\dots\\u_n(x,t)\end{pmatrix},\boldsymbol{i}(x,t)=\begin{pmatrix}i_1(x,t)\\i_2(x,t)\\\dots\\i_n(x,t)\end{pmatrix},$$

$$L=\begin{pmatrix}L_{11}&L_{12}&\dots&L_{1n}\\L_{12}&L_{22}&\dots&L_{2n}\\\dots&\dots&\dots&\dots\\L_{1n}&L_{2n}&\dots&L_{nn}\end{pmatrix},$$

$$C=\begin{pmatrix}C_{11}&C_{12}&\dots&C_{1n}\\C_{12}&C_{22}&\dots&C_{2n}\\\dots&\dots&\dots&\dots\\C_{1n}&C_{2n}&\dots&C_{nn}\end{pmatrix}, \qquad (2)$$

$$R=\begin{pmatrix}R_{11}&0&\dots&0\\0&R_{22}&\dots&0\\\dots&\dots&\dots&\dots\\0&0&\dots&R_{nn}\end{pmatrix},$$

$$G=\begin{pmatrix}G_{11}&0&\dots&0\\0&G_{22}&\dots&0\\\dots&\dots&\dots&\dots\\0&0&\dots&G_{nn}\end{pmatrix}.$$

To obtain a unique solution, we must add the initial (when $t=0$) and boundary (when $x=0$ and $x=l$) conditions. We assume that at the initial time $t=0$ there are no voltages and currents in the line

$$\boldsymbol{u}(x,t)=\boldsymbol{i}(x,t)=0,\ x\in[0,1]\ , \qquad (3)$$

at the input of the line, at $x=0$, voltages are given, and at the output for $x=l$ we have a load with resistance R_S

$$\boldsymbol{u}(0,t)=\boldsymbol{U}_0(t),\quad \boldsymbol{u}(l,t)=R_S\boldsymbol{i}(l,t)\,. \qquad (4)$$

We assume further that we have a 6-wire line (n = 6) with three phases A, B, C (Fig. 1) and the values $\boldsymbol{U}_0$ and R_S from (4) are the following

$$\boldsymbol{U}_0(t)=U_m\begin{pmatrix}\sin\omega t\\\sin(\omega t+\varphi_a)\\\sin(\omega t-2\pi/3)\\\sin(\omega t-2\pi/3+\varphi_b)\\\sin(\omega t+2\pi/3)\\\sin(\omega t+2\pi/3+\varphi_c)\end{pmatrix},$$

$$\boldsymbol{R_S}=\begin{pmatrix}Z_{11}&0&0&0&0&0\\0&Z_{22}&0&0&0&0\\0&0&Z_{33}&0&0&0\\0&0&0&Z_{44}&0&0\\0&0&0&0&Z_{55}&0\\0&0&0&0&0&Z_{66}\end{pmatrix}, \qquad (5)$$

Where $\omega=2\pi f$ is the circular frequency (1/sec); f is the oscillation frequency (1/sec); $T=\frac{2\pi}{\omega}=\frac{1}{f}$ is the period of oscillations (sec); φ_a, φ_b, φ_c are the phase shifts in the wires; the values $Z_{11}=\sqrt{\frac{L_{11}}{C_{11}}}$, $Z_{22}=\sqrt{\frac{L_{22}}{C_{22}}}$, ..., $Z_{66}=\sqrt{\frac{L_{66}}{C_{66}}}$ represent the characteristic impedance (or characteristic resistance); $U_m=110\sqrt{2/3}$ is the value of phase voltage amplitude for 110 kV line.

After solving the formulated problem, the active power $\boldsymbol{P}$ can be calculated as the average value for the period T of instantaneous power oscillations $\boldsymbol{p}(x,t)=\boldsymbol{u}(x,t)\boldsymbol{i}(x,t)$ using the formula

$$\boldsymbol{P}(x,t)=\frac{1}{T}\int_{t-T/2}^{t+T/2}\boldsymbol{u}(x,\tau)\boldsymbol{i}(x,\tau)\,d\tau. \quad (6)$$

Reactive power in the line $\boldsymbol{Q}$ can be calculated by the formula

$$\boldsymbol{Q}(x,t)=\frac{1}{\omega T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}}\boldsymbol{u}(x,\tau)\frac{d\boldsymbol{i}(x,\tau)}{d\tau}d\tau=$$

$$=-\frac{1}{\omega T}\int_{t-T/2}^{t+T/2}\boldsymbol{i}(x,\tau)\frac{d\boldsymbol{u}(x,\tau)}{d\tau}d\tau. \quad (7)$$

In (6) and (7) the component-wise multiplication of vectors, i.e. $p_k=u_k i_k, k=\overline{1,n}$ is used.

III. Characteristics of 110 kV electrical line and conditions of transposition

The geometry of 110 kV electrical two-circuit line with the horizontal phase alignment is represented in fig. 1.

In the mathematical model we take into account the transposition of the phases of transmission lines, which is performed to reduce the asymmetry of voltages and currents in the electrical system under normal operation of power transmission and to limit the interfering effects of power lines on low-frequency communication channels. We carry out the transposition according to the scheme shown in Fig. 2.

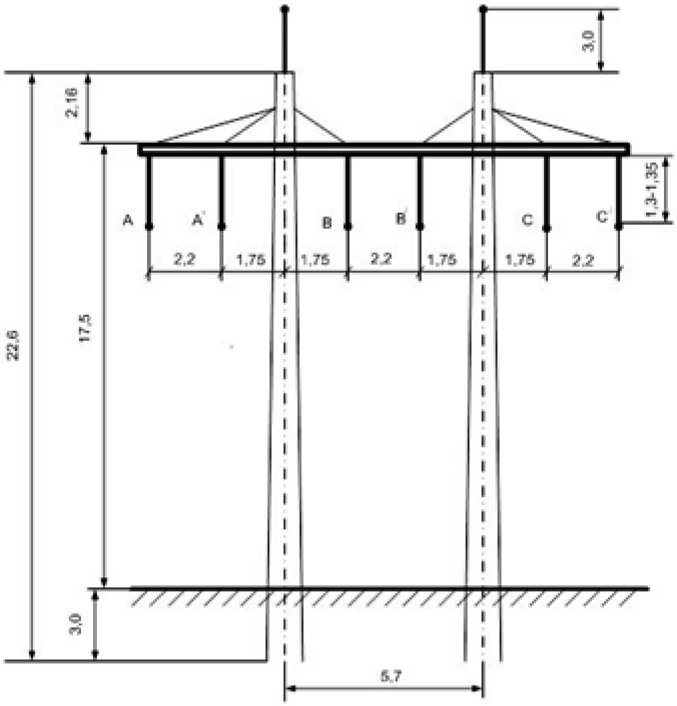

Fig. 1. Intermediate SPB 110-2M reinforced concrete support for 110 kV transmission line with adjacent phases. Dimensions are in meters. The diameter of the conductors is 17.1 mm.

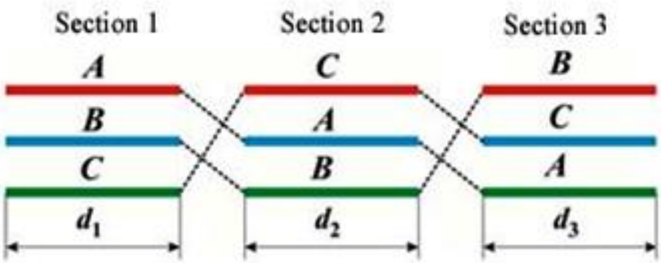

Fig. 2. The transposition scheme of a three-phase power transmission line

In this case the line with total length l is divided in three sections with length d_1, d_2, d_3 respectively. Let consider $d_1 = d_2 = d_3 = d$ and $\sum_{k=1}^{3} d_k = l$. The boundary conditions of the form (4) are given at the input and the output of the line, and the conjugation conditions are formulated at the transposition points (preservation of continuous voltages and currents in phases A, B and C) as follows

$$\boldsymbol{u}(kd-0,t) = T\boldsymbol{u}(kd+0,t),$$
$$\boldsymbol{i}(kd-0,t) = T\boldsymbol{i}(kd+0,t), k = 1,2. \quad (7)$$

Here T is the transposition matrix, which for the scheme in Fig. 2 has the form

$$T = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$T^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}. \quad (8)$$

IV. Solution of the problem by method of complex amplitudes

Since a sinusoidal voltage with frequency ω (5) is given at the input of the line, the formulated problem can be solved exactly by the method of complex amplitudes (MCA). Let represent the vector functions of voltage $\boldsymbol{u}(x,t)$ and current $\boldsymbol{i}(x,t)$ in the form of complexes $\boldsymbol{U}(x)$ and $\boldsymbol{I}(x)$ as follows: $\boldsymbol{u}(x,t) = \boldsymbol{U}(x)e^{j\omega t}$ and $\boldsymbol{i}(x,t) = \boldsymbol{I}(x)e^{j\omega t}$. After corresponding substitution the system (1) takes the form

$$\frac{d\boldsymbol{U}}{dx} + (R + j\omega L)\boldsymbol{I} = 0,$$
$$\frac{d\boldsymbol{I}}{dx} + (G + j\omega C)\boldsymbol{U} = 0 \quad (9)$$

or

$$\frac{d^2\boldsymbol{U}}{dx^2} - (R + j\omega L)(G + j\omega C)\boldsymbol{U} = 0,$$
$$\frac{d^2\boldsymbol{I}}{dx^2} - (G + j\omega C)(R + j\omega L)\boldsymbol{I} = 0. \quad (10)$$

Firstly, we reduce the matrices L and C to the diagonal form. Since in the transmission line the electromagnetic waves propagate at the same speed $a \le c_0$ (c_0 is the speed of the light), then the condition $LC = CL = \frac{1}{a^2}E$ (E is the unit matrix) must be satisfied [17, 18]. This means that the symmetric matrices L and C can be reduced to diagonal form by the same orthogonal matrix Q:

$$L = Q\Lambda_L^2 Q', C = Q\Lambda_C^2 Q', \quad (11)$$

where Λ_L and Λ_C are diagonal matrices with the elements $\lambda_{L,k}\lambda_{C,k}, k = \overline{1,n}$. The values $\lambda_{L,k}$ and $\lambda_{C,k}$ are equal to square roots of the eigenvalues of the matrices L and C, respectively. The elements $\lambda_{L,k}$ and $\lambda_{C,k}$ are located in the matrices Λ_L and Λ_C in such a way that their product with the same number k is equal to the velocity of the electromagnetic wave propagation, i.e.

$$\frac{1}{\lambda_{L,k}\lambda_{C,k}} = a, k = \overline{1,n}. \quad (12)$$

Next, instead of current and voltage complexes, we introduce new functions $\boldsymbol{W}_1 = Q'\boldsymbol{U}$, $\boldsymbol{W}_2 = Q'\boldsymbol{I}$ and transform the systems (9), (10) to the form

$$\frac{d\boldsymbol{W}_1}{dx} + \Lambda_Z \boldsymbol{W}_2 = 0, \Lambda_Z = R + j\omega\Lambda_L,$$
$$\frac{d\boldsymbol{W}_2}{dx} + \Lambda_Y \boldsymbol{W}_1 = 0, \Lambda_Y = G + j\omega\Lambda_C \quad (13)$$

or

$$\frac{d^2\boldsymbol{W}_1}{dx^2} - \Lambda^2 \boldsymbol{W}_1 = 0, \Lambda^2 = \Lambda_Z\Lambda_Y,$$
$$\frac{d^2\boldsymbol{W}_2}{dx^2} - \Lambda^2 \boldsymbol{W}_2 = 0, \quad (14)$$

where Λ is the diagonal matrix with elements

$$\lambda_k = \sqrt{(R + j\omega\lambda_{L,k})(G + j\omega\lambda_{C,k})}, k = \overline{1,n}.$$

The general solution for the function $\boldsymbol{W}_1$ on the interval $x \in [0, d]$ is represented in the form

$$\boldsymbol{W}_1(x) = E_p(x)\boldsymbol{c}_1 + E_m(x)\boldsymbol{c}_2, \quad (15)$$

where $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ are vectors of unknown constants, $E_p(x)$ and $E_m(x)$ are diagonal matrices with elements $e^{\lambda_k x}$ and $e^{-\lambda_k x}$. Further, from the first equation (13) we obtain a solution for the function $\boldsymbol{W}_2$:

$$\boldsymbol{W}_2(x) = -\Lambda_Z^{-1}\frac{d\boldsymbol{W}_1}{dx} =$$

$$= -\Lambda_Z^{-1}\left(\Lambda E_p(x)\boldsymbol{c}_1 - \Lambda E_m(x)\boldsymbol{c}_2\right) =$$

$$= -\Lambda_B^{-1}\left(E_p(x)\boldsymbol{c}_1 - E_m(x)\boldsymbol{c}_2\right). \quad (16)$$

Here Λ_B is a diagonal matrix with elements $\sqrt{\frac{R+j\omega\lambda_{L,k}}{G+j\omega\lambda_{C,k}}}, k = \overline{1,n}$. This matrix represents the diagonal form of the wave impedance matrix

$$Z_B = [(G + j\omega C)^{-1}(R + j\omega L)]^{1/2},$$

$$Z_B = Q\Lambda_B Q'. \quad (17)$$

On the next two sections of the line the general solution is obtained in an analogous way and has the form

$$\boldsymbol{W}_1(x) = E_p(x)\boldsymbol{c}_3 + E_m(x)\boldsymbol{c}_4,$$

$$\boldsymbol{W}_2(x) = -\Lambda_B^{-1}\left(E_p(x)\boldsymbol{c}_3 - E_m(x)\boldsymbol{c}_4\right),$$

$$x \in [d, 2d], \quad (18)$$

$$\boldsymbol{W}_1(x) = E_p(x)\boldsymbol{c}_5 + E_m(x)\boldsymbol{c}_6,$$

$$\boldsymbol{W}_2(x) = -\Lambda_B^{-1}\left(E_p(x)\boldsymbol{c}_5 - E_m(x)\boldsymbol{c}_6\right),$$

$$x \in [2d, l]. \quad (19)$$

To calculate the values of the constant vectors $\boldsymbol{c}_k, k = \overline{1,6}$, we rewrite the boundary conditions and conjugation conditions (4), (5) for the functions $\boldsymbol{W}_1(x)$ and $\boldsymbol{W}_2(x)$ as follows

$$\boldsymbol{W}_1(0) = Q'\boldsymbol{U}_0,$$

$$\boldsymbol{W}_1(l) = Q'R_s Q\boldsymbol{W}_2(l) \quad (20)$$

$$\boldsymbol{U}_0 = U_m\left(1,\ e^{j\varphi_a}, e^{-j2\pi/3}, e^{j(-2\pi/3+\varphi_b)},\right.$$

$$\left.e^{j2\pi/3},\ e^{j(2\pi/3+\varphi_c)}\right), \quad (21)$$

$$\boldsymbol{W}_1(d-0) = T_Q\boldsymbol{W}_1(d+0),$$

$$\boldsymbol{W}_2(d-0) = T_Q\boldsymbol{W}_2(d+0),$$

$$T_Q = Q'TQ, \quad (22)$$

$$\boldsymbol{W}_1(2d-0) = T_Q\boldsymbol{W}_1(2d+0),$$

$$\boldsymbol{W}_2(2d-0) = T_Q\boldsymbol{W}_2(2d+0). \quad (23)$$

Now we substitute $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ from (15) – (19) in relations (20) – (23) and obtain

$$\boldsymbol{c}_1 + \boldsymbol{c}_2 = Q'\boldsymbol{U}_0,$$

$$E_p(d)\boldsymbol{c}_1 + E_m(d)\boldsymbol{c}_2 =$$

$$= T_Q[E_p(d)\boldsymbol{c}_3 + E_m(d)\boldsymbol{c}_4],$$

$$E_p(d)\boldsymbol{c}_1 - E_m(d)\boldsymbol{c}_2 =$$

$$= \Lambda_B T_Q \Lambda_B^{-1}[E_p(d)\boldsymbol{c}_3 - E_m(d)\boldsymbol{c}_4],$$

$$E_p(2d)\boldsymbol{c}_3 + E_m(2d)\boldsymbol{c}_4 =$$

$$= T_Q[E_p(2d)\boldsymbol{c}_5 + E_m(2d)\boldsymbol{c}_6], \quad (24)$$

$$E_p(2d)\boldsymbol{c}_3 - E_m(2d)\boldsymbol{c}_4 =$$

$$= \Lambda_B T_Q \Lambda_B^{-1}[E_p(2d)\boldsymbol{c}_5 - E_m(2d)\boldsymbol{c}_6],$$

$$E_p(l)\boldsymbol{c}_5 + E_m(l)\boldsymbol{c}_6 =$$

$$= -Q'R_s Q\Lambda_B^{-1}[E_p(l)\boldsymbol{c}_5 - E_m(l)\boldsymbol{c}_6].$$

From the last equation of the system (24), we express the vector $\boldsymbol{c}_5$ in terms of $\boldsymbol{c}_6$. Then, successively, from the second to the fifth equations, we express the remaining vectors also in terms of $\boldsymbol{c}_6$. As a result we have

$$\boldsymbol{c_5} = T_{56}\boldsymbol{c_6}, \boldsymbol{c_4} = T_{46}\boldsymbol{c_6}, \boldsymbol{c_3} = T_{36}\boldsymbol{c_6},$$

$$\boldsymbol{c_2} = T_{26}\boldsymbol{c_6}, \boldsymbol{c_1} = T_{16}\boldsymbol{c_6}, \qquad (25)$$

where

$$T_{56} = E_m(l)(E + Q'R_sQ\Lambda_B^{-1})^{-1}(-E + Q'R_sQ\Lambda_B^{-1})E_m(l),$$

$$T_{46} = 0.5E_p(2d)\left[T_Q\left(E_p(2d)T_{56} + E_m(2d)\right) - \Lambda_B T_Q \Lambda_B^{-1}\left(E_p(2d)T_{56} - E_m(2d)\right)\right],$$

$$T_{36} = 0.5E_m(2d)\left[T_Q\left(E_p(2d)T_{56} + E_m(2d)\right) + \Lambda_B T_Q \Lambda_B^{-1}\left(E_p(2d)T_{56} - E_m(2d)\right)\right],$$

$$T_{26} = 0.5E_p(d)\left[T_Q\left(E_p(d)T_{36} + E_m(d)T_{46}\right) - \Lambda_B T_Q \Lambda_B^{-1}\left(E_p(d)T_{36} - E_m(d)T_{46}\right)\right],$$

$$T_{16} = 0.5E_m(d)\left[T_Q\left(E_p(d)T_{36} + E_m(d)T_{46}\right) + \Lambda_B T_Q \Lambda_B^{-1}\left(E_p(d)T_{36} - E_m(d)T_{46}\right)\right].$$

After substituting (25) into the first equation from (24), we obtain a system of equations for determining the unknown vector $\boldsymbol{c}_6$

$$(T_{16} + T_{26})\boldsymbol{c_6} = Q'\boldsymbol{U_0}. \qquad (26)$$

Solving the system (26) we determine the vector $\boldsymbol{c}_6$ and then from (25) we can obtain the remaining vectors $\boldsymbol{c}_i$, i = 5, 4, 3, 2, 1. The solution of the problem for functions $\boldsymbol{W}_1(x)$ and $\boldsymbol{W}_2(x)$ is determined by the formulas (15), (16) and (18), (19). Now the complexes $\boldsymbol{U}$ and $\boldsymbol{I}$ can be determined by transition formulas $\boldsymbol{U} = Q\boldsymbol{W}_1$ and $\boldsymbol{I} = Q\boldsymbol{W}_2$.

V. Numerical results

The method described above has been implemented as a complex of programs in the Matlab system. The elaborated software was tested and used for calculating the operating modes in the three phase 110 kV line with two circuits with the horizontal position of the conductors (fig.1). The length of this line was taken equal to 150 km. The operating mode has been calculated for two constructive embodiments of the power line: without transposition and with transposition of the phase conductors.

The proposed calculation model does not include in consideration the protection sections, because the sections are not subject to transposition. If necessary, the proposed mathematical model of the three-phase line can be complemented with passive construction components within the power line (a section or two sections of protection). It is to mention that protection sections can have an influence on the parameters of the line as well as on the operating mode. These issues are not the subject of this paper.

The proposed mathematical model can also be used to investigate the permanent regime of compact lines and lines conducted in the phased coordinate system. In the calculations, the values of line parameters were used for the geometry shown in fig. 1. The values of the line dissipative parameters were considered identical for both circuits. The conductors of the phase of this electric line are assigned the following values: R = 0.4218 mOhm/m, G = 2.05 pSm/m.

The model takes into account the mutual influence of phase conductors, which is determined by the inductances and line capacities of these conductors. The 110 kV transmission line is the line with the nearby conductors in the aperture of the line (fig. 3, fig. 4). The phases denoted by 1, 3, 5 form the first circuit, and the phases denoted by 2, 4, 6 form the second circuit. This structure allows modeling of the regime in the case when the angle φ of phase difference between the voltage inputs is also adjusted. This adjustment is realized in the second circuit (conductors 2, 4, 6). This ensures the operation of the power line in the self-compensated mode [19].

The values of the parameters and the mutual inductive and capacitive values for the geometry in fig. 2 are the following

$$L=\begin{pmatrix}1.6188 & 0.6475 & 0.3190 & 0.2860 & 0.1920 & 0.1768\\ 0.6475 & 1.6188 & 0.3597 & 0.3190 & 0.2093 & 0.1920\\ 0.3190 & 0.3597 & 1.6188 & 0.6475 & 0.3190 & 0.2860\\ 0.2860 & 0.3190 & 0.6475 & 1.6188 & 0.3597 & 0.3190\\ 0.1920 & 0.2093 & 0.3190 & 0.3597 & 1.6188 & 0.6475\\ 0.1768 & 0.1920 & 0.2860 & 0.3190 & 0.6475 & 1.6188\end{pmatrix}\ \mu\text{H/m};$$

$$C=\begin{pmatrix}8.3667 & -3.0349 & -0.6807 & -0.5043 & -0.2623 & -0.2291\\ -3.0349 & 8.4908 & -0.9222 & -0.6484 & -0.3069 & -0.2623\\ -0.6807 & -0.9222 & 8.5846 & -2.8883 & -0.6484 & -0.5043\\ -0.5043 & -0.6484 & -2.8883 & 8.5846 & -0.9222 & -0.6807\\ -0.2623 & -0.3069 & -0.6484 & -0.9222 & 8.4908 & -3.0349\\ -0.2291 & -0.2623 & -0.5043 & -0.6807 & -3.0349 & 8.3667\end{pmatrix}\ \text{pF/m}.$$

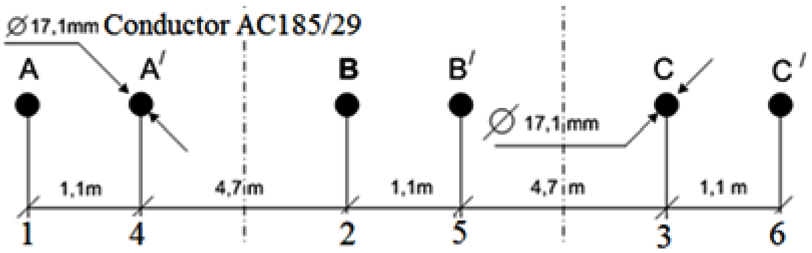

Fig.3 Reciprocal positioning of the 110 kV line conductors with two circuits in the aperture of the line

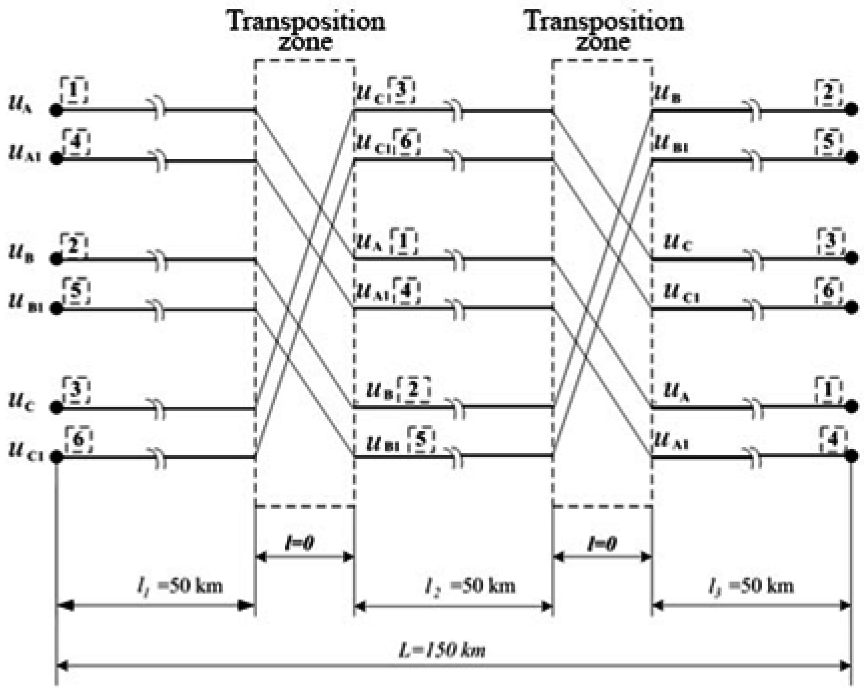

Fig.4. Scheme of conductor transposition

The results of the calculations of the evolution of the active power values in different sections of the line with and without transposition of the phase conductors are shown in Fig. 5, 6. Fig. 7 presents the characteristics of the active power evolution at the input and output of the two-circuit line with the cyclic transposition of the phases at the simultaneous adjustment of the phase shift angle of the input voltage.

It is to mention that in the case of the operation of two-phase circuit in phase control mode, the individual behavior of the active power fluxes transmitted through the phases (the line without transposition of conductors) is maintained, and in case of transposition this difference of power flows is kept for line circuits (fig. 7). The same phenomenon is also characteristic for reactive power of the line in the case of connected load (fig. 5) and of not connected load (fig. 6, 7).

In fig. 7 the active power transmission characteristics through the two-circuit line to phase shift adjustment of the input-output voltage vectors correspond to the uncharged state of load. The load has an active-reactive character, and the inductive component exceeds the value of the active component. When calculating the line mode, the phase shift angle was changed in the interval $0^0 \le \varphi \le 180^0$. The value of the load was determined by the relation $Z_S = R_S(1 + j1.5)$, therefore $R_S = \sqrt{L_{kk}/C_{kk}}$, $k = \overline{1,6}$ and $|X_S| = 1.5R_S$.

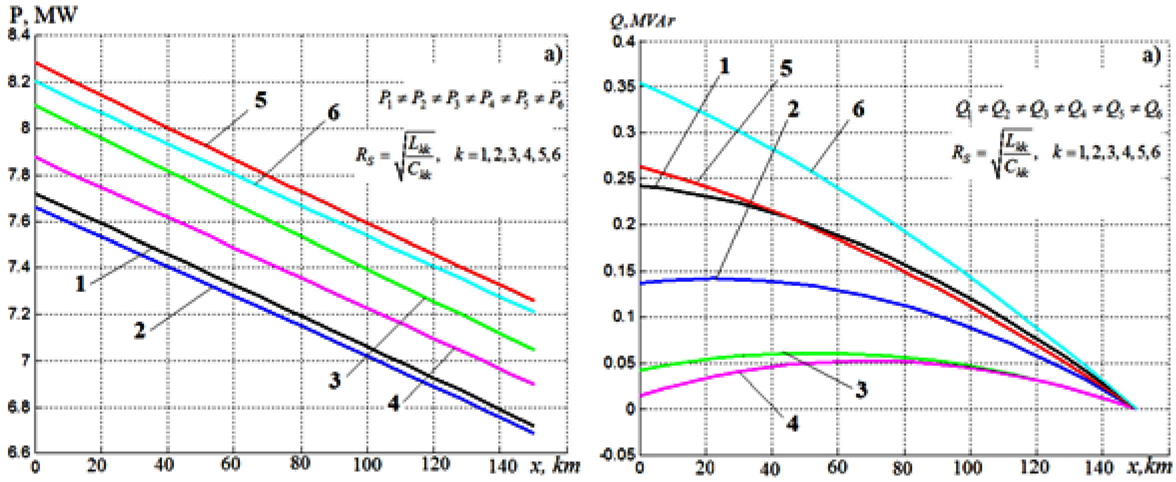

Fig.5. Distribution of active *P* (a) and reactive *Q* (b) power in 110 kV line with connected load $R_S = Z_0$, manifestation of mutual influence between the phases

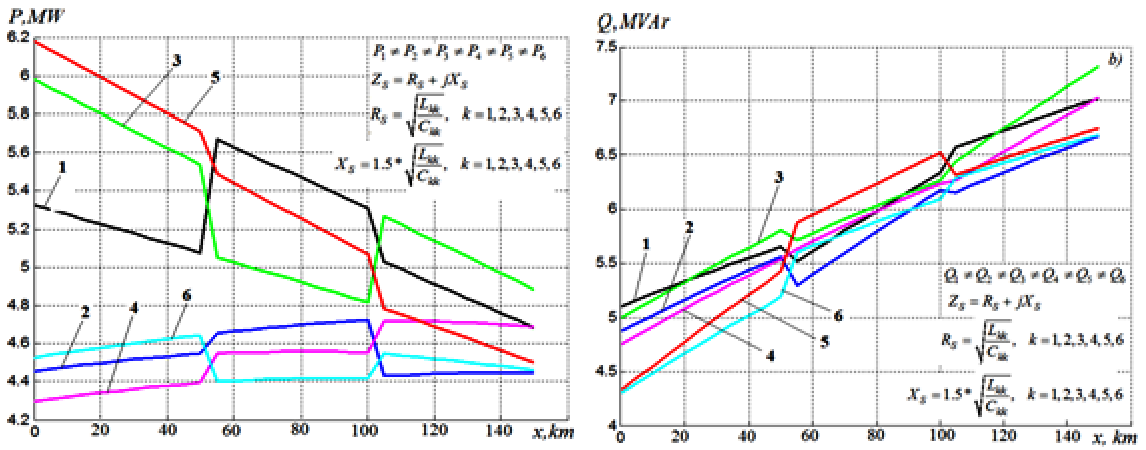

Fig.6. Distribution of active *P* (a) and reactive *Q* (b) power in 110 kV line with not connected load and transposition of phase conductors

Analyzing the obtained results, we can mention that the processes in the multi-phase or multi-conductor power lines are much more complex if the reciprocal mutual influence of both phases and line circuits is taken into account. So regarding the "independence" of the regime in the phases of multi-conductor line it is possible to speak only in the case when these conductors are located at great distances from one another. In fact, the tendency of compaction of the power lines leads to the increase in mutual influence of conductors and leads to the complexity of the calculation methods needed to be applied in order to obtain more accurate results.

Transposition of conductors results in full equilibrium power transmission through three-phase circuits. In two-circuit electric lines, the mechanical transposition does not ensure the balancing of power transmitted through circuits in the context of the electrical line examination as a constructive element integrated with the use of traditional transposition solutions. This is quite obvious for the case when these lines are realized as lines with capability to achieve the controlled operating mode, for example, by adjusting the phase shift angle in self-compensated lines [10, 20].

In the case of self-compensated electric power lines, the power transmission characteristics of the various sections are more complex compared to the case of a compact line without the use of the phase-shift rotation technology of the incoming voltage vectors. In the case of a two-circuit line without conductor transposition, the deviation between the transmitted powers in the connected load mode is estimated at approx. 7 – 8%.

It can also be seen (fig. 7) that the power injection characteristics in the multi-circuit line and the absorption of this power by the load of the line differ at the phase shift angle adjustment. This change is conditioned by the electrical line itself. Thus, in case when the line has the input active-inductive load $Z_S = R_S(1 + j1.5)$, the maximum magnitude of active power deviation at line input (x = 0 and $\varphi = 80 - 90^0$) is about 1,6 MW per phase. This maximum deviation is approximately 30% in comparison with the value of the input power for the angle $\varphi = 0^0$. At the output of the line (x = l = 150 km) with active-reactive load (fig. 6, b) the maximum magnitude of the deviation of the active power absorbed by the load does not exceed 0.15 MW. This deviation is about 3.3% of the active power value when $\varphi = 0^0$. Let mention that these aspects can only be studied using the mathematical models of multi-conductor lines realized in the phasor coordinate system.

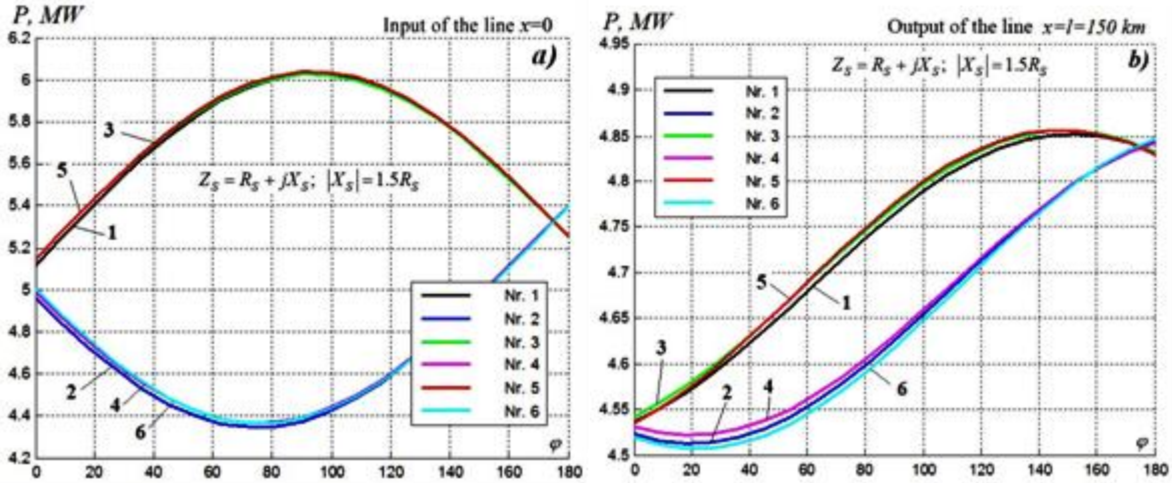

Fig.7. Evolution of the active power at the line input (x = 0) and the active power discharged to the load Z_S (x = l = 150 km) in relation to conductor transposition and phase shift angle of the input voltages

$$0^0 \le \varphi \le 180^0$$

As a recommendation, it can be assumed that in order to achieve a good result of balancing power flows through phases and circuits of multi-conductor lines, it is necessary to study other algorithms for conductor transposition to have a higher degree of equilibrium for the entire line.

It can be seen that the proposed calculation process is robust to analyze the particularities of the operation of multi-conductor electric lines with conductor transposition, including the case of adjusting the transmission line capacity by modifying the phase shift angle of the input voltage.

VI. Conclusions

1. The use of the mathematical model of the multi-conductor electric line based on the telegraph equations opens new horizons to study

the peculiarities of the operation of these electric lines and the possibilities of switching from qualitative estimations to estimates in quantitative terms, taking into account the actual constructive realization of the power lines. The proposed calculation model and procedure is robust also in the case of jumping of the phase parameter values.

2. The mutual influence of the phases of lines with many conductors leads to their different loading, even if at the output of the line the load is symmetrical. Unbalanced phase loading takes place for both the active power and the reactive power of the line. The reactive power distribution is more complex in comparison to the active power distribution and depends on the character of the load in the line.

3. Transposition of the phase conductors ensures the balancing of the power transmitted through the phases of the three-phase circuit and in the case of the two-circuit line for the wide range of phase-angle adjustment of the input voltages of both circuits. However, phase transposition in circuits does not completely solve the problem of balancing the operating circuitry, and this imbalance depends on the value of the phase angle, reaching the maximum deviation magnitude for $\varphi = 80° - 90°$.

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