Electromagnetic field calculation for 110 kV power line

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***Abstract*.** The paper studies the evolution of values for the characteristic electric field and magnetic field generated by power line voltages and currents voltage 110 kV their value based snapshots. These evolutions we examined in changing the value of the angle of the voltage vector and current vector within $0^0 \le \varphi \le 180^0$. The conductors are placed horizontally and triangle tops with different lengths of the sides. The electric field distribution was calculated with finite volume method. Since the electric field distributions were determined parameter values of LEA110 kV. The values of the line parameters, which were determined by the finite volume method, difference from the values calculated by the traditional method. In this context finite volume method presents attractive enough to determine the parameters of power lines and spatial distribution of the electric field in three-phase lines.
***Keywords*:** electric field, method of finite volumes, electric line parameters.

Calculul câmpului electromagneric pentru linia 110 kV

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***Rezumat*.** În lucrare se studiază evoluția valorilor mărimilor ce caracterizează câmpul electric și câmpul magnetic generat de tensiunile și curenții liniei electrice cu tensiunea de 110 kV în funcție valoarea lor instantanee. Aceste evoluții sunt examinate în funcție de schimbarea valorii unghiului decalajului de fază dintre vectorii tensiunii și curentului în conductoarele liniei trifazate în limitele $0^0 \le \varphi \le 180^0$. Conductoarele sunt amplasate în plan orizontal și în vârfurile triunghiului cu lungimi diferite ale laturilor. Repartiția câmpului electric s-a calculat cu metoda volumelor finite.. Din repartițiile câmpului electric s-au determinat valorile parametrilor lineică ale LEA 110kV. Valorile numerice ale parametrilor liniei, care au fost determinate cu metoda volumelor finite diferă de valorile calculate cu metodele tradiționale. În acest context metoda volumelor finite se prezintă destul de atractivă pentru determinarea parametrilor liniilor electrice și repartiției spațiale ale câmpului electric în liniile cu trei faze.
Cuvinte-cheie: câmp electric, metoda volumelor finite, parametrii liniei electrice.

Расчет электромагнитного поля линии электропередачи 110 кВ

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***Аннотация*.** В работе представлены результаты расчета распределения электрического поля воздушной линии ВЛ 110кВ, для случая изменения мгновенных значений напряжений и токов в проводах. При этом рассматривается случай изменения угла сдвига фазы между векторами напряжения и тока в пределах $0^0 \le \varphi \le 180^0$. Проводники размещаются горизонтально и в треугольник. Распределение электрического поля рассчитывается методом конечных объемов. По распределению электрического поля были определены значения параметров линии LEA110 кВ. Численные значения параметров линии, которые были определены методом конечных объемов, отличается от значений, рассчитанных по традиционному методу. В этом контексте, метод конечных объемов представляет достаточно привлекательным, чтобы определить параметры линий электропередач и пространственное распределение электрического поля в трехфазной линии.
***Ключевые слова*:** электрическое поле, метод конечных объемов, параметры электрической линии.

I. INTRODUCTION

Power lines 110 kV are of major significance both for the electricity transmission system and for distribution of electricity to consumers. The lines have different constructive implementations concerning the location of the phase conductors in space. Thus, it is expected that the phase parameters will be different, including the electromagnetic field distribution in the transverse and longitudinal sections of the line. In most cases, it is considered that the phase parameters are equivalent, and the lines as the power system objects are represented as monofilament lines. In these cases, it is usually operated in calculations with effective values or with the amplitude values of voltage and current. In reality, both the phase voltages and the currents are values that are changing in time with industrial frequency. During such period, both the voltage and the current are changing from plus to minus of the values of the actual amplitudes. Since the phase voltages and currents in three-phase systems have the phase lag equal to $2\pi/3$ and phase conductors are spatially distributed, it is naturally to expect that such structures have the electric and magnetic fields of rotation. So the evolution in time of the maximum values of the electric and magnetic field strength in the transverse section of the line must be expected.

Fig. 1 represents two conventional solutions for achieving 110 kV overhead transmission line with one circuit. We denote by 1 – phase A; 2 – phase B; 3 – phase C, then we have for the instantaneous values of voltages and currents the following relations:

$$u_A = U_m \sin \omega t; \;\; u_B = U_m \sin\left(\omega t - \frac{2\pi}{3}\right);$$

$$u_C = U_m \sin\left(\omega t + \frac{2\pi}{3}\right);$$

$$i_A = I_m \sin(\omega t + \varphi_A); \;\; i_B = I_m \sin\left(\omega t - \frac{2\pi}{3} + \varphi_B\right);$$

$$i_C = I_m \sin\left(\omega t + \frac{2\pi}{3} + \varphi_C\right);$$

$$\varphi = \varphi_A = \varphi_B = \varphi_C .$$

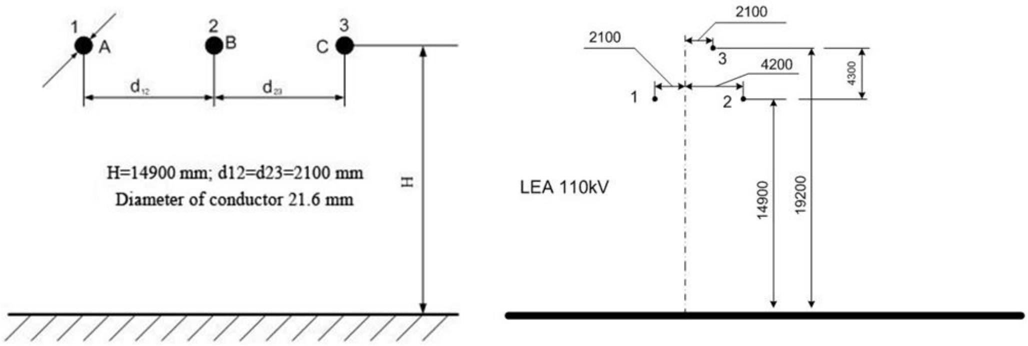


Fig. 1. Variants of spatial location of phase conductors in 110 kV overhead line

We consider the power line with conductors of type AC240/39 with external diameter 21.6 mm. The section of the conductor is equal to 274.6 mm2, including the aluminum and steel components Al/Fe = 236/38.6 mm2. The value of the intensity of the effective current in phase is $j = 1.1$ A/mm^2. In these conditions the maximum effective value of the current in phase $I = jS = 1.1 \text{ x } 274.6 = 302\,\text{A};$ and the phase power $S = UI = 100/1.73 \approx 19$ MW.

The task is to calculate the electromagnetic field and the linear parameters (linear capacitance and linear inductance) of the phases, taking into account the fact that the potential of the phase conductors are determined by the system of equations for phase voltages that have the angles lags and are variable in time. At the first stage we consider that the phase lag between the voltages and currents is zero ($\varphi = 0$). As the independent variable we choose the time t which will change with discrete step Δt within the interval $0 \le t \le 20$ ms. The potential of conductors should be determined by the instantaneous values of the phase voltages $u_A(t),\, u_B(t),\, u_C(t)$, based on the current values of the independent variable t. The amplitudes values of phase voltages and currents are the following $U_m = 89.65$ kV and $I_m = 425.8$ A; the angular frequency is $\omega = 2\pi f = 2\pi \cdot 50 = 314.$ rad/s. The calculation is performed for the stationary regime, but for different ratios of the instantaneous values of voltage phases in accordance with the discretization step $\Delta t = 1 \cdot 10^{-3}$ s or $\Delta t = 2 \cdot 10^{-3}$ s. We will also calculate the linear inductance of the phase conductors for selected time intervals.

The second step will estimate the influence of varying of the phase lag between the phase voltages and the currents that can change within the range $0 \le \phi \le \pi$.

Let consider the problem of determination of two-dimensional potential distribution $u(x, y)$ of electrostatic field in the multiply-connected domain $\Omega = (-L_x \le x \le L_x, 0 \le y \le L_y)$ with piecewise constant permittivity $\varepsilon_a(x,y)$. This formulation is a particular case of the three-dimensional problem for an infinite (along the z axis) cylinder with a cross section Ω. Within the Ω, the function $u(x,y)$ satisfies Poisson equation

$$\mathrm{div}(\varepsilon_a \mathrm{grad} u) = -\sigma(x,y),$$

where $\sigma(x,y)$ is the density of free charge distribution. If within Ω there are no any of such charges, then the equation turns into Laplace equation $\mathrm{div}(\varepsilon_a \mathrm{grad} u) = 0$. The values of $u(x,y)$ on the boundary $\Gamma = \partial\Omega$ of the Ω are known

$$u(x,y)\big|_\Gamma = \mu(x,y).$$

The electric field intensity $\bar{E}$ is defined by the formula $\bar{E} = -\mathrm{grad}\, u$ and the electric displacement field – by the formula $\bar{D} = \varepsilon_a \bar{E}$. On the boundary interfaces between the heterogeneous media the continuity conditions $[u] = 0$ and $[(\bar{D},\bar{n})] = 0$ hold. Here the square brackets denote the difference between the limit values at the left and at the right of the boundary interface and $\bar{n}$ is the normal vector to this interface.

The investigation will be performed by means of the finite volume method [1].

II. FINITE VOLUME METHOD

For the numerical solution of the Dirichlet problem for the Poisson equation, we divide the domain $\bar{\Omega} = \Omega + \Gamma$ into a finite set of small triangles. All their vertices form a discrete set of grid points, which is superimposed on a continuum $\bar{\Omega}$. The grid is constructed in such a way that the sides of the triangles coincide with the interface of heterogeneous media. Let denote by T_h the set of triangles, where h is the maximal value of the triangles side lengths. Let introduce also the dual grid T_h^* that consists of so-called Voronoi cells (see fig. 2, a). Let denote by P_0 the basic node and by $K_{P_0}^*$ – the Voronoi cell. The vertices of Voronoi cell $K_{P_0}^*$ we denote by Q_i. These vertices Q_i are the centers of the circles circumscribed around the triangles having the point P_0 as a vertex.

As an approximate solution of the Dirichlet problem we consider the piecewise linear function $u_h(x,y)$ that must be continuous in $\bar{\Omega}$ and linear on every triangle $K \in T_h$. The function $u_h(x,y)$ on the set of triangles T_h can be defined in the following manner.

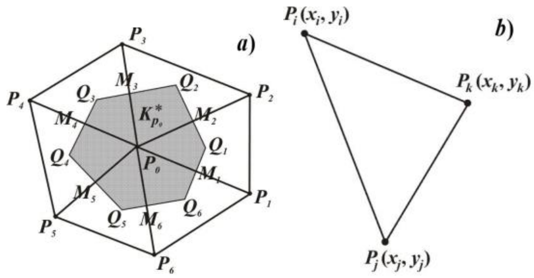


Fig. 2. The neighborhood of the grid node P_0 and Voronoi cell K_{P0}^* (a), the triangle $\Delta P_i P_j P_k$ (b).

Let the triangle $K = \Delta P_i P_j P_k$ (fig. 2,b) be some element of the set T_h and $P(x,y)$ be an arbitrary point of this element. In this triangle for each vertex we introduce the shape functions $N_i(x,y)$, $N_j(x,y)$ and $N_k(x,y)$. These functions should verify the following conditions: the functions are linear and their values at the triangle vertices are equal to 0 or 1, i.e.:

$$N_i(P_i) = 1;\ N_i(P_j) = N_i(P_k) = 0;$$
$$N_j(P_j) = 1;\ N_j(P_i) = N_j(P_k) = 0;$$
$$N_k(P_k) = 1;\ N_k(P_i) = N_k(P_j) = 0.$$

The shape functions can be represented in the explicit form through the coordinates of the vertices

$$N_i(x,y) = \frac{1}{2A}(a_i + b_i x + c_i y),$$
$$a_i = x_j y_k - x_k y_j,\ b_i = y_j - y_k,\ c_i = x_k - x_j;$$
$$N_j(x,y) = \frac{1}{2A}(a_j + b_j x + c_j y),$$
$$a_j = x_k y_i - x_i y_k,\ b_j = y_k - y_i,\ c_j = x_i - x_k;$$
$$N_k(x,y) = \frac{1}{2A}(a_k + b_k x + c_k y),$$
$$a_k = x_i y_j - x_j y_i,\ b_k = y_i - y_j,\ c_k = x_j - x_i.$$

Here A is the area of the triangle

$$2A = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}.$$

Using the shape functions for every grid node (internal or boundary) we introduce the basis function $\phi_i(x,y)$, i = 1, 2, …, n, n+1, …, n_1 (n and n_1 represent here the number of internal nodes and the total number of nodes respectively). The function $\phi_i(x,y)$ is piecewise linear, i.e. it is continuous and linear on each triangle with unit value in the node P_i and with zero values in all other nodes. Then the approximate solution $u_h(x,y)$ can be represented as a linear combination of the basis functions

$$u_h(x,y) = \sum_{i=1}^{n_1} u_i \varphi_i(x,y).$$

Let integrate the Poisson equation $\mathrm{div}(\varepsilon_a \mathrm{grad} u) = -\sigma(x,y,z)$ over the volume of the Voronoi cell $K^*_{P_0}$. We obtain the following integral relation

$$\int_{\partial K^*_{P_0}} \varepsilon_a \frac{\partial u}{\partial n} dl = -\int_{K^*_{P_0}} \sigma(x,y) dS. \quad (1)$$

To obtain the system of equations for the approximate solution we proceed as follows. Let's denote in the Voronoi cell $K^*_{P_0}$ (see fig. 2,a) by $P_m, m = \overline{0,6}$ – the grid nodes; by $Q_m, m = \overline{1,6}$ – the vertices of $K^*_{P_0}$ for the node P_0, by M_m, $m = \overline{1,6}$ – the intersection points of the segment $\overline{P_0 P_m}$ and $\overline{Q_{m-1} Q_m}$. Then the integral from (1) over the contour $\partial K^*_{P_0}$ can be approximated as follows (taking in consideration that $P_7 = P_1,\ Q_7 = Q_1,\ M_7 = M_1$):

$$\int_{\partial K^*_{P_0}} \varepsilon_a \frac{\partial u}{\partial n} dl = \sum_{i=1}^{6} \int_{\overline{Q_i Q_{i+1}}} \varepsilon_a \frac{\partial u}{\partial n} dl \cong$$

$$\sum_{i=1}^{6} \varepsilon_a (M_{i+1}) \frac{u(P_{i+1}) - u(P_0)}{\left|\overline{P_0 P_{i+1}}\right|} \left|\overline{Q_i Q_{i+1}}\right|,$$

where $\left|\overline{P_0 P_{i+1}}\right|$ and $\left|\overline{Q_i Q_{i+1}}\right|$ are the lengths of the segments $\left|\overline{P_0 P_{i+1}}\right|$ and $\left|\overline{Q_i Q_{i+1}}\right|$.

The integral from the right-hand member of (1) we approximate by formula:

$$\int_{K^*_{P_0}} \sigma(x,y) dS = \sigma(P_0) S_0,$$

where S_0 is the area of the Voronoi cell $K^*_{P_0}$. Then the approximation of the equation (1) can be represented in the following form

$$\sum_{i=1}^{6} \varepsilon_a (M_{i+1}) \frac{u(P_{i+1}) - u(P_0)}{\left|\overline{P_0 P_{i+1}}\right|} \left|\overline{Q_i Q_{i+1}}\right| = -\sigma(P_0) S_0.$$

So the final equation for the grid node P_0 takes the form

$$\alpha_0 u(P_0) + \sum_{i=1}^{6} \alpha_i u(P_{i+1}) = -\sigma(P_0) S_0; \quad (2)$$

$$\alpha_i = \varepsilon_a (M_{i+1}) \frac{\left|\overline{Q_i Q_{i+1}}\right|}{\left|\overline{P_0 P_{i+1}}\right|},\ i = \overline{1,6};\ \ \alpha_0 = -\sum_{i=1}^{6} \alpha_i$$

$$(P_1 = P_7,\ M_1 = M_7,\ Q_1 = Q_7).$$

Now we can write out the equation of type (2) for each internal grid node and we use the known conditions for the boundary nodes. As a result, we obtain the system of linear algebraic equations with symmetrical matrix. It is to mention that when solving the practically important problems the number of equations in such systems amounts to thousands or dozens of thousands. However, since each equation of the type (2) contains only some nonzero elements (usually there are from 3 to 9 nonzeros) then it turns out that the final matrix is sufficiently sparse matrix. For inversion of such matrices (of the band type) the Gauss method or the square root method are commonly used.

III. FLOW OF THE INTENSITY VECTOR

The obtained solution $u_h(x,y)$ for field potential distribution in $\overline{\Omega}$ permits to construct the flow of the intensity vector $\overline{E} = (E_x, E_y) = -\mathrm{grad}\, u$. Let denote by V the flow of vector $\overline{E}$, passing through the unit area element that is parallel with axis z and on the surface of which the condition $u(x,y) = \mathrm{const}$ is fulfilled. The functions u and V satisfy the Cauchy-Riemann equations

$$E_x = -\frac{\partial u}{\partial x} = \frac{\partial V}{\partial y};\ E_y = -\frac{\partial u}{\partial y} = -\frac{\partial V}{\partial x},$$

then the level curves (isolines) $u(x,y)=\text{const}$ and $V(x,y)=\text{const}$ generate mutually orthogonal families. The function $V(x,y)$ can be obtained by calculation of the following contour integral

$$V(x,y)=\int_{(x_0,y_0)}^{(x,y)}\left(\frac{\partial u}{\partial y}dx-\frac{\partial u}{\partial x}dy\right),$$

where x_0, y_0 are the coordinates of an arbitrary fixed point from Ω, and the patch of integration is situated inside of Ω. In case of multiply-connected domain the patch of integration also can not intersect the cuts of the domain that bring it to simply connected structure.

IV. DETERMINATION OF LINEAR CAPACITANCE AND INDUCTANCE

The capacitance C between two conducting bodies can be computed by the formula

$$C=\frac{q}{u_1-u_2},$$

where (u_1-u_2) potential difference of these bodies. The charge q of the body located inside of the some three-dimensional domain V can be computed in accordance with Gauss' law of flux as a surface integral of the field intensity vector $\overline{E}$ over surface $S=\partial V$

$$q=\varepsilon\int_S \overline{E}\cdot\overline{dS}=-\varepsilon\int_S \text{grad}u\cdot\overline{dS}=$$

$$=-\varepsilon\int_S(\text{grad}u\cdot\overline{n})dS=-\varepsilon\int_S\frac{\partial u}{\partial n}dS.$$

Here by S we denoted an arbitrary surface containing the charged body, by $\overline{n}$ – the exterior normal vector to the surface S and by ε – the permittivity.

Linear inductances for the system of wires can be calculated according to formulas [2, §28.1]. In the case when the transverse wire sizes are very small in comparison with contours length and with the distance between them, the linear self-inductance L and the mutual linear inductance M_{12} are calculated by the formulas:

$$L=\frac{\mu_0}{4\pi l_1}\int_0^{l_1}\int_0^{l_1}\frac{dx_1dx_2}{r}+\frac{\mu_0}{8\pi},\quad M_{12}=\frac{\mu_0}{4\pi l_1}\int_0^{l_1}\int_0^{l_2}\frac{dx_1dx_2}{r}.$$

Here r is the distance between two points on the centerline of the conductors with elements dx_1 and dx_2, $\mu_0=4\pi\cdot10^{-7}$ is the magnetic constant, $l_1=l_2=1$ m. For the mutual inductance, the elements dx_1 and dx_2 belong respectively to the conductors with the numbers 1 and 2, and for the self-inductance these elements are chosen on the same conductor.

V. NUMERIC RESULTS

The calculation domain (see fig. 1) represents the square $\Omega=(-L_x\le x\le L_x, 0\le y\le L_y)$ with $L_x=50$ m, $L_y=100$ m, y = 0 is the earth's surface. We suppose the value of potential of the electric field tends to zero at the boundaries of the square.

In order to optimize the number of nodes it is typically used the computational grid with variable dimensions of the cells. The density of cells in the computational grid is higher and increases when approaching the conductor.

The examples of computational grids for different constructive variants of spatial location of phase conductors in 110 kV overhead line are represented in fig. 3.1, fig. 3.2 (phase conductors are placed horizontally in a line) and fig. 4.1, fig. 4.2 (phase conductors are placed at the vertices of a triangle).

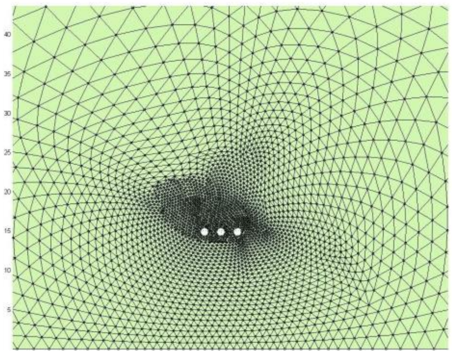


Fig. 3.1. Computational grid in the square Ω for three-conductor overhead line of 110 kV when phase conductors are placed horizontally in a line. General view.

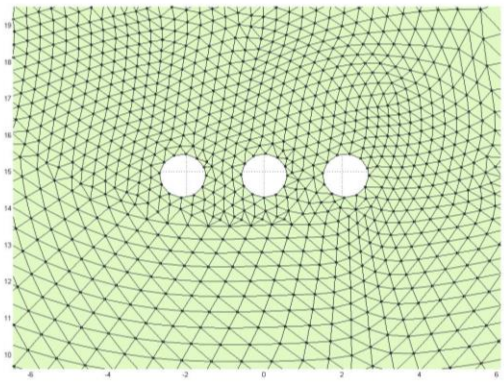


Fig. 3.2. Computational grid in the vicinity of conductors when phase conductors are placed horizontally in a line.

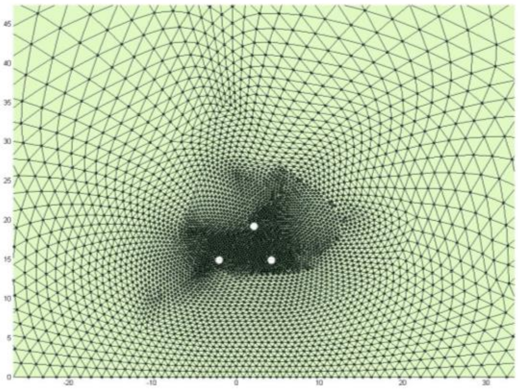


Fig. 4.1. Computational grid in the square Ω for three-conductor overhead line of 110 kV when phase conductors are placed at the vertices of a triangle.

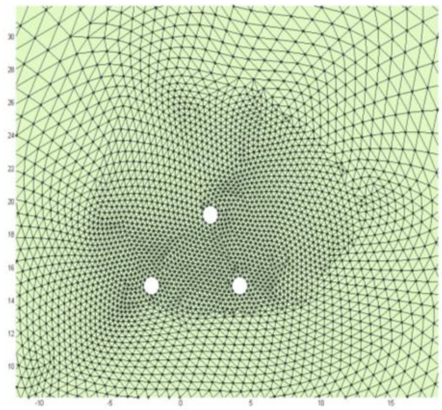
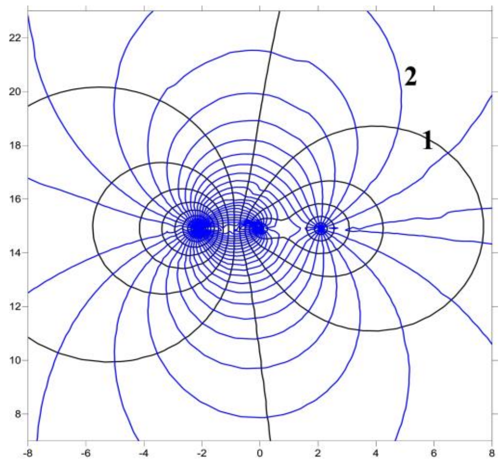


Fig. 4.2. Computational grid in the vicinity of conductors when phase conductors are placed at the vertices of a triangle.



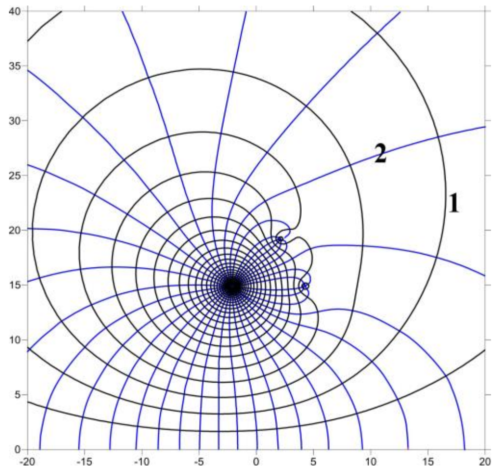


Fig. 5. Potential distribution (curves 1) and the flow of electric field intensity (curves 2).

The step size of the grid in areas close to the boundaries of the square constitutes 8 m, and as we get closer to the conductor, the step size decreases to the conductor radius, i.e. to 0,0108 m. Total number of computing nodes of the grid is equal to 5160, and the number of triangular elements is equal to 10164. Let note that to obtain one variant of the numerical solution we need about 200 minutes using 2GHz frequency processor.

Figure 5 illustrates the distributions of the electric field in 110 kV lines for examined constructive variants. These results are obtained for the case when the phase voltages have the following values:

$$U_A = 89.81 \text{ kV};\ U_B = U_C = -44.91 \text{ kV}.$$

On the basis of the electromagnetic field distributions the matrices of electrostatic induction coefficients as well as the partial capacitances and inductances have been calculated. The matrices with values of linear parameter for 110 kV line are shown in Table 1.

TABLE I. MATRICES OF LINEAR PARAMETERS FOR DIFFERENT CONSTRUCTIVE VARIANTS OF 110 KV LINE

Parameters	Location of phase conductors in parallel with the ground	Location of phase conductors at the vertices of a triangle
Electrostatic induction coefficients, pF/m	$\beta = \begin{pmatrix} 8.31 & -2.36 & -1.25 \\ -2.36 & 8.80 & -2.36 \\ -1.25 & -2.36 & 8.31 \end{pmatrix}$	$\beta = \begin{pmatrix} 7.73 & -1.33 & -1.18 \\ -1.33 & 7.67 & -1.60 \\ -1.18 & -1.60 & 7.84 \end{pmatrix}$

Partial capacitances, pF/m	$C=\begin{pmatrix}4.70 & 2.36 & 1.25\\ 2.36 & 4.09 & 2.36\\ 1.25 & 2.36 & 4.70\end{pmatrix}$	$C=\begin{pmatrix}5.22 & 1.33 & 1.18\\ 1.33 & 4.74 & 1.60\\ 1.18 & 1.60 & 5.06\end{pmatrix}$
Self and mutual inductances, $\mu H/m$	$L=\begin{pmatrix}1.54 & 0.51 & 0.38\\ 0.51 & 1.54 & 0.51\\ 0.38 & 0.51 & 1.54\end{pmatrix}$	$L=\begin{pmatrix}1.54 & 0.33 & 0.30\\ 0.33 & 1.59 & 0.37\\ 0.30 & 0.377 & 1.54\end{pmatrix}$

VI. CONCLUSIONS

By applying the finite volume method we have determined the electromagnetic field distribution of three-phase AC line, taking into account the temporal variation of the phase voltages determined by the frequency of the alternating current. We have determined the capacitance and inductance values of the three-phase line for different suspension configurations of conductors. We have obtained the numerical values of the linear parameters showing the obvious differences depending on the constructive realization and differences in parameter values calculated by traditional methods and values determined on the basis of the proposed technique. Finite volume method is quite attractive and well suited when modeling electrostatic problems, especially the problems for which the flux is of importance.

References

[1] V. Patsiuk, *Mathematical methods for electrical circuits and fields calculation.* – Chişinău, Centrul Editorial-Poligrafic al USM, 2009 – 442 p.

[2] Demirchian K.S., Neiman L.P., Korovkin N.V. *Theoretical Foundations of Electrical Engineering:* A Textbook for Universities. 5th edition. T. 2 – SPb, 2009. –432 c.

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