Numerical Method for Determining Potential Coefficients Matrix for Multiconductor Transmission Line

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Abstract. Consider the propagation of electromagnetic energy through multiconductor three-phase high-voltage transmission line with arbitrary number of conductors. The mathematical formulation of the problem represents the system of partial differential equations known as transmission line equations. When constructing the capacitance matrix of an electric line, it is necessary to solve the boundary problems for the Laplace equation with nonlocal boundary conditions. The aim of this research is to develop a new method for constructing a potential coefficients matrix, which allows us to express the voltage vector in the wires through the charge vector. The application of this method leads to the necessity of solving a boundary value problem for Laplace equations with nonlocal boundary conditions. A new type of non-local boundary conditions has been identified, which has not been previously investigated. The boundary conditions of this type are represented by the contour integral over the boundary of the region from an unknown potential. In the general case, the obtained problem does not have a unique solution. The theorem of existence and uniqueness of the solution for the boundary value problem for the Laplace equation with nonlocal boundary conditions is proved. However, the requirement that the potential on the wire surface is constant allows us to prove the existence and uniqueness of the solution for considered problem. In order to illustrate the application of the developed approach we have solved numerically the problem for the sector-shaped cable with three cores. The values of potential and capacitive coefficients obtained by calculation are given. The error of the values of the diagonal elements of the matrices of potential and capacitive coefficients is estimated as less than 0.1%.

Keywords: telegraph equations, linear capacitance matrix, Laplace equation, nonlocal boundary conditions, electrical cable.

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Metoda numerică pentru determinarea matricei coeficienţilor potenţiali a liniei de transmisie cu multe conductoare

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Rezumat. În lucrare se consideră problema de propagare a energiei electromagnetice prin intermediul unei linii de transmisie de înaltă tensiune cu un număr arbitrar de conductoare. Formularea matematică a problemei reprezintă sistemul de ecuații diferențiale cu derivate parţiale, cunoscut ca ecuații de linie de transmisie. Valorile capacităţilor liniare ale liniilor cu mai multe conductoare sunt determinate folosind valorile sarcinii electrice şi tensiunilor conductoarelor liniilor. Atunci când se construieşte matricea de capacitate a unei linii electrice cu multe conductoare, este necesară rezolvarea problemelor la limită pentru ecuaţia Laplace cu condiţii la limită nelocale. Scopul acestei lucrări constă în dezvoltarea unei noi metode de construire a matricei coeficienţilor potenţiali, care permite să exprimăm vectorul tensiunii în conductoarele electrice prin intermediul vectorului de sarcină electrică. Această matrice este inversă la matricea capacităţilor lineică, care este utilizată în ecuaţiile telegrafiştilor. Abordarea propusă permite compararea valorilor exacte ale coeficienţilor matricei cu valorile aproximative obţinute fără a lua în considerare interacţiunea potenţialelor pe suprafaţa conductoarelor. Aplicarea acestei metode conduce la necesitatea rezolvării unor probleme la limită pentru ecuaţiile Laplace cu condiţii de limită nelocale. Acest tip de condiţii de limită nelocale nu a fost investigat anterior. Ele se reprezentă de un integral pe contur al potenţialului necunoscut. S-a demonstrat teorema existenţei şi unicitatea soluţiei privind problema valorii limită pentru ecuaţia Laplace cu condiţii de limită nelocale prin utilizarea ipotezei, că potenţialul pe suprafaţa conductorului este constant. Pentru a ilustra aplicarea abordării dezvoltate, a fost rezolvată numeric problema pentru cablul cu trei faze de tip ААБл, conductoarele căruia prezintă în secţiune sectoare curbliniare. Sunt date valorile coeficienţilor potenţiali şi capacitivi obţinuţi prin calcul. Eroarea valorilor coeficienţilor diagonali ai matricelor coeficienţilor potenţiali şi capacitivi este estimată ca fiind mai mică de 0.1%.

Cuvinte cheie: ecuaţii telegrafice, matricea capacităţilor liniare, ecuaţia Laplace, condiţii la limită nelocale, cablu electric.

Численный метод определения матрицы потенциальных коэффициентов для многопроводной линии

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Аннотация. Рассматривается задача о распространении электромагнитной энергии по многопроводной трехфазной высоковольтной линии электропередачи с произвольным количеством проводников. Математическая постановка задачи представляет собой систему уравнений с частными производными, описывающих распределение напряжения и тока по времени и расстоянию в линии и известную как телеграфные уравнения. Значения погонных емкостей многопроводных линий определяются, используя значения заряда и напряжений на проводах линий. При построении матрицы емкостей многопроводной электрической решены краевые задачи для уравнения Лапласа с нелокальными граничными условиями. Целью данной работы является разработка нового метода построения матрицы погонных потенциальных коэффициентов, которая позволяет выразить вектор напряжений в проводах через вектор зарядов. Эта матрица является обратной к матрице погонных коэффициентов электростатической индукции, которая используется в телеграфных уравнениях. Поставленная цель достигается путем решения задачи Дирихле с нелокальными граничными условиями. Предлагаемый подход позволяет проводить сравнение точных значений коэффициентов матрицы с приближенными, полученными без учета взаимовлияния величины потенциалов на поверхности проводов. Применение данного метода приводит к необходимости решения краевой задачи для уравнений Лапласа с нелокальными граничными условиями. Рассмотрен новый тип нелокальных граничных условий, который не был ранее исследован. Граничные условия этого типа представляют собой контурный интеграл по границе рассматриваемой области от неизвестного потенциала. В общем случае, полученная задача не имеет единственного решения. Наложение требования постоянства потенциала на поверхности проводов кабеля позволяет доказать теорему существования и единственности решения поставленной задачи. Приведено численное решение сформулированной задачи для кабеля типа ААБл с тремя жилами секторной формы. Приведены значения потенциальных и емкостных коэффициентов, полученных расчетным путем. Погрешность значений диагональных коэффициентов матриц потенциальных и емкостных коэффициентов оценивается, как меньше 0.1 %.

Ключевые слова: телеграфные уравнения, матрица погонных емкостей, уравнение Лапласа, нелокальные граничные условия, электрический кабель.

INTRODUCTION

In this paper, we propose a new method for calculating the coefficients of the matrix β of linear capacities. The conventional method consists in applying the relation $Q=\beta U$, which expresses the dependence of the charge vector Q through the voltage vector ***U*** in the wires. This approach is described in [1-4]. In [1] a physical experiment is proposed to determine the coefficients of the matrix β, in [2, 3] well-known commercial software products ELKUT and QuickField are used for this purpose, in [4] the initial problem is reduced to solving integral equations. We propose to determine the values of linear capacities from potential coefficients matrix α that allows expressing the vector of potentials U through the charge vector. In this case, the exact values of the coefficients α_{ij} can be compared with the approximate values obtained in [1] using analytical formulas:

$$\alpha_{ii}=\frac{1}{2\pi\in l}\ln\frac{2h_i}{R_i},\ \alpha_{ij}=\frac{1}{2\pi\in l}\ln\frac{r_{i'j}}{r_{ij}},\ i\neq j.$$

Such a method for constructing this matrix requires solving some boundary value problems for the Laplace equation with nonlocal boundary conditions. In [5–10], the problems with different types of nonlocal boundary conditions were investigated, but the problem presented in this paper was not reflected.

I. MATHEMATICAL MODEL

The mathematical model that describes the propagation of electromagnetic energy over a multiconductor three-phase high-voltage transmission line is represented by the system of differential equations for voltage $u(x,t)$ and current $i(x,t)$ vectors:

$$L \cdot \frac{\partial i}{\partial t} + \frac{\partial u}{\partial x} + R \cdot i = 0, \quad (1.1)$$

$$C \cdot \frac{\partial u}{\partial t} + \frac{\partial i}{\partial x} + G \cdot u = 0. \quad (1.2)$$

These equations are derived from Maxwell's equations for a multiconductor power transmission line [15, 16].

The domain of the solution is the rectangle $D = \{(x,t) : x \in (0,l), t \in (0,T_{\max})\}$, where l is the length of the transmission line and $T_{\max}$ is the maximal time of calculating for vectors $u(x,t)$ and $i(x,t)$. In a *n*-conductor line the vector functions $u(x,t)$ and $i(x,t)$ have *n* components each, but *L*, *C*, *R* and *G* in (1.1), (1.2) are symmetrical matrices $(n \text{ x } n)$ of linear inductances, capacitances, wire resistances and conductivities of insulation (vector objects are denoted by boldface)

$$u(x,t) = \begin{pmatrix} u_1(x,t) \\ u_2(x,t) \\ \dots \\ u_n(x,t) \end{pmatrix}, \; i(x,t) = \begin{pmatrix} i_1(x,t) \\ i_2(x,t) \\ \dots \\ i_n(x,t) \end{pmatrix},$$

$$C = \begin{pmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{12} & C_{22} & \dots & C_{2n} \\ \dots & \dots & \dots & \dots \\ C_{1n} & C_{2n} & \dots & C_{nn} \end{pmatrix},$$

$$L = \begin{pmatrix} L_{11} & L_{12} & \dots & L_{1n} \\ L_{12} & L_{22} & \dots & L_{2n} \\ \dots & \dots & \dots & \dots \\ L_{1n} & L_{2n} & \dots & L_{nn} \end{pmatrix}, \quad (1.3)$$

$$R = \begin{pmatrix} R_{11} & 0 & \dots & 0 \\ 0 & R_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & R_{nn} \end{pmatrix},$$

$$G = \begin{pmatrix} G_{11} & 0 & \dots & 0 \\ 0 & G_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & G_{nn} \end{pmatrix}, .$$

The system (1.1), (1.2) is a system of partial differential equations of hyperbolic type.

To obtain a unique solution, we must add the initial (when $t = 0$) and boundary (when $x = 0$ and $x = l$) conditions. We assume that at the initial time $t = 0$ there are no voltages and currents in the line

$$u(x,0) = i(x,t) = 0, x \in [0,l], \quad (1.4)$$

at the input of the line, at $x = 0$, voltages are given, and at the output for $x = l$ we have a load with resistance R_S

$$u(0,t) = U_0(t), \;\; u(l,t) = R_S i(l,t). \quad (1.5)$$

II. MATRIX OF COEFFICIENTS OF ELECTROSTATIC INDUCTION AND POTENTIAL COEFFICIENTS MATRIX

The matrix *C* in (1.2) is called the matrix of coefficients of electrostatic induction and in [1] it is noted by β. For multiconductor line, this matrix allows to express the charge vector ***Q*** through the vector of potentials ***U*** in each conductor as follows

$$Q = \beta U \,;$$

$$\begin{cases} Q_1 = \beta_{11}U_1 + \beta_{12}U_2 + \dots + \beta_{1n}U_n \\ Q_2 = \beta_{21}U_1 + \beta_{22}U_2 + \dots + \beta_{2n}U_n \\ \dots \\ Q_n = \beta_{n1}U_1 + \beta_{n2}U_2 + \dots + \beta_{nn}U_n \end{cases}. \quad (2.1)$$

In [1, §25.2] a method for calculating the exact values of the coefficients β_{ij} is proposed. To this end, it is required to solve *n* Dirichlet problems with the following boundary conditions. Consider the domain *D*, which is the outer region for *n* wires (Fig. 1)

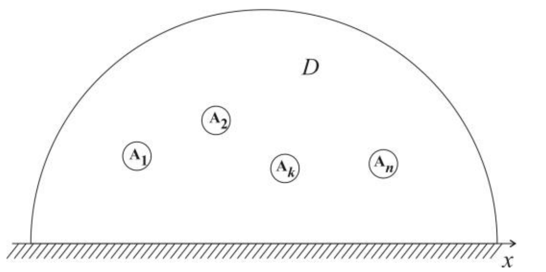

Fig. 1. Domain *D*, where the Dirichlet problems must be solved.

When solving the problem with the number *k*, as the boundary conditions on the surface of all wires, except the wire A_k, a zero potential is given, and on the wire A_k - a nonzero potential of the value U_k is given. On the outer boundary and on the surface of the earth the potential is considered to be zero. Thus, the following Dirichlet problems are to be solved:

$$\Delta u = L\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, (x, y) \in D, \quad (2.2)$$

$$u(x, y) = U_i = \begin{cases} U_k \neq 0, i = k \\ 0, i \neq k \end{cases},$$

$$(x, y) \in \Gamma_i, i = \overline{i, n}, \quad (2.3)$$

$$u(x, y) = 0, (x, y) \in \Gamma_в. \quad (2.4)$$

Here by Γ_i the surface of the *i*-th wire is denoted and $\Gamma_в$ is the outer boundary of the domain $\overline{D}$, $\overline{D} = D \cup \Gamma, \Gamma = \Gamma_в \cup \sum_{i=1}^{n} \Gamma_i$.

After solving the Dirichlet problem and determining the potential distribution $u(x, y)$, for each wire the charge $Q_i, i = \overline{1, n}$ is calculated through the electric field strength vector ***E*** applying the Gauss theorem

$$Q_i = \varepsilon_0 l \oint_{\Gamma_i} (E, n) dl,$$

$$E = -grad\ u = -\begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{pmatrix}. \quad (2.5)$$

Here $\varepsilon = 8{,}85418782 \cdot 10^{-12} [F/m]$ is the electrical constant, Γ_i is the contour surrounding Then the coefficients β_{ki} can be calculated by formulas the wire A_i, $n = \begin{pmatrix} n_x \\ n_y \end{pmatrix}$ is the vector of outer normal to contour Γ_i.

$$\beta_{ik} = \frac{Q_i}{U_k}, i, k = \overline{1, n}. \quad (2.6)$$

At the same time in [1, §25.3] it is proposed numerical method for calculating the potential coefficients matrix α, which allows to express the vector of potentials ***U*** through the charge vector ***Q*** in each conductor by formulas

$$U = \alpha Q;$$

$$\begin{cases} U_1 = \alpha_{11} Q_1 + \alpha_{12} Q_2 + \ldots + \alpha_{1n} Q_n \\ U_2 = \alpha_{21} Q_1 + \alpha_{22} Q_2 + \ldots + \alpha_{2n} Q_n \\ \ldots \\ U_n = \alpha_{n1} Q_1 + \alpha_{n2} Q_2 + \ldots + \alpha_{nn} Q_n \end{cases}. \quad (2.7)$$

As follows from (2.1) and (2.7) the matrices α and β are mutually inverse. To determine the coefficients $\alpha_{ki}, i = \overline{1, n}$, it is assumed that all charges except the charge with the number *k* are equal to zero $\left(Q_i = \begin{cases} Q_k \neq 0, i = k \\ 0, i \neq k \end{cases} \right)$. It is assumed also that the field of the charged wire will be the same as the field of a single wire stretched above the earth (since the distortion of the field due to the existence of other wires can be neglected because of the smallness of their cross sections and of the absence of the charges on them). Then, using the exact solution for one wire, the values α_{kk} and $\alpha_{kp}, k, p = \overline{1, n}$ (fig. 2) can be calculated. Here ε is absolute permittivity, l, R_k are the length and the radius of the wire, r_{pk} is the distance between wires with numbers *p* and *k*, $r_{p'k}$ is the distance between the wire numbered by *k* and the mirror image of the wire numbered by *p*. It follows from the foregoing that in order to determine the exact values of the coefficients of the matrix α it is necessary to solve problems analogous to problems (2.2) – (2.4), only at the boundaries Γ_i, (on the surfaces of the wires) the known value of the charges $Q_i = \begin{cases} Q_k \neq 0, i = k \\ 0, i \neq k \end{cases}, i, k = \overline{1, n}$ must be setted (not the potential values).

$$\alpha_{kk} = \frac{1}{2\pi \in l} \ln \frac{2h_k}{R_k},$$

$$\alpha_{kp} = \frac{1}{2\pi \in l} \ln \frac{r_{p'k}}{r_{pk}}, \; p \neq k. \qquad (2.8)$$

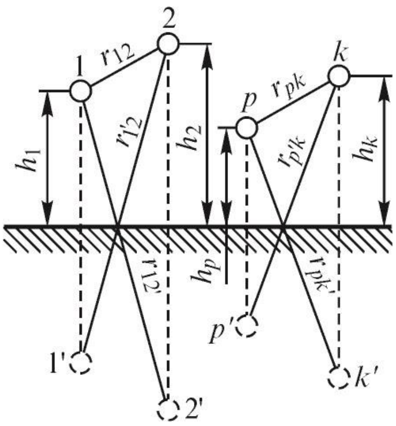

Fig. 2. Wires stretched above the earth and their mirror images.

Hence these problems are formulated as follows. In the domain $\overline{D} = D \bigcup \Gamma$, it is required to determine the field of the potential $u(x, y)$ that inside of the domain D satisfies the Laplace equation (2.2), on the outer boundary $\Gamma_в$ satisfies the condition (2.4), and on the wire surface potential takes the constant value and satisfies the conditions

$$\oint_{\Gamma_i} (E, n) dl = -\oint_{\Gamma_i} \frac{\partial u}{\partial n} dl = \frac{Q_i}{\varepsilon_0} \qquad (2.9)$$

or

$$\oint_{\Gamma_i} \left(\frac{\partial u}{\partial x} n_x + \frac{\partial u}{\partial y} n_y \right) dl = -\frac{Q_i}{\varepsilon_0} \qquad (2.10)$$

After solving the problem (2.2), (2.4), (2.9), the exact values of the coefficients α_{ik} are calculated as the ratio between the found values of the potentials U_k on the wire surface and the specified charge values Q_i by formulas

$$\alpha_{ik} = \frac{U_k}{Q_i}, i, k = \overline{1, n}. \qquad (2.11)$$

The problem (2.2), (2.4), (2.9) differs from the classical Dirichlet problem for the Laplace equation. The difference consists in replacing the Dirichlet condition (2.3) by the boundary condition (2.9) or (2.10), which contains integrals over the boundary of the domain from the values of the unknown function $u(x, y)$. Such problems are called problems with nonlocal boundary conditions [5–10]. Let prove the solvability of this problem through the solution of the classical Dirichlet problem for the Laplace equation (2.2) – (2.4). It is known that the exact solution of problem (2.2) – (2.4) exists, is unique [11-16] and can be obtained using the Green's function. The Green's function $G(M, M_0)$ for problem (2.2) – (2.4) is the solution of the following problem for each fixed point $M_0 \in D$.

$$\Delta_M G(M, M_0) = \delta(M, M_0),$$

$$M, M_0 \in D, \qquad (2.12)$$

$$G(P, M_0) = 0, P \in \Gamma = \partial D. \qquad (2.13)$$

Here M, M_0 are two-dimensional points belonging to the domain D, $\delta(M, M_0)$ is the Dirac delta function, $\Gamma = \Gamma_в \cup \sum_{i=1}^{n} \Gamma_i$ is the boundary of the domain D, Δ_M is the Laplace operator, where variables x and y are the coordinates of the point $M(x, y)$.

If the Green's function is known, then the solution of problem (2.2) – (2.4) can be represented in the form

$$u(M) = -\oint_{\Gamma} U(p) \frac{\partial G(P, M)}{\partial n_p} dl_p,$$

$$M \in D, P \in \Gamma. \qquad (2.14)$$

Here $U(p)$ are the given values of the potential on the boundary Γ of the domain D, n_p is the vector of the outer normal to the contour Γ. Taking into account the form of the boundary conditions (2.3), (2.4), formula (2.14) can be rewritten as follows

$$u(M) = -\sum_{i=1}^{n} U_i \oint_{\Gamma} \frac{\partial G(P_i, M)}{\partial n_{P_i}} dl_{p_i},$$

$$M \in D, P_i \in \Gamma_i. \qquad (2.15)$$

To obtain the solution of the problem (2.2), (2.4), (2.9), we substitute (2.15) in (2.9) and obtain a system of algebraic equations for determining the unknown values of the potential on the wire surface U_i in terms of the known values of the charges Q_i

$$\sum_{i=1}^{n} U_i \oint_{\Gamma_k} \frac{\partial}{\partial n_{P_k}} \left(\oint_{\Gamma_i} \frac{\partial G(P_i, M)}{\partial n_{P_i}} dl_{p_i} \right) dl_{p_k} = \frac{Q_k}{\varepsilon_0}, k = \overline{1,n}. \quad (2.16)$$

Comparing the obtained system (2.16) with (2.1), it is easy to note that the coefficients of the system (2.16) coincide with the values of the coefficients of the matrix of electrostatic induction β

$$\beta_{ki} = \oint_{\Gamma_k} \frac{\partial}{\partial n_{P_k}} \left(\oint_{\Gamma_i} \frac{\partial G(P_i, M)}{\partial n_{P_i}} dl_{p_i} \right) dl_{p_k}, \quad k, i = \overline{1,n}. \quad (2.17)$$

Thus, to solve the problem (2.2), (2.4), (2.9) it is required to prove that there exists a unique solution of the system of equations (2.16). This affirmation will be proved in the form of a theorem.

Theorem. The system (2.16) and therefore the problem (2.2), (2.4), (2.9) has a unique solution.

Proof. We prove the theorem by using the Fredholm alternative for a system of algebraic equations, i.e. we prove that the corresponding homogeneous system for (2.16) does not have nonzero solutions. We carry out the proof by contradiction. Suppose that the system has a nonzero solution. This means that the problem (2.2), (2.4), (2.9) has a nonzero solution for given zero charges on all wires. It is known that the solution of the Laplace equation (2.2) satisfies the strong maximum principle, i. e. the solution of the Dirichlet problem for the Laplace equation attains its maximum and minimum values on the boundaries of the solution domain, and within the interior of the domain the solution value is strictly less than the maximum value and strictly greater than the minimum value. In our case, the solution is different from the constant, since we assumed that the solution of the homogeneous problem (2.2), (2.4), (2.9) is nonzero and on the outer boundary $\Gamma_в$ it accepts the zero condition (2.4). It means that the maximum positive value and the minimum negative value of the potential are attained at the boundaries $\Gamma_i, i = \overline{1,n}$. Let the maximum positive value of the constant potential $U_{\max} > 0$ is attained on the wire . Then the derivative of potential with respect to normal takes only strictly negative values at all points of the contour Γ_p, i.e. $\left.\frac{\partial u}{\partial n}\right|_{\Gamma_p} < 0$. Then the charge of this wire, which is calculated by the formula (2.9), $Q_p = -\varepsilon_0 \oint_{\Gamma_p} \frac{\partial u}{\partial n} dl$, is strictly positive, that contradicts the assumption that all charges in the homogeneous problem are zero. A similar argument can be drawn for a wire on which a minimum negative value of the potential is attained.

So, the original assumption of the existence of a nonzero solution for a homogeneous problem is incorrect. Hence, by the Fredholm alternative, the nonhomogeneous problem (2.2), (2.4), (2.9) has a unique solution.

III. CALCULATION THE POTENTIAL COEFFICIENTS MATRIX FOR A THREE-PHASE CABLE

Numerical experiments have shown that for overhead power line with real values R_k, r_{pk} and $r_{p'k}$, approximate formulas (2.8) for determining the potential coefficients give values with high accuracy. Therefore, the determination of the potential coefficients by numerical solution of problems (2.2), (2.4), (2.9) or (2.2), (2.3), (2.4) does not make sense. At the same time, there are no acceptable approximate formulas [17] for determination of the potential coefficients of power cables, therefore, below we give the results of calculating a specific cable.

As an example we have chosen the sector-shaped cable with three cores (fig. 3). In the figure, the metallic parts of the cable (3 cores and aluminum screen) are light gray.

Since the zero potential is given on the screen, the domain D of the solution to the problem (2.2), (2.4), (2.9) will be the region shown in fig. 4.

Fig. 3. Cable with three triangular cores.

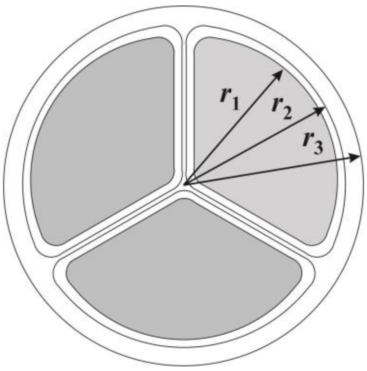

Fig. 4. Domain of the solution: domain *D* – unpainted inner part of the circle

The domain D is represented by unpainted inner part of the circle, being determined by the following radii values:

$$r_1 = 15\ mm,\ r_2 = 16{,}9\ mm,\ r_3 = 18{,}45\ mm.$$

Each sector-shaped core is surrounded by insulation and all three cores are surrounded by another layer of insulation. In general, these insulations can have different permittivity. Therefore, to the relations (2.2), (2.4), (2.9), we must add the conjugation conditions on the interface between two media

$$u^- = u^+;\ \varepsilon^- \frac{\partial u^-}{\partial n} = \varepsilon^+ \frac{\partial u^+}{\partial n}. \quad (3.1)$$

Here, signs "+" and "–" denote values from one and the other side of the media interface.

The problem (2.2), (2.4), (2.9), (3.1) was solved numerically by the finite volume method. Constructed grid consists of 30853 triangular elements (fig. 5).

Based on the results of calculations, using formulas (2.11), a matrix α was constructed and the matrix β was calculated:

$$\alpha = \begin{pmatrix} 3.9689 & 1.0822 & 1.0822 \\ 1.0829 & 3.9689 & 1.0822 \\ 1.0822 & 1.0829 & 3.9689 \end{pmatrix} \cdot 10^9 [1/F], \quad (3.2)$$

$$\beta = \alpha^{-1} =$$

$$= \begin{pmatrix} 0.2853 & -0.0611 & -0.0612 \\ -0.0612 & 0.2853 & -0.0611 \\ -0.0611 & -0.0612 & 0.2853 \end{pmatrix} [nF]. \quad (3.3)$$

From the geometry of the domain D it is clear that the matrices should have the same values on the diagonal and all the off-diagonal elements should be the same. From the presented results it is seen that the diagonal elements coincide, and there is a difference in the fourth sign after the comma in the off-diagonal elements. The difference is about 0.06% and is caused by numerical method error.

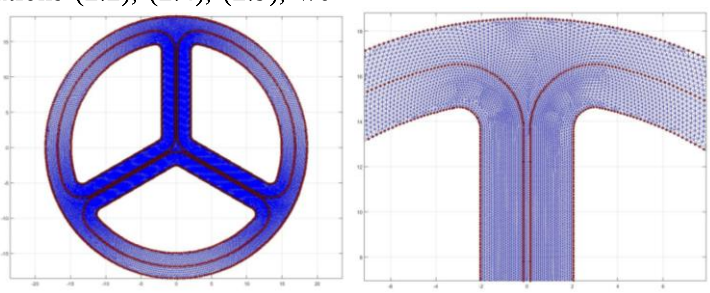

Fig. 5. A grid of triangular elements in the domain *D*. The red dots denote the boundary nodes of the grid and the nodes on the media interfaces.

IV. CONCLUSIONS

1. A method for constructing the potential coefficients matrix is proposed.
2. The theorem of existence and uniqueness of the solution for the boundary value problem for the Laplace equation with nonlocal boundary conditions is proved.
3. The developed method is applied for determining the potential field distribution and the potential coefficients matrix in a three-core cable.

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